STATE OF MICHIGAN
DEPARTMENT OF EDUCATION
LANSING

JENNIFER M. GRANHOLM
GOVERNOR

MICHAEL P. FLANAGAN
SUPERINTENDENT OF
PUBLIC INSTRUCTION

# MEMORANDUM

**TO:** State Board of Education

**FROM:** Michael P. Flanagan, Chairman

**DATE:** April 3, 2006

**SUBJECT:** Approval of the High School Content Expectations in English Language Arts and Mathematics

At the November 2005 State Board of Education meeting, you accepted the draft version of the High School Content Expectations for English Language Arts and Mathematics that would guide a rigorous high school experience. These documents were then reviewed by educators, professional organizations, and community members via the web and formal reviews. Comments and suggestions from the reviews were incorporated into the documents by the original work group of scholars and practitioners chaired by Dr. Rebecca Sipe, of Eastern Michigan University, for English Language Arts and Dr. Joan Ferrini-Mundy, of Michigan State University, for Mathematics.

In addition, the Michigan Department of Education's Office of School Improvement worked closely with Achieve, Inc., to align the draft content expectations with national standards and assessments. Through this partnership, both the draft High School English Language Arts and Mathematics Content Expectations have been reviewed by a group of university and community college representatives, with comments and suggestions being incorporated. Following the alignment study, the edited documents were reviewed by Achieve, Inc., and the suggested improvements have been made.

It is therefore recommended that the State Board of Education approve the High School Content Expectations for English Language Arts (Attachment A) and Mathematics (Attachment B), and direct the department to proceed with the development of companion documents, including, but not limited to, high school course content expectations that are aligned to the Michigan Merit Core.

Attachments



608 WEST ALLEGAN STREET • P.O. BOX 30008 • LANSING, MICHIGAN 48909
www.michigan.gov/mde • (517) 373-3324

# High School Content Expectations

# ENGLISH LANGUAGE ARTS

- Writing, Speaking, and Expressing
- Reading, Listening, and Viewing
- Literature and Culture
- Language

NCE • **RIGOR** • RELEVANCE • RELATIONSHIPS • RIGOR • RELEVAN
HIPS • RELATIONSHIPS • RIGOR • **RELEVANCE** • RELATIONS
NCE • RIGOR • RELEVANCE • RELATIONSHIPS • RIGOR • RELEVAN
HIPS • **RELATIONSHIPS** • RIGOR • RELEVANCE • RELATIONS



MICHIGAN Department of Education


Office of School Improvement

## Michigan State Board of Education

**Kathleen N. Straus, President**
Bloomfield Township

**John C. Austin, Vice President**
Ann Arbor

**Carolyn L. Curtin, Secretary**
Evart

**Marianne Yared McGuire, Treasurer**
Detroit

**Nancy Danhof, NASBE Delegate**
East Lansing

**Elizabeth W. Bauer**
Birmingham

**Reginald M. Turner**
Detroit

**Eileen Lappin Weiser**
Ann Arbor

**Governor Jennifer M. Granholm**
*Ex Officio*

**Michael P. Flanagan, Chairman**
*Superintendent of Public Instruction*

**Jeremy M. Hughes, Ph.D.**
*Deputy Superintendent and*
*Chief Academic Officer*

**Dr. Yvonne Caamal Canul, Director**
*Office of School Improvement*

## English Language Arts Work Group

### *Academic Review*

**Rebecca Sipe, Chair**
*Eastern Michigan University*

**Linda Adler-Kassner**
*Eastern Michigan University*

**Ellen Brinkley**
*Western Michigan University*

**Lynne Listeman**
*Michigan Dept. of Labor and Economic Growth/ Career and Technical Education*

**Charles Peters**
*University of Michigan*

**Linda Stokes Smith**
*Michigan Department of Education*

**Hugh Spagnuolo**
*Lansing School District*

**Allen Webb**
*Western Michigan University*

### *University Review*

**Susan Steffel**
*Central Michigan University*

**Marilyn Wilson**
*Michigan State University*

### *Internal Review*

**Jane Hesse**
*Novi Schools/Achieve**

**Cynthia Clingman**
*Ottawa ISD/MRA**

**Rita Maddox**
*Gratiot/Isabella RESD**

**Gale Sharpe**
*Michigan Department of Education**

**Linda Stokes Smith**
*Michigan Department of Education*

**Betty Underwood**
*Michigan Department of Education**

**Elaine Weber**
*Macomb ISD**

### *External Review*

**Maureen Baker**
*Michigan Council of Teachers of English*

**Marilyn Brooks**
*Michigan Reading Association*

### *Project Coordinator*

**Susan Codere Kelly**
*Michigan Department of Education**

**Grade Level Content Expectations Committee*

# Welcome to Michigan's High School English Language Arts Content Standards and Expectations

## Why Develop Content Standards and Expectations for High School?

In 2004, the Michigan Department of Education embraced the challenge to initiate a "high school redesign" project. Since then, the national call to create more rigorous learning for high school students has become a major priority for state leaders across the country. The Cherry Commission Report highlighted several goals for Michigan including the development of high school content expectations that reflect both a rigorous and a relevant curricular focus. Dovetailing with this call to "curricular action" is Michigan's legislative change in high school assessment. The Michigan Merit Exam, based on rigorous high school learning standards, is to be fully implemented by 2007.

Given these two catalysts, the Michigan Department of Education's Office of School Improvement led the development of high school content expectations for English Language Arts and Mathematics. Content area work groups of academicians chaired by a nationally known scholar in the respective field, were commissioned to conduct a scholarly review and identify content standards and expectations. These content standards and expectations have gone through an extensive field and national review and are presented to educators in this document.

## An Overview

The expectations contained in this document reflect best practices and current research in the teaching and learning of English language, the craft of writing, and literature. They not only build from the *Michigan English Language Arts Curriculum Framework Standards and Benchmarks* (1996), the *Career and Employability Skills Content Standards and Benchmarks* (2001), but extend the *Michigan K-8 English Language Arts Grade Level Content Expectations* (2004) as appropriate for grades 9-12. These standards and expectations represent a vision for a rigorous and relevant high school experience for all Michigan students, with special attention being paid to national research and support for the skills that prepare students for succcessful post-secondary engagement in the workplace. The standards and expectations are closely aligned with national standards as described in College Board's *Standards for College Success* (2005), ACT's *College Readiness Standards*, American Diploma Project's *Ready or Not: Creating a High School DiplomaThat Counts* (2004), NCTE/IRA Standards for the English Language Arts (1996), the National Communication Association Guiding Principles for Speaking and Listening (1996), and the National Assessment Governing Board's *Reading Framework for the 2009 National Assessment of Educational Progress* (NAEP, 2005).

