Michigan Merit Curriculum

Course/Credit Requirements

GEOMETRY

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1 Credit

MICHIGAN Department of Education

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Welcome

This guide was developed to assist teachers in successfully implementing the Michigan Merit Curriculum. The identified content expectations and guidelines provide a useful framework for designing curriculum, assessments and relevant learning experiences for students. Through the collaborative efforts of Governor Jennifer M. Granholm, the State Board of Education, and the State Legislature, these landmark state graduation requirements are being implemented to give Michigan students the knowledge and skills to succeed in the 21st Century and drive Michigan's economic success in the global economy. Working together, teachers can explore varied pathways to help students demonstrate proficiency in meeting the content expectations and guidelines.

Curriculum Unit Design

One of the ultimate goals of teaching is for students to acquire transferable knowledge. To accomplish this, learning needs to result in a deep understanding of content and mastery level of skills. As educational designers, teachers must use both the art and the science of teaching. In planning coherent, rigorous instructional units of study, it is best to *begin with the end in mind*.

Engaging and effective units include

- appropriate content expectations
- students setting goals and monitoring own progress
- a focus on big ideas that have great transfer value
- focus and essential questions that stimulate inquiry and connections
- identified valid and relevant skills and processes
- purposeful real-world applications
- relevant and worthy learning experiences
- varied flexible instruction for diverse learners
- research-based instructional strategies
- explicit and systematic instruction
- adequate teacher modeling and guided practice
- substantial time to review or apply new knowledge
- opportunities for revision of work based on feedback
- student evaluation of the unit
- culminating celebrations

Relevance

Instruction that is clearly relevant to today's rapidly changing world is at the forefront of unit design. Content knowledge cannot by itself lead all students to academic achievement. Classes and projects that spark student interest and provide a rationale for why the content is worth learning enable students to make connections between what they read and learn in school, their lives, and their futures. An engaging and effective curriculum provides opportunities for exploration and exposure to new ideas. Real-world learning experiences provide students with opportunities to transfer and apply knowledge in new, diverse situations.

Student Assessment

The assessment process can be a powerful tool for learning when students are actively involved in the process. Both assessment *of* learning and assessment *for* learning are essential. Reliable formative and summative assessments provide teachers with information they need to make informed instructional decisions that are more responsive to students' needs. Engagement empowers students to take ownership of their learning and builds confidence over time.

Sound assessments:

- align with learning goals
- vary in type and format
- use authentic performance tasks
- use criteria scoring tools such as rubrics or exemplars
- allow teachers and students to track growth over time
- validate the acquisition of transferable knowledge
- give insight into students' thinking processes
- cause students to use higher level thinking skills
- address guiding questions and identified skills and processes
- provide informative feedback for teachers and students
- ask students to reflect on their learning

High School Content Expectation Codes

To allow for ease in referencing expectations, each mathematics expectation has been coded by strand, standard, topic, and expectation. For example:

A1.2.3

- **A:** Algebra and Functions strand
- **A1:** Standard 1 of the Algebra and Functions strand
- **A1.2:** Topic 2 in Standard A1
- **A.1.2.3:** 3rd expectation in the 2nd topic of Standard A1

Organizational Structure

STRAND 1 **Quantitative Literacy and Logic (L)**	**STRAND 2** **Algebra and Functions (A)**
STANDARDS (and number of core expectations in each standard)	
L1: Reasoning About Numbers, Systems and Quantitative Situations (9) **L2: Calculation, Algorithms, and Estimation** (9) **L3: Measurement and Precision** (5) **L4: Mathematical Reasoning, Logic, and Proof** (10)	**A1: Expressions, Equations, and Inequalities** (16) **A2: Function** (39) **A3: Mathematical Modeling** (3)
Recommended Quantitative Literacy and Logic Expectations (3)	*Recommended Algebra and Functions Expectations (5)*

STRAND 3 **Geometry and Trigonometry (G)**	**STRAND 4** **Statistics and Probability (S)**
STANDARDS (and number of core expectations in each standard)	
G1: Figures and Their Properties (29) **G2: Relationships Between Figures** (10) **G3: Transformations of Figures in the Plane** (5)	**S1: Univariate Data—Examining Distributions** (9) **S2: Bivariate Data—Examining Relationships** (6) **S3: Samples, Surveys, and Experiments** (3) **S4: Probability Models and Probability Calculation** (4)
Recommended Geometry and Trigonometry Expectations (3)	*Recommended Statistics and Probability Expectations (6)*

Recommended Expectations

At the end of each strand, a set of recommended expectations may be listed. These extensions represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the core expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning. Recommended expectations will not be tested on the Michigan Merit Exam or on future high school subject credit assessments.

Coding Note: Recommended expectations have an * preceding their code.

Organization of this Document

In the mathematics credit requirement documents, the expectations are organized by strand and standard underneath topic headings. The organization in no way implies an instructional sequence. Curriculum personnel or teachers are encouraged to organize these topics and expectations in a manner that encourages connections between strands and among topics with a strand.