### *Understanding the Organizational Structure*

The expectations in this document are divided into four strands with multiple standards within each, as shown below. The skills and content addressed in these standards will, in practice, be woven together into a coherent, integrated English language arts curriculum. While the standards are comprehensive, they are not meant to be used as a proportional guide to curriculum development. For example, students and teachers are not expected to spend equal time on each strand or standard, and content should logically be divided among courses (e.g., not every class must address American, British, and world literature). Writing, reading, speaking, listening, viewing, and visually expressing are recursive and reinforcing processes; students learn by engaging in and reflecting on these processes at increasingly complex levels over time. Many of the skills addressed in Language Arts classes will also be re-inforced by teachers in other disciplines across the curriculum, while beyond the English language arts curriculum, students will use the English language arts processes to support their learning in all content areas.

| **STRAND 1** Writing, Speaking, and Expressing | **STRAND 2** Reading, Listening, and Viewing | **STRAND 3** Literature and Culture | **STRAND 4** Language |
|---|---|---|---|
| **STANDARDS** (and number of expectations in each standard) | | | |
| | *Narrative and Informational (Literary and Expository) Text* | | |
| **1.1: Writing Process** (6)<br>**1.2: Personal Growth** (4)<br>**1.3: Purpose and Audience** (9)<br>**1.4: Inquiry and Research** (7)<br>**1.5: Finished Products** (5) | **2.1: Strategy Development** (12)<br>**2.2: Meaning Beyond the Literal Level** (3)<br>**2.3: Independent Reading** (8) | **3.1: Close Literary Reading** (10)<br>**3.2: Reading and Response *(varied genres and time periods)*** (5)<br>**3.3: Text Analysis** (6)<br>**3.4: Mass Media** (4) | **4.1: Effective Use of the English Language** (5)<br>**4.2: Language Variety** (5) |

## Beliefs

### *Standards for English language arts are developed with certain beliefs in mind:*

- Listening, speaking, reading, writing, viewing, and expressing are critical for all students.
- With appropriate instruction, all students can be successful listeners, speakers, readers, and writers.
- Acquaintance with content, strategies, and skills does not equal mastery. The English language arts are highly recursive and must be continuously developed as students engage with more complex ideas, texts, and tasks.
- Students learn best by being actively involved in high quality, challenging experiences; they demonstrate their learning best in authentic contexts. Not all skills are easily testable, especially on standardized tests; therefore, the curriculum must not be limited to teaching skills that are so tested.
- High standards in the English language arts support the acquisition of skills and strategies for reading and writing in a variety of genres and for varying purposes: skills and strategies that students can transfer independently to writing beyond the classroom.
- Conventions for edited and finished texts are important and help to shape the way a message is received. Students need to understand and be able to use Standard English as appropriate for composing and speaking tasks.
- Classroom teachers have extensive content knowledge, an ability to make on-going, data-driven curriculum decisions, and the ability to adapt curriculum to student needs. Teacher passion and creativity is essential to learning.

## Curriculum and Assessment

This document is intended to support conversations at the school and district levels that result in rigorous and relevant curriculum incorporating these content expectations. The expectations should be addressed recursively and with increasing complexity throughout the high school language arts curriculum.

### *As stakeholders (e.g., teachers, administrators, school board members, parents, community members, students, local legislative representatives) work with these standards, they should consider the following questions:*

- How are these content standards and expectations reflected in our curriculum and instruction already?
- Where do we need to strengthen our curriculum and instruction to more fully realize the intent of these standards and expectations?
- What opportunities do these standards and expectations present to develop new and strengthen existing curriculum, leading to instructional excellence and college/workplace readiness?
- How do we implement these standards and expectations taking into account what we know about our students, school, and community?
- How will we assess the effectiveness with which our students and schools are meeting these standards and content expectations?
- How can we use school-based assessments (e.g., student portfolios, school-based writing assessments, teacher or classroom research, district-level assessments) to make data-driven decisions about teaching and learning?

Through conversations about questions such as these, and building upon the multitude of existing strengths in our current high schools, voices of all stakeholders will participate in the important and continuing process of shaping instructional excellence in Michigan schools and preparing Michigan students for college and the workplace.

## English Language Arts*

The English language arts are the vehicles of communication by which we live, work, share, and build ideas and understandings of the present, reflect on the past, and imagine the future. Through the English language arts, we learn to appreciate, integrate, and apply what is learned for real purposes in our homes, schools, communities, and workplaces.

The English language arts encompass process and content – how people communicate as well as what they communicate. Process includes skills and strategies used in listening, speaking, reading, writing, viewing, and expressing. Content includes the ideas, themes, issues, problems, and conflicts found in classical and contemporary literature and other texts, such as technical manuals, periodicals, speeches, and videos. Ideas, experiences, and cultural perspectives we discover in texts help us shape our vision of the world. The insights we gain enable us to understand our cultural, linguistic, and literary heritages.

The ultimate goal for all English language arts learners is personal, social, occupational, and civic literacy. Literacy goes beyond the ability to read and write at basic levels. Literate individuals understand the different functions of English language arts for personal, social, and political purposes (e.g., for personal enjoyment and interest; for communicating with and understanding others; for accomplishing goals, understanding others' perspectives, shaping opinions and attitudes, and controlling behaviors).

### *As a contributing citizen, a literate individual:*

- communicates skillfully and effectively through printed, visual, auditory, and technological media in the home, school, community and workplace;
- thinks analytically and creatively about important themes, concepts, and ideas;
- uses the English language arts to identify and solve problems;
- uses the English language arts to understand and appreciate the commonalities and differences within social, cultural, and linguistic communities;
- understands and appreciates the aesthetic elements of oral, visual, and written texts;
- uses the English language arts to develop insights about human experiences;
- uses the English language arts to develop the characteristics of lifelong learners and workers, such as curiosity, patience, flexibility, and reflection; and,
- connects all knowledge from all curriculum areas to enhance understanding of the world.

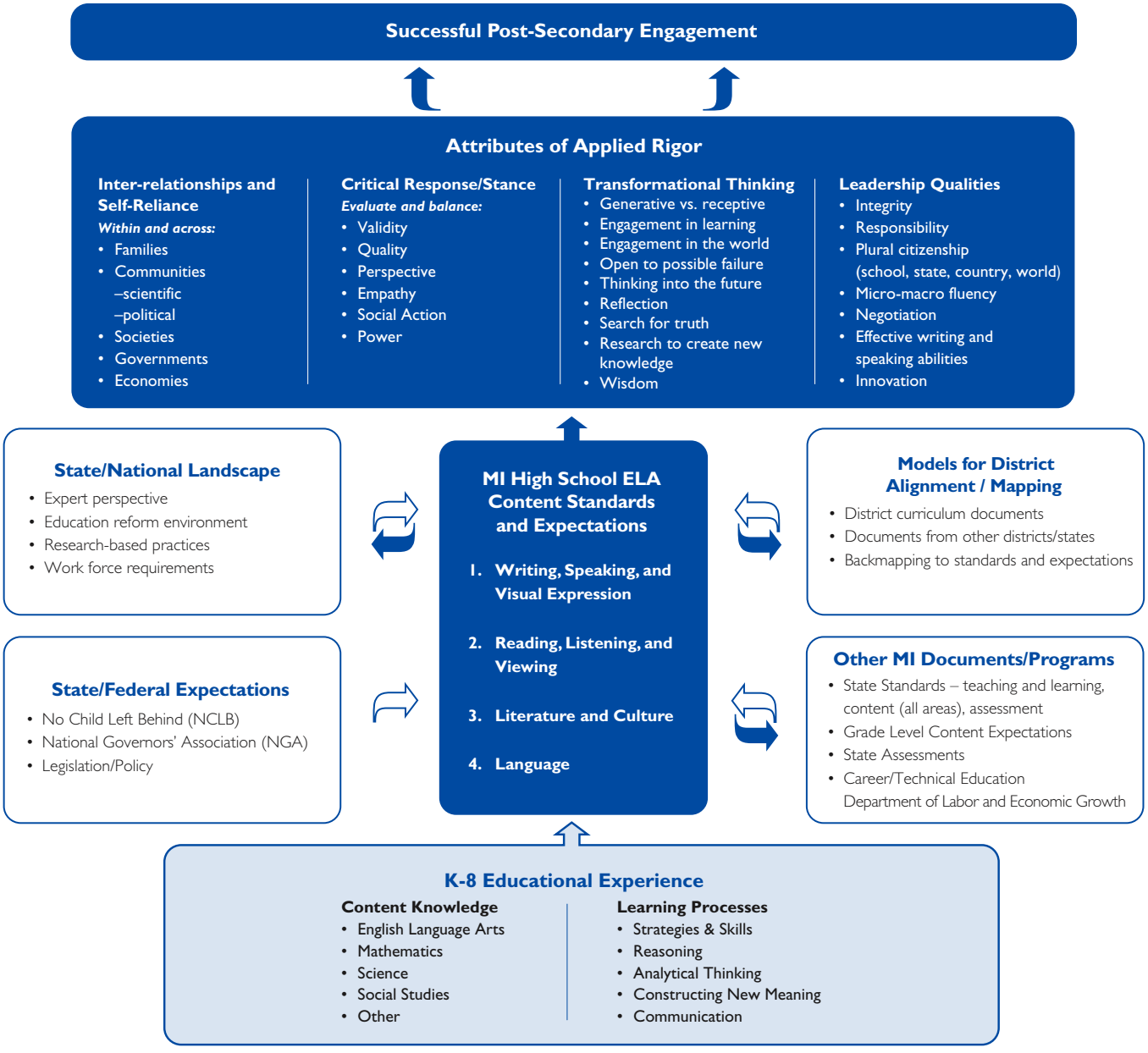
**Adapted from Michigan Curriculum Framework 1996*

## Preparing Students for Successful Post-Secondary Engagement

As educators use these standards and expectations to develop rigorous and relevant units of instruction, powerful and engaging learning activities, and challenging high school curricula, it is critical to keep in mind that content knowledge alone will not provide adequate preparation for success in entry-level university courses or entry-level positions in today's workforce.

Successful post-secondary engagement requires that students must be able to apply knowledge in new situations; to solve problems by generating new ideas; to make connections between what they read and hear in class, the world around them, and the future; and through their work, develop leadership qualities while still in high school.

Therefore, educators must model for and develop in students the cognitive skills and habits of mind that will result in attributes of applied rigor as indentified in the chart below.




This chart includes talking points for the professional development model.

# STRAND 1: WRITING, SPEAKING, AND VISUAL EXPRESSION

*Writing and speaking involve a complex process of inquiry and the discovery of meaning. Through writing, speaking, and visually expressing, students understand themselves, communicate with others, advance personal and professional goals, and participate in a democratic society. Effective communication requires an understanding of purpose and audience, and reflects well-developed ideas using appropriate conventions of genre, content, form, style, voice, and mechanics.*

## STANDARD 1.1 *Understand and practice writing as a recursive process.*

**CE 1.1.1** Demonstrate flexibility in using independent and collaborative strategies for planning, drafting, revising, and editing complex texts.

**CE 1.1.2** Know and use a variety of prewriting strategies to generate, focus, and organize ideas (e.g., free writing, clustering/mapping, talking with others, brainstorming, outlining, developing graphic organizers, taking notes, summarizing, paraphrasing).

**CE 1.1.3** Select and use language that is appropriate (e.g., formal, informal, literary, or technical) for the purpose, audience, and context of the text, speech, or visual representation (e.g., letter to editor, proposal, poem, or digital story).

**CE 1.1.4** Compose drafts that convey an impression, express an opinion, raise a question, argue a position, explore a topic, tell a story, or serve another purpose, while simultaneously considering the constraints and possibilities (e.g., structure, language, use of conventions of grammar, usage, and mechanics) of the selected form or genre.

**CE 1.1.5** Revise drafts to more fully and/or precisely convey meaning—drawing on response from others, self-reflection, and reading one's own work with the eye of a reader; then refine the text—deleting and/or reorganizing ideas, and addressing potential readers' questions.

**CE 1.1.6** Reorganize sentence elements as needed and choose grammatical and stylistic options that provide sentence variety, fluency, and flow.

**CE 1.1.7** Edit for style, tone, and word choice (specificity, variety, accuracy, appropriateness, conciseness) and for conventions of grammar, usage and mechanics that are appropriate for audience.

**CE 1.1.8** Proofread to check spelling, layout, and font; and prepare selected pieces for a public audience.

## STANDARD 1.2 *Use writing, speaking, and visual expression for personal understanding and growth.*

**CE 1.2.1** Write, speak, and use images and graphs to understand and discover complex ideas.

**CE 1.2.2** Write, speak, and visually represent to develop self-awareness and insight (e.g., diary, journal writing, portfolio self-assessment).

**CE 1.2.3** Write, speak, and create artistic representations to express personal experience and perspective (e.g., personal narrative, poetry, imaginative writing, slam poetry, blogs, webpages).

**CE 1.2.4** Assess strengths, weaknesses, and development as a writer by examining a collection of own writing.

## STANDARD 1.3 *Communicate in speech, writing, and multimedia using content, form, voice, and style appropriate to the audience and purpose (e.g., to reflect, persuade, inform, analyze, entertain, inspire).*

**CE 1.3.1** Compose written, spoken, and/or multimedia compositions in a range of genres (e.g., personal narrative, biography, poem, fiction, drama, creative nonfiction, summary, literary analysis essay, research report, or work-related text): pieces that serve a variety of purposes (e.g., expressive, informative, creative, and persuasive) and that use a variety of organizational patterns (e.g., autobiography, free verse, dialogue, comparison/contrast, definition, or cause and effect).

**CE 1.3.2** Compose written and spoken essays or work-related text that demonstrate logical thinking and the development of ideas for academic, creative, and personal purposes: essays that convey the author's message by using an engaging introduction (with a clear thesis as appropriate), well-constructed paragraphs, transition sentences, and a powerful conclusion.

**CE 1.3.3** Compose essays with well-crafted and varied sentences demonstrating a precise, flexible, and creative use of language.

**CE 1.3.4** Develop and extend a thesis, argument, or exploration of a topic by analyzing differing perspectives and employing a structure that effectively conveys the ideas in writing (e.g. resolve inconsistencies in logic; use a range of strategies to persuade, clarify, and defend a position with precise and relevant evidence; anticipate and address concerns and counterclaims; provide a clear and effective conclusion).

**CE 1.3.5** From the outset, identify and assess audience expectations and needs; consider the rhetorical effects of style, form, and content based on that assessment; and adapt communication strategies appropriately and effectively.

**CE 1.3.6** Use speaking, writing, and visual presentations to appeal to audiences of different social, economic, and cultural backgrounds and experiences (e.g., include explanations and definitions according to the audience's background, age, or knowledge of the topic; adjust formality of style; consider interests of potential readers).

**CE 1.3.7** Participate collaboratively and productively in groups (e.g., response groups, work teams, discussion groups, and committees)—fulfilling roles and responsibilities, posing relevant questions, giving and following instructions, acknowledging and building on ideas and contributions of others to answer questions or to solve problems, and offering dissent courteously.

**CE 1.3.8** Evaluate own and others' effectiveness in group discussions and formal presentations (e.g., considering accuracy, relevance, clarity, and delivery; types of arguments used; and relationships among purpose, audience, and content).

**CE 1.3.9** Use the formal, stylistic, content, and mechanical conventions of a variety of genres in speaking, writing, and multimedia presentations.

## STANDARD 1.4 *Develop and use the tools and practices of inquiry and research—generating, exploring, and refining important questions; creating a hypothesis or thesis; gathering and studying evidence; drawing conclusions; and composing a report.*

**CE 1.4.1** Identify, explore, and refine topics and questions appropriate for research.

**CE 1.4.2** Develop a system for gathering, organizing, paraphrasing, and summarizing information; select, evaluate, synthesize, and use multiple primary and secondary (print and electronic) resources.

**CE 1.4.3** Develop and refine a position, claim, thesis, or hypothesis that will be explored and supported by analyzing different perspectives, resolving inconsistencies, and writing about those differences in a structure appropriate for the audience (e.g., argumentative essay that avoids inconsistencies in logic and develops a single thesis; exploratory essay that explains differences and similarities and raises additional questions).