Introduction to Geometry

In our ever-increasing technological world, a rich study of logic and mathematical proof is fundamental for reasoning and good decision making. The study of geometry offers students the opportunity to develop skill in reasoning and formal proof. Additionally, it helps students to describe, analyze, and recognize the underlying beauty in the structures that compose our world. Geometric thinking is a powerful tool for understanding and solving both mathematical and applied problems and offers alternate ways of reasoning mathematically beyond algebra, including analytical and spatial reasoning.

Geometry Goal Statement

Geometry builds on a number of key geometric topics developed in the middle grades, namely relationships between angles, triangles, quadrilaterals, circles, and simple three-dimensional shapes. It is expected that students beginning geometry are able to recognize, classify, and apply properties of simple geometric shapes, know and apply basic similarity and congruence theorems, understand simple constructions with a compass and straight edge, and find area and volume of basic shapes.

Students studying geometry in high school further develop analytic and spatial reasoning. They apply what they know about two-dimensional figures to three-dimensional figures in real-world contexts, building spatial visualization skills and deepening their understanding of shape and shape relationships. Geometry includes a study of right triangle trigonometry that is developed through similarity relationships. These topics allow for many rich real-world problems to help students expand geometric reasoning skills. It is critical that connections are made from algebraic reasoning to geometric situations. Connections between transformations of linear and quadratic functions to geometric transformations should be made. Earlier work in linear functions and coordinate graphing leads into coordinate geometry.

The study of formal logic and proof helps students to understand the axiomatic system that underlies mathematics through the presentation and development of postulates, definitions, and theorems. It is essential that students develop deductive reasoning skills that can be applied to both mathematical and real-world problem contexts.

Throughout geometry, students will experience geometric thinking and reasoning techniques as accessible and powerful tools that can be used to explore the concept of mathematical proofs as well as to model and solve real-world problems.

Geometry Content Expectations Outline

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

- L1.1 Number Systems and Number Sense
- L1.2 Representations and Relationships

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

- L2.1 Calculation Using Real and Complex Numbers

STANDARD L3: MEASUREMENT AND PRECISION

- L3.1 Measurement Units, Calculations, and Scales

STANDARD L4: MATHEMATICAL REASONING, LOGIC, AND PROOF

- L4.1 Mathematical Reasoning
- L4.2 Language and Laws of Logic
- L4.3 Proof

STANDARD G1: FIGURES AND THEIR PROPERTIES

- G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry
- G1.2 Triangles and Their Properties
- G1.3 Triangles and Trigonometry
- G1.4 Quadrilaterals and Their Properties
- G1.5 Other Polygons and Their Properties
- G1.6 Circles and Their Properties
- G1.8 Three-Dimensional Figures

STANDARD G2: RELATIONSHIPS BETWEEN FIGURES

G2.1 Relationships Between Area and Volume Formulas

G2.2 Relationships Between Two-dimensional and Three-dimensional Representations

G2.3 Congruence and Similarity

STANDARD G3: TRANSFORMATIONS OF FIGURES IN THE PLANE

G3.1 Distance-preserving Transformations: Isometries

G3.2 Shape-preserving Transformations: Dilations and Isometries

CONTENT EXPECTATIONS FOR GEOMETRY

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

L1.1 Number Systems and Number Sense

L1.1.6 Explain the importance of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ in basic right triangle trigonometry, the importance of π because of its role in circle relationships, and the role of e in applications such as continuously compounded interest.

L1.2 Representations and Relationships

L1.2.3 Use vectors to represent quantities that have magnitude and direction, interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.

L2.1 Calculation Using Real and Complex Numbers

L2.1.6 Recognize when exact answers aren't always possible or practical. Use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).

L3.1 Measurement Units, Calculations, and Scales

L3.1.1 Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.

L4.1 Mathematical Reasoning

L4.1.1 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

L4.1.2 Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.

L4.1.3 Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics. Identify and give examples of each.

L4.2 Language and Laws of Logic

L4.2.1 Know and use the terms of basic logic (e.g., proposition, negation, truth and falsity, implication, if and only if, contrapositive, and converse).

L4.2.2 Use the connectives "not," "and," "or," and "if..., then," in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.

L4.2.3 Use the quantifiers "there exists" and "all" in mathematical and everyday settings and know how to logically negate statements involving them.

L4.2.4 Write the converse, inverse, and contrapositive of an "If..., then..." statement. Use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original while the inverse and converse are not.

L4.3 Proof

L4.3.1 Know the basic structure for the proof of an "If..., then..." statement (assuming the hypothesis and ending with the conclusion) and that proving the contrapositive is equivalent.

L4.3.2 Construct proofs by contradiction. Use counter-examples, when appropriate, to disprove a statement.

L4.3.3 Explain the difference between a necessary and a sufficient condition within the statement of a theorem. Determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.

STANDARD G1: FIGURES AND THEIR PROPERTIES

G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry

G1.1.1 Solve multistep problems and construct proofs involving vertical angles, linear pairs of angles, supplementary angles, complementary angles, and right angles.