**CE 1.4.4** Interpret, synthesize, and evaluate information/findings in various print sources and media (e.g., fact and opinion, comprehensiveness of the evidence, bias, varied perspectives, motives and credibility of the author, date of publication) to draw conclusions and implications.

**CE 1.4.5** Develop organizational structures appropriate to the purpose and message, and use transitions that produce a sequential or logical flow of ideas.

**CE 1.4.6** Use appropriate conventions of textual citation in different contexts (e.g., different academic disciplines and workplace writing situations).

**CE 1.4.7** Recognize the role of research, including student research, as a contribution to collective knowledge, selecting an appropriate method or genre through which research findings will be shared and evaluated, keeping in mind the needs of the prospective audience. (e.g., presentations, online sharing, written products such as a research report, a research brief, a multi-genre report, I-Search, literary analysis, news article).

## STANDARD 1.5 *Produce a variety of written, spoken, multigenre, and multimedia works, making conscious choices about language, form, style, and/or visual representation for each work (e.g., poetry, fiction and creative nonfiction stories, academic and literary essays, proposals, memos, manifestos, business letters, advertisements, prepared speeches, group and dramatic performances, poetry slams, and digital stories).*

**CE 1.5.1** Use writing, speaking, and visual expression to develop powerful, creative and critical messages.

**CE 1.5.2** Prepare spoken and multimedia presentations that effectively address audiences by careful use of voice, pacing, gestures, eye contact, visual aids, audio and video technology.

**CE 1.5.3** Select format and tone based on the desired effect and audience, using effective written and spoken language, sound, and/or visual representations (e.g., focus, transitions, facts, detail and evidence to support judgments, skillful use of rhetorical devices, and a coherent conclusion).

**CE 1.5.4** Use technology tools (e.g, word processing, presentation and multimedia software) to produce polished written and multimedia work (e.g., literary and expository works, proposals, business presentations, advertisements).

**CE 1.5.5** Respond to and use feedback to strengthen written and multimedia presentations (e.g., clarify and defend ideas, expand on a topic, use logical arguments, modify organization, evaluate effectiveness of images, set goals for future presentations).

# STRAND 2: READING, LISTENING, AND VIEWING

*In constructing meaning while reading, listening, or viewing, students draw upon prior knowledge and engage complex skills and strategies of comprehension and interpretation, and critical thinking. They develop skill, confidence, and independence in understanding narrative and expository texts, including aural, visual, and multimodal works. Students synthesize information through reading, listening, and viewing and also generate new thinking.*

## STANDARD 2.1 *Develop critical reading, listening, and viewing strategies.*

**CE 2.1.1** Use a variety of pre-reading and previewing strategies (e.g., acknowledge own prior knowledge, make connections, generate questions, make predictions, scan a text for a particular purpose or audience, analyze text structure and features) to make conscious choices about how to approach the reading based on purpose, genre, level of difficulty, text demands and features.

**CE 2.1.2** Make supported inferences and draw conclusions based on informational print and multimedia features (e.g., prefaces, appendices, marginal notes, illustrations, bibliographies, author's pages, footnotes, diagrams, tables, charts, maps, timelines, graphs, and other visual and special effects) and explain how authors and speakers use them to infer the organization of text and enhance understanding, convey meaning, and inspire or mislead audiences.

**CE 2.1.3** Determine the meaning of unfamiliar words, specialized vocabulary, figurative language, idiomatic expressions, and technical meanings of terms through context clues, word roots and affixes, and the use of appropriate resource materials such as print and electronic dictionaries.

**CE 2.1.4** Identify and evaluate the primary focus, logical argument, structure, and style of a text or speech and the ways in which these elements support or confound meaning or purpose.

**CE 2.1.5** Analyze and evaluate the components of multiple organizational patterns (e.g., compare/contrast, cause/effect, problem/solution, fact/opinion, theory/evidence).

**CE 2.1.6** Recognize the defining characteristics of informational texts, speeches, and multimedia presentations (e.g., documentaries and research presentations) and elements of expository texts (e.g., thesis, supporting ideas, and statistical evidence); critically examine the argumentation and conclusions of multiple informational texts.

**CE 2.1.7** Demonstrate understanding of written, spoken, or visual information by restating, paraphrasing, summarizing, critiquing, or composing a personal response; distinguish between a summary and a critique.

**CE 2.1.8** Recognize the conventions of visual and multimedia presentations (e.g., lighting, camera angle, special effects, color, and soundtrack) and how they carry or influence messages.

**CE 2.1.9** Examine the intersections and distinctions between visual (media images, painting, film, and graphic arts) and verbal communication.

**CE 2.1.10** Listen to and view speeches, presentations, and multimedia works to identify and respond thoughtfully to key ideas, significant details, logical organization, fact and opinion, and propaganda.

**CE 2.1.11** Demonstrate appropriate social skills of audience, group discussion, or work team behavior by listening attentively and with civility to the ideas of others, gaining the floor in respectful ways, posing appropriate questions, and tolerating ambiguity and lack of consensus.

**CE 2.1.12** Use a variety of strategies to enhance listening comprehension (e.g., monitor message for clarity and understanding, ask relevant questions, provide verbal and nonverbal feedback, notice cues such as change of pace or emphasis that indicate a new point is about to be made; and take notes to organize essential information).

## STANDARD 2.2 *Use a variety of reading, listening, and viewing strategies to construct meaning beyond the literal level (e.g., drawing inferences; confirming and correcting; making comparisons, connections, and generalizations; and drawing conclusions).*

**CE 2.2.1** Recognize literary and persuasive strategies as ways by which authors convey ideas and readers make meaning (e.g., imagery, irony, satire, parody, propaganda, overstatement/understatement, omission, and multiple points of view).

**CE 2.2.2** Examine the ways in which prior knowledge and personal experience affect the understanding of written, spoken, or multimedia text.

**CE 2.2.3** Interpret the meaning of written, spoken, and visual texts by drawing on different cultural, theoretical, and critical perspectives.

## STANDARD 2.3 *Develop as a reader, listener, and viewer for personal, social, and political purposes, through independent and collaborative reading.*

**CE 2.3.1** Read, listen to, and view diverse texts for multiple purposes such as learning complex procedures, making work-place decisions, or pursuing in-depth studies.

**CE 2.3.2** Read, view, and/or listen independently to a variety of fiction, nonfiction, and multimedia genres based on student interest and curiosity.

**CE 2.3.3** Critically read and interpret instructions for a variety of tasks (e.g., completing assignments, using software, writing college and job applications).

**CE 2.3.4** Critically interpret primary and secondary research-related documents (e.g., historical and government documents, newspapers, critical and technical articles, and subject-specific books).

**CE 2.3.5** Engage in self-assessment as a reader, listener, and viewer, while monitoring comprehension and using a variety of strategies to overcome difficulties when constructing and conveying meaning.

**CE 2.3.6** Reflect on personal understanding of reading, listening, and viewing; set personal learning goals; and take responsibility for personal growth.

**CE 2.3.7** Participate as an active member of a reading, listening, and viewing community, collaboratively selecting materials to read or events to view and enjoy (e.g., book talks, literature circles, film clubs).

**CE 2.3.8** Develop and apply personal, shared, and academic criteria to evaluate own and others' oral, written, and visual texts.

# STRAND 3: LITERATURE AND CULTURE

*Students study and appreciate a rich and varied selection of classical and contemporary literary, cultural, and historical texts from American, British, and world traditions. They learn to make meaning from the experiences, ideas, and emotions of others across the ages, applying their understanding to contemporary circumstances.*

## STANDARD 3.1 *Develop the skills of close and contextual literary reading.*

**CE 3.1.1** Interpret literary language (e.g., imagery, allusions, symbolism, metaphor) while reading literary and expository works.

**CE 3.1.2** Demonstrate an understanding of literary characterization, character development, the function of major and minor characters, motives and causes for action, and moral dilemmas that characters encounter by describing their function in specific works.

**CE 3.1.3** Recognize a variety of plot structures and elements (e.g., story within a story, rising action, foreshadowing, flash backs, cause-and-effect relationships, conflicts, resolutions) and describe their impact on the reader in specific literary works.

**CE 3.1.4** Analyze characteristics of specific works and authors (e.g., voice, mood, time sequence, author vs. narrator, stated vs. implied author, intended audience and purpose, irony, parody, satire, propaganda, use of archetypes and symbols) and identify basic beliefs, perspectives, and philosophical assumptions underlying an author's work.

**CE 3.1.5** Comparatively analyze two or more literary or expository texts, comparing how and why similar themes are treated differently, by different authors, in different types of text, in different historical periods, and/or from different cultural perspectives.

**CE 3.1.6** Examine differing and diverse interpretations of literary and expository works and explain how and why interpretation may vary from reader to reader.

**CE 3.1.7** Analyze and evaluate the portrayal of various groups, societies, and cultures in literature and other texts.

**CE 3.1.8** Demonstrate an understanding of historical, political, cultural, and philosophical themes and questions raised by literary and expository works.

**CE 3.1.9** Analyze how the tensions among characters, communities, themes, and issues in literature and other texts reflect human experience.

**CE 3.1.10** Demonstrate an understanding of the connections between literary and expository works, themes, and historical and contemporary contexts.

## STANDARD 3.2 *Read and respond to classic and contemporary fiction, literary nonfiction, and expository text, from a variety of literary genres representing many time periods and authors (e.g., myth, epic, folklore, drama, poetry, autobiography, novels, short stories, philosophical pieces, science fiction, fantasy, young adult literature, creative non-fiction, hypertext fiction).*

**CE 3.2.1** Recognize a variety of literary genres and forms (e.g., poetry, drama, novels, short stories, autobiographies, biographies, multi-genre texts, satire, parody, allegory) and demonstrate an understanding of the way in which genre and form influence meaning.

**CE 3.2.2** Identify different types of poetry (e.g., epic, lyric, sonnet, free verse) and explain how specific features (e.g., figurative language, imagery, rhythm, alliteration, etc.) influence meaning.

**CE 3.2.3** Identify how elements of dramatic literature (e.g., dramatic irony, soliloquy, stage direction, and dialogue) illuminate the meaning of the text.

**CE 3.2.4** Respond by participating actively and appropriately in small and large group discussions about literature (e.g., posing questions, listening to others, contributing ideas, reflecting on and revising initial responses).

**CE 3.2.5** Respond to literature in a variety of ways (e.g., dramatic interpretation, reader's theatre, literature circles, illustration, writing in a character's voice, engaging in social action, writing an analytic essay) providing examples of how texts affect their lives, connect them with the contemporary world, and communicate across time.

## STANDARD 3.3 *Use knowledge of literary history, traditions, and theory to respond to and analyze the meaning of texts.*

**CE 3.3.1** Explore the relationships among individual works, authors, and literary movements in English and American literature (e.g., Romanticism, Puritanism, the Harlem Renaissance, Postcolonial), and consider the historical, cultural, and societal contexts in which works were produced.

**CE 3.3.2** Read and analyze classic and contemporary works of literature (American, British, world) representing a variety of genres and traditions and consider their significance in their own time period as well as how they may be relevant to contemporary society.

**CE 3.3.3** Draw on a variety of critical perspectives to respond to and analyze works of literature (e.g., religious, biographical, feminist, multicultural, political).

**CE 3.3.4** Demonstrate knowledge of American minority literature and the contributions of minority writers.

**CE 3.3.5** Demonstrate familiarity with world literature, including authors beyond American and British literary traditions.

**CE 3.3.6** Critically examine standards of literary judgment (e.g., aesthetic value, quality of writing, literary merit, social significance) and questions regarding the inclusion and/or exclusion of literary works in the curriculum (e.g., canon formation, "classic" vs. "popular" texts, traditional vs. non-traditional literature, the place of literature by women and/or minority writers).

## STANDARD 3.4 *Examine mass media, film, series fiction, and other texts from popular culture.*

**CE 3.4.1** Use methods of close and contextualized reading and viewing to examine, interpret, and evaluate print and visual media and other works from popular culture.

**CE 3.4.2** Understand that media and popular texts are produced within a social context and have economic, political, social, and aesthetic purposes.

**CE 3.4.3** Understand the ways people use media in their personal and public lives.

**CE 3.4.4** Understand how the commercial and political purposes of producers and publishers influence not only the nature of advertisements and the selection of media content, but the slant of news articles in newspapers, magazines, and the visual media.

# STRAND 4: LANGUAGE

*Language is an evolving tool with powerful personal, cultural, economic, and political implications. Knowledge of the structures of language (e.g., the history, meaning, and use of words; varying sentence structures and patterns of language; the conventions of standard English) is essential for the effective use of language for varying purposes (e.g., the development of a rich vocabulary, sentence structures for different rhetorical purposes, appropriate speech patterns for different social contexts). Understanding the political implications of language use is also critical for fostering a democratic society in which all voices are valued.*

## STANDARD 4.1 *Understand and use the English language effectively in a variety of contexts and settings.*

**CE 4.1.1** Use sentence structures and vocabulary effectively within different modes (oral and written, formal and informal) and for various rhetorical purposes.

**CE 4.1.2** Use resources to determine word meanings, pronunciations, and word etymologies (e.g., context, print and electronic dictionaries, thesauruses, glossaries, and others).

**CE 4.1.3** Use a range of linguistic applications and styles for accomplishing different rhetorical purposes (e.g., persuading others to change opinions, conducting business transactions, speaking in a public forum, discussing issues informally with peers).

**CE 4.1.4** Control standard English structures in a variety of contexts (e.g., formal speaking, academic prose, business, and public writing) using language carefully and precisely.

**CE 4.1.5** Demonstrate use of conventions of grammar, usage, and mechanics in written texts, including parts of speech, sentence structure and variety, spelling, capitalization, and punctuation.

## STANDARD 4.2 *Understand how language variety reflects and shapes experience.*

**CE 4.2.1** Understand how languages and dialects are used to communicate effectively in different roles, under different circumstances, and among speakers of different speech communities (e.g., ethnic communities, social groups, professional organizations).

**CE 4.2.2** Understand the implications and potential consequences of language use (e.g., appropriate professional speech; sexist, racist, homophobic language).

**CE 4.2.3** Recognize and appreciate language variety, understand that all dialects are rule-governed, and respect the linguistic differences of other speech communities.

**CE 4.2.4** Understand the appropriate uses and implications of casual or informal versus professional language; understand, as well, the implications of language designed to control others and the detrimental effects of its use on targeted individuals or groups (e.g., propaganda, homophobic language, and racial, ethnic, or gender epithets).

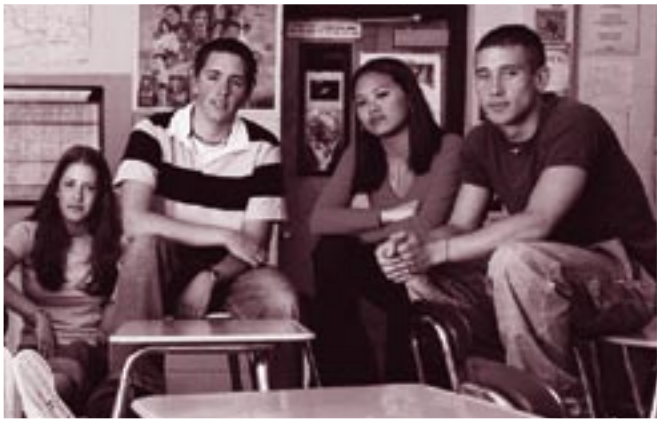
**CE 4.2.5** Recognize language bias in one's community, school, textbooks, the public press, and in one's own use of language.