G1.1.2 Solve multistep problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.

G1.1.3 Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using straightedge and compass.

G1.1.4 Given a line and a point, construct a line through the point that is parallel to the original line using straightedge and compass. Given a line and a point, construct a line through the point that is perpendicular to the original line. Justify the steps of the constructions.

G1.1.5 Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.

G1.1.6 Recognize Euclidean geometry as an axiom system. Know the key axioms and understand the meaning of and distinguish between undefined terms (e.g., point, line, and plane), axioms, definitions, and theorems.

G1.2 Triangles and Their Properties

G1.2.1 Prove that the angle sum of a triangle is 180° and that an exterior angle of a triangle is the sum of the two remote interior angles.

G1.2.2 Construct and justify arguments and solve multistep problems involving angle measure, side length, perimeter, and area of all types of triangles.

G1.2.3 Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem and its converse to solve multistep problems.

G1.2.4 Prove and use the relationships among the side lengths and the angles of 30°- 60°- 90° triangles and 45°- 45°- 90° triangles.

G1.2.5 Solve multistep problems and construct proofs about the properties of medians, altitudes perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle. Using a straightedge and compass, construct these lines.

G1.3 Triangles and Trigonometry

G1.3.1 Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides. Solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.

G1.3.2 Know and use the Law of Sines and the Law of Cosines and use them to solve problems. Find the area of a triangle with sides a and b and included angle θ using the formula Area = (1/2) $a\ b \sin\theta$.

G1.3.3 Determine the exact values of sine, cosine, and tangent for 0°, 30°, 45°, 60°, and their integer multiples and apply in various contexts.

G1.4 Quadrilaterals and Their Properties

G1.4.1 Solve multistep problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.

G1.4.2 Solve multistep problems and construct proofs involving quadrilaterals (e.g., prove that the diagonals of a rhombus are perpendicular) using Euclidean methods or coordinate geometry.

G1.4.3 Describe and justify hierarchical relationships among quadrilaterals (e.g., every rectangle is a parallelogram).

G1.4.4 Prove theorems about the interior and exterior angle sums of a quadrilateral.

G1.5 Other Polygons and Their Properties

G1.5.1 Know and use subdivision or circumscription methods to find areas of polygons (e.g., regular octagon, nonregular pentagon).

G1.5.2 Know, justify, and use formulas for the perimeter and area of a regular n-gon and formulas to find interior and exterior angles of a regular n-gon and their sums.

G1.6 Circles and Their Properties

G1.6.1 Solve multistep problems involving circumference and area of circles.

G1.6.2 Solve problems and justify arguments about chords (e.g., if a line through the center of a circle is perpendicular to a chord, it bisects the chord) and lines tangent to circles (e.g., a line tangent to a circle is perpendicular to the radius drawn to the point of tangency).

G1.6.3 Solve problems and justify arguments about central angles, inscribed angles, and triangles in circles.

G1.6.4 Know and use properties of arcs and sectors and find lengths of arcs and areas of sectors.

G1.8 Three-dimensional Figures

G1.8.1 Solve multistep problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

G1.8.2 Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

STANDARD G2: RELATIONSHIPS BETWEEN FIGURES

G2.1 Relationships Between Area and Volume Formulas

G2.1.1 Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.

G2.1.2 Know and demonstrate the relationships between the area formulas of various quadrilaterals (e.g., explain how to find the area of a trapezoid based on the areas of parallelograms and triangles).

G2.1.3 Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).

G2.2 Relationships Between Two-dimensional and Three-dimensional Representations

G2.2.1 Identify or sketch a possible three-dimensional figure, given two-dimensional views (e.g., nets, multiple views). Create a two-dimensional representation of a three-dimensional figure.

G2.2.2 Identify or sketch cross sections of three-dimensional figures. Identify or sketch solids formed by revolving two-dimensional figures around lines.

G2.3 Congruence and Similarity

G2.3.1 Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria and that right triangles are congruent using the hypotenuse-leg criterion.

G2.3.2 Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.

G2.3.3 Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.

G2.3.4 Use theorems about similar triangles to solve problems with and without use of coordinates.

G2.3.5 Know and apply the theorem stating that the effect of a scale factor of k relating one two-dimensional figure to another or one three-dimensional figure to another, on the length, area, and volume of the figures is to multiply each by k, k^2, and k^3, respectively.

G3.1 Distance-preserving Transformations: Isometries

G3.1.1 Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.

G3.1.2 Given two figures that are images of each other under an isometry, find the isometry and describe it completely.

G3.1.3 Find the image of a figure under the composition of two or more isometries and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.

G3.2 Shape-preserving Transformations: Dilations and Isometries

G3.2.1 Know the definition of dilation and find the image of a figure under a given dilation.

G3.2.2 Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.

RECOMMENDED:

***G1.4.5** Understand the definition of a cyclic quadrilateral and know and use the basic properties of cyclic quadrilaterals.

***G3.2.3** Find the image of a figure under the composition of a dilation and an isometry.

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