# NOTES


**Michigan Department of Education**
Office of School Improvement
Dr. Yvonne Caamal Canul, Director
(517) 241-3147  www.michigan.gov/mde

# High School Content Expectations





# MATHEMATICS

- Quantitative Literacy
- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability

CE • **RIGOR** • RELEVANCE • RELATIONSHIPS • RIGOR • RELEVAN
IPS • RELATIONSHIPS • RIGOR • **RELEVANCE** • RELATIONSH
CE • RIGOR • RELEVANCE • RELATIONSHIPS • RIGOR • RELEVAN
IPS • **RELATIONSHIPS** • RIGOR • RELEVANCE • RELATIONSH



MICHIGAN Department of Education

MICHIGAN Department of Education
Office of School Improvement

## Michigan State Board of Education

**Kathleen N. Straus, President**
Bloomfield Township

**John C. Austin, Vice President**
Ann Arbor

**Carolyn L. Curtin, Secretary**
Evart

**Marianne Yared McGuire, Treasurer**
Detroit

**Nancy Danhof, NASBE Delegate**
East Lansing

**Elizabeth W. Bauer**
Birmingham

**Reginald M. Turner**
Detroit

**Eileen Lappin Weiser**
Ann Arbor

**Governor Jennifer M. Granholm**
*Ex Officio*

**Michael P. Flanagan, Chairman**
*Superintendent of Public Instruction*

**Jeremy M. Hughes, Ph.D.**
*Deputy Superintendent and*
*Chief Academic Officer*

**Dr. Yvonne Caamal Canul, Director**
*Office of School Improvement*

## Mathematics Work Group

### *Academic Review*

**Joan Ferrini-Mundy, Chair**
*Michigan State University*
**Kathy Berry**
*Monroe Public Schools*
**Christian Hirsch**
*Western Michigan University*
**Ruth Anne Hodges**
*Michigan Department of Education*
**Sharon Senk**
*Michigan State University*
**Patricia Shure**
*University of Michigan*
**Clifford Weil**
*Michigan State University*
**Michael Weiskopf**
*Walled Lake Public Schools*
**Glenna Zollinger-Russell**
*Michigan Department of Education/CTE*

### *Graduate Student Assistants*

**Gregory Larnell**
*Michigan State University*
**Aaron Mosier**
*Michigan State University*
**Jill Newton**
*Michigan State University*

### *Planned University Review*

**Sharif Shakrani**
*Michigan State University/NAGB*
**William Schmidt**
*Michigan State University*

### *Internal Review*

**Ruth Anne Hodges**
*Michigan Department of Education/Achieve**
**Theron Blakeslee**
*Ingham County ISD**
**Marie Copeland**
*Macomb ISD**
**Stephen Frank**
*Gull Lake Schools**
**Deborah Ferry**
*Macomb ISD**
**Jennifer Nimtz**
*Washtenaw ISD**
**Carol Pinneo**
*Birmingham Schools/Michigan Council of Teachers of Mathematics*
**Dan Schab**
*Michigan Teacher of the Year 2005-2006*

### *External Review*

**Charles Allan**
*Michigan Council of Teachers of Mathematics**
**Terry Parks**
*Michigan Mathematics Leadership Academy*
**Carolyn Siebers**
*Wayne RESA*
**Libby Trenkle**
*Michigan Math and Science Centers*
**Judy Wheeler**
*Michigan Council of Teachers of Mathematics**

### *Project Coodinator*

**Susan Codere Kelly**
*Michigan Department of Education**

**Grade Level Content Expectations Committee*

# Welcome to Michigan's High School Mathematics Content Standards and Expectations

## Why Develop Content Standards and Expectations for High School?

In 2004, the Michigan Department of Education embraced the challenge to initiate a "high school redesign" project. Since then, the national call to create more rigorous learning for high school students has become a major priority for state leaders across the country. The Cherry Commission Report highlighted several goals for Michigan including the development of high school content expectations that reflect both a rigorous and a relevant curricular focus. Dovetailing with this call to "curricular action" is Michigan's legislative change in high school assessment. The Michigan Merit Exam, based on rigorous high school learning standards, is to be fully implemented by 2007.

Given these two catalysts, the Michigan Department of Education's Office of School Improvement led the development of high school content expectations for English Language Arts and Mathematics. Content area work groups of academicians chaired by a nationally known scholar in the respective field, were commissioned to conduct a scholarly review and identify content standards and expectations. These content standards and expectations have gone through an extensive field and national review and are presented to educators in this document.

## An Overview

The expectations contained in this document reflect best practices and current research in the teaching and learning of mathematics. They build from the *Michigan Mathematics Curriculum Framework Standards and Benchmarks* (1996), the *Career and Employability Skills Content Standards and Benchmarks (2001)*, and extend the *Michigan K-8 Mathematics Grade Level Content Expectations* (2004) as appropriate for grades 9-12. These standards and expectations represent a vision for a rigorous and relevant high school experience for all Michigan students over the next five to ten years. Special attention has been paid to national research and support for the skills that prepare students for successful post-secondary engagement and the workplace.

The standards and expectations are closely aligned with national standards as described in ACT's *College Readiness Standards*®, American Diploma Project's *Ready or Not: Creating a High School Diploma That Counts* (2004), the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics (2000)*, and the National Assessment Governing Board's *Mathematics Framework for the 2003 National Assessment of Educational Progress (NAEP)*. Students whose work is guided by these standards and expectations will be prepared both for college and for the workplace.

## Understanding the Organizational Structure

The expectations in this document are divided into four strands with multiple standards within each, as shown below. The skills and content addressed in these standards will, in practice, be woven together into a coherent, integrated Mathematics curriculum. The standards are comprehensive and are meant to be used as a guide to curriculum development.

| STRAND 1<br>Quantitative Literacy<br>(L) | STRAND 2<br>Algebra & Functions<br>(A) | STRAND 3<br>Geometry & Trigonometry<br>(G) | STRAND 4<br>Statistics & Probability<br>(S) |
|---|---|---|---|
| **STANDARDS** (and number of expectations for each standard) | | | |
| **L1: Reasoning About Numbers, Systems and Quantitative Situations** (17)<br>**L2: Calculation, Algorithms, and Estimation** (9)<br>**L3: Measurement and Precision** (7) | **A1: Expressions, Equations, and Inequalities** (17)<br>**A2: Function** (43)<br>**A3: Mathematical Modeling** (5) | **G1: Figures and Their Properties** (29)<br>**G2: Relationships Between Figures** (10)<br>**G3: Transformations of Figures in the Plane** (6) | **S1: Univariate Data—Examining Distributions** (11)<br>**S2: Bivariate Data—Examining Relationships** (6)<br>**S3: Samples, Surveys, and Experiments** (6)<br>**S4: Probability Models and Probability Calculation** (8) |

## Curriculum and Assessment

This document is intended to support conversations at the school and district level that result in rigorous and relevant curriculum that incorporates these content expectations.

***As stakeholders (e.g., teachers, administrators, school board members, parents, community members, students, local legislative representatives) work with these standards, they should consider the following questions:***

- How are these content standards and expectations reflected in our curriculum and instruction already?
- Where do we need to strengthen our curriculum and instruction to more fully realize the intent of these standards and expectations?
- What opportunities do these standards and expectations present to develop new and strengthen existing curriculum, leading to instructional excellence?
- How do we implement these standards and expectations taking into account what we know about our students, school, and community?
- How will we assess the effectiveness with which our students and schools are meeting these standards and content expectations?
- How can we use school-based assessments (e.g., student portfolios, school-based writing assessments, teacher or classroom research, district-level assessments) to make data-driven decisions about teaching and learning?

Through conversations about questions such as these, and building upon the multitude of existing strengths in our current high schools, voices of all stakeholders will participate in the important and continuing process of shaping instructional excellence in Michigan schools and preparing students in Michigan schools for college and the workplace.

## Mathematics

Mathematical understandings and skills are essential elements for meaningful participation in the global information society. US expectations in mathematics for high school students have not kept pace with expectations in high-achieving countries around the world. And, expectations about who can do mathematics in the US have led to inequitable and unacceptably low opportunities to learn for students living in poor and urban communities. In Michigan, the K-8 Mathematics Grade Level Expectations represent a major step forward in raising expectations in mathematics for all students. These high school expectations assume the ambitious foundation of the K-8 GLCEs and are intended to equip all students with a solid background for continued postsecondary study in any area, as well as with skills and knowledge essential for the workplace. It is essential to hold high expectations in mathematics for all students for completion of high school, whether they will enter the workforce or go on to postsecondary education.

The high school mathematics content expectations are organized in four strands: Quantitative Literacy, Algebra and Functions, Geometry and Trigonometry, and Statistics and Probability. The topics within each strand have been arranged to show mathe-matical growth and to illustrate mathematical trajectories of ideas that build on one another, when possible. The expectations in these four strands, are not mapped into course arrangements in this document. Such mapping, whether to traditional course titles like Algebra I, Geometry, or Algebra II, or into courses that integrate the material, is a complex process.

### *Decisions about the inclusion of topics were based on the following five criteria:*

- how well the topic connects to other mathematical areas
- the mathematical centrality of the topic
- the standing of the topic as a cultural accomplishment
- the relevance of the topic for secondary school students
- the importance of the topic in the workplace or for informed citizenship

There is a strong emphasis on mathematical reasoning throughout all of these strands. It is also important for high school students to become successful in applying mathematical concepts and processes to solve complex problems. Technological advances affect what is possible to learn, and what is necessary to learn, in high school mathematics, and these expectations reflect this trend.

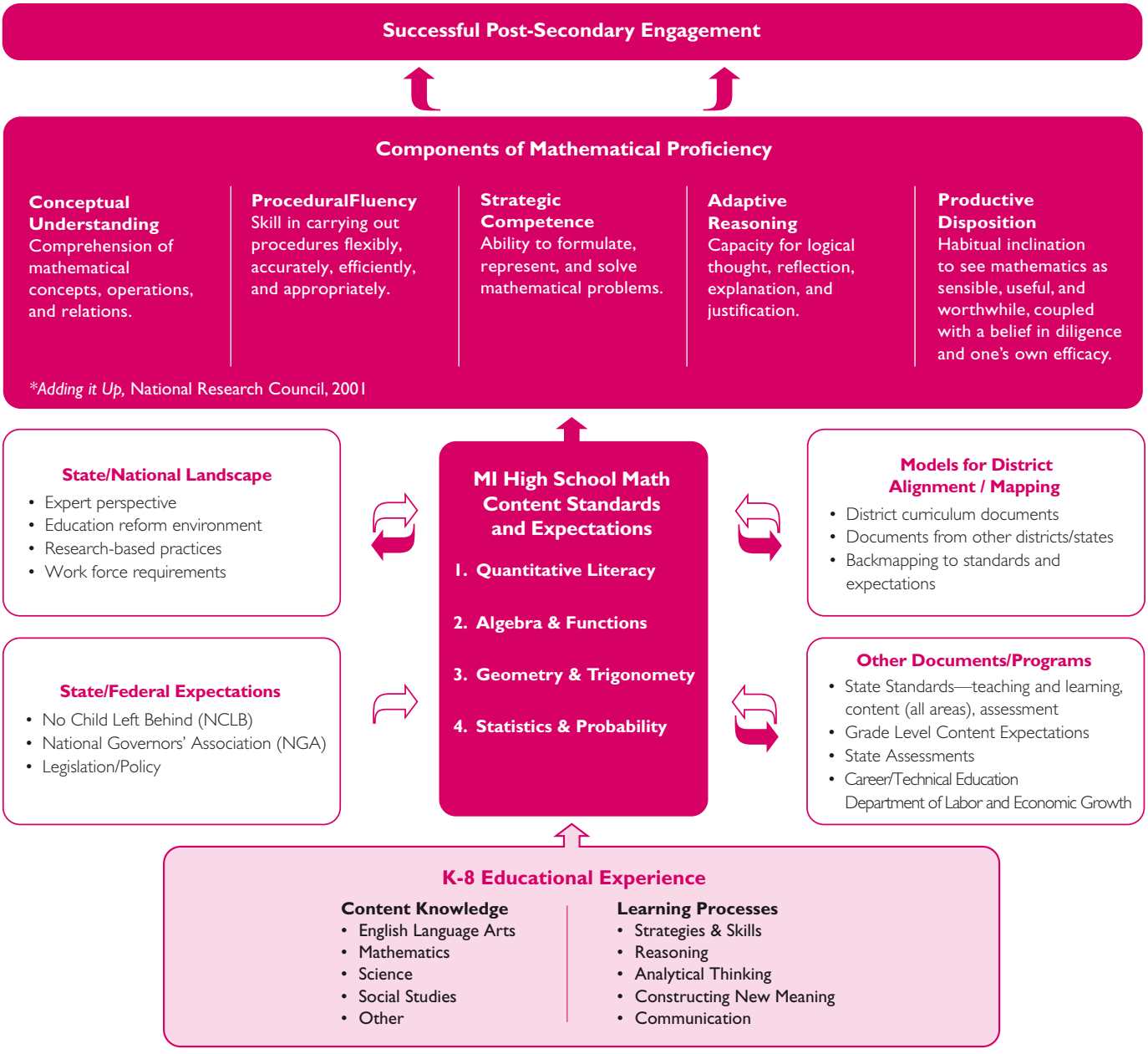
These four strands are fundamentally interconnected and also arranged to reflect the sequencing and emphases in the mathematical ideas that are central to high school. We believe that all students should take mathematics in their fourth year of high school to ensure a smooth transition to their next set of experiences.

## Preparing Students for Successful Post-Secondary Engagement

As educators use these standards and expectations to develop rigorous and relevant units of instruction, powerful and engaging learning activities, and challenging high school curricula, it is critical to keep in mind that content knowledge alone will not provide adequate preparation for success in entry-level university courses or entry-level positions in today's workforce.

Successful post-secondary engagement requires that students must be able to apply knowledge in new situations; to solve problems by generating new ideas; to make connections between what they read and hear in class, the world around them, and the future; and through their work, develop leadership qualities while still in high school.

Therefore, educators must model for and develop in students the cognitive skills and habits of mind that will result in mathematical proficiency and successful post-secondary engagement.




This chart includes talking points for the professional development model.

# STRAND 1: QUANTITATIVE LITERACY (L)

*"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas."*[1]

Historically, for many US students, a significant component of their high school mathematics education included a revisiting of the processes of arithmetic. The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this Quantitative Literacy strand assume this strong foundation in number and build on it, as well as expand it, into a definition of secondary school quantitative literacy for all students. In particular, these expectations assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators.

The K-8 expectations are reinforced and applied in more complex situations through these Quantitative Literacy expectations, particularly through connections and applications of number ideas to other areas of mathematics, such as algebra, geometry, and statistics. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as to other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation reflect important uses of number in a range of real-life situations. Ideas about measurement and precision tie closely to geometry.

These Quantitative Literacy expectations also include focus on the kinds of numerical and quantitative arguments and representations that arise on a daily basis for citizens in a technological, global society. It is important for the high school students of Michigan to be adept in such modes of reasoning and able to understand the presentations of quantitative information that they will encounter in their daily lives.

[1] Estry, D., & Ferrini-Mundy, J. (January, 2005). Quantitative Literacy Task Force Final Report and Recommendations. East Lansing: Michigan State University.

## STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations. They construct and evaluate logical arguments, and can interpret arguments made about quantitative situations in the popular media.

### L1.1 Number Systems and Number Sense

**L1.1.1** Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.

**L1.1.2** Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.

**L1.1.3** Understand how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.

**L1.1.4** Reason about the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.

**L1.1.5** Justify numerical relationships (e.g., show that the sum of even integers is even; that every integer can be written as $3m+k$, where $k$ is 0, 1, or 2; or that the sum of the first $n$ integers is $n(n+1)/2$).

**L1.1.6** Understand the importance of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ in basic right triangle trigonometry; recognize the role of $\pi$ because of its role in circle relationships; understand the role of $e$ in applications such as continuously compounded interest.

### L1.2 Representations and Relationships

**L1.2.1** Use mathematical symbols (e.g., interval notation, set notation, summation notation) to represent quantitative relationships and situations.

**L1.2.2** Interpret absolute value relationships and notation (e.g. $|x - a| \leq b$) in such contexts as error tolerance.

**L1.2.3** Use vectors to represent quantities that have magnitude and direction; interpret direction and magnitude of a vector numerically.

**L1.2.4** Read and interpret representations from various technological sources, such as contour or isobar diagrams, Geographic Information System data, or spreadsheet outputs.

**L1.2.5** Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.

### L1.3 Logical Arguments and Reasoning

**L1.3.1** Construct and evaluate inductive and deductive arguments.

**L1.3.2** Use terms and concepts from logic (e.g., axiom, definition, theorem (proposition), negation, conjunctions, truth and falsity, implication, if-and-only-if, converse, inverse, contrapositive) to reason about mathematical relationships and quantitative situations.

**L1.3.3** Distinguish between inferences based on empirical data or patterns and inferences based on deductive arguments (i.e., based on principles of logic); identify relevant, irrelevant, and missing information in arguments; recognize flawed reasoning.

### L1.4 Counting and Probabilistic Reasoning

**L1.4.1** Describe, explain, and apply various counting techniques (e.g., number of different 4-letter passwords, permutations, and combination; relate combinations to Pascal's triangle): know when to use each technique.

**L1.4.2** Define and interpret commonly used expressions of probability (e.g., chances of an event, likelihood, odds).

**L1.4.3** Recognize and explain common probability misconceptions such as "hot streaks," "being due," and "number of successes must approach expected number of successes."

## STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

Students calculate fluently, estimate proficiently, and describe and use algorithms in appropriate situations (e.g., approximating solutions to equations.) They understand the basic ideas of iteration and algorithms.

### L2.1 Calculation Using Real and Complex Numbers

**L2.1.1** Calculate fluently with exponential expressions using the rules of exponents and evaluate numerical expressions involving (in particular, rational and negative) exponents; transition easily between roots and exponents.

**L2.1.2** Understand the exponential relationship between a number and its logarithm (e.g., base 10), and use it to relate rules of logarithms to those of exponents.

**L2.1.3** Know that the complex number $i$ is one of two solutions to $x^2 = -1$.

**L2.1.4** Understand that exact answers aren't always possible or practical; use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).

**L2.1.5** Know the meaning and uses of weighted averages (e.g., grading schemes, consumer price index, GNP); understand the mathematics underlying basic voting procedures (e.g., majority, plurality, runoff).

### L2.2 Sequences and Iteration

**L2.2.1** Find the $n$th term in arithmetic, geometric, or other simple sequences.

**L2.2.2** Compute sums of finite arithmetic sequences.

**L2.2.3** Compute sums of finite and infinite geometric sequences.

**L2.2.4** Comprehend the idea of an iterative process, in such examples as computer programming, finding prime factorizations of integers, or approximation procedures.

## STANDARD L3: MEASUREMENT AND PRECISION

Students apply measurement units and calculations, and understand the concept of error.

### L3.1 Measurement Units, Calculations, and Scales

**L3.1.1** Convert units of measurement between systems.

**L3.1.2** Explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.

**L3.1.3** Describe and interpret logarithmic scales such as the Richter scale, or the pH scale (e.g., explain why a small change in the independent variable can represent a large change in the dependent variable).

### L3.2 Understanding Error

**L3.2.1** Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation.

**L3.2.2** Describe and explain round-off error, rounding, and truncating, and know how to compensate for the inaccuracies to produce a desired degree of accuracy.

**L3.2.3** Understand and use concepts of accuracy, error tolerance, and accumulated error in applied situations.

**L3.2.4** Interpret quantitative terms (e.g., statistical significance, margin of error, confidence level, and Type I and Type II errors) as they appear in media reports.

# STRAND 2: ALGEBRA AND FUNCTIONS (A)

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of the building-block functions, for example, linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining the underlying structure of expressions, equations, and functions, and applying properties of structure to solve equations. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric functions. They will be able to reason about functions and their properties and solve multi-step problems that involve both functions and equation-solving.

Students will view algebra not only as a theoretical tool for analyzing and describing mathematical relationships, but they will also experience the power of algebraic thinking in the context of applications by studying the mathematical modeling of real-world problems. They will use deductive reasoning to justify algebraic processes.

## STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.

### A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

**A1.1.1** Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

**A1.1.2** Know the definitions and properties of exponents and roots, transition fluently between them, and apply them in algebraic expressions.

**A1.1.3** Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).

**A1.1.4** Add, subtract, multiply, and simplify polynomials and rational expressions (e.g., $(x - 1)(1 - x^2 + 3)$; $\frac{9x - x^3}{x + 3}$)

**A1.1.5** Divide a polynomial by a monomial.

**A1.1.6** Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms including the inverse relationship between exponents and logarithms.

**A1.1.7** Transform trigonometric expressions into equivalent forms using basic identities such as $\sin^2\theta + \cos^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\tan^2\theta + 1 = \sec^2\theta$

### A1.2 Solutions of Equations and Inequalities (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

**A1.2.1** Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.

**A1.2.2** Understand and describe the relationships between the solutions of an equation and the zeroes of the associated function.

**A1.2.3** Solve (and justify steps in the solutions) linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns; apply the quadratic formula appropriately.

**A1.2.4** Solve absolute value equations and inequalities, (e.g. $|x - 3| \leq 6$), and justify steps in the solution.

**A1.2.5** Solve polynomial equations and equations involving rational expressions (e.g. solve $-2x(x^2 + 4x+3) = 0$; $x - \frac{1}{x+6} = 3$), and justify steps in the solution.

**A1.2.6** Solve power equations (e.g., solve $(x + 1)^3 = 8$) and equations including radical expressions (e.g., solve $\sqrt{3x - 7} = 7$), justify steps in the solution, and explain how extraneous solutions may arise.

**A1.2.7** Solve exponential and logarithmic equations (e.g., $3(2^x) = 7$), $2\ ln(x + 1) = 4$), and justify steps in the solution.

**A1.2.8** Solve basic trigonometric equations over specific intervals and justify steps in the solution (e.g., $2\ sin\ x - 1 = 0$ for $-\pi \leq x \leq 3\pi$).

**A1.2.9** Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.

**A1.2.10** Know common formulas (e.g., slope, distance between two points, quadratic formula, compound interest, distance = rate • time), and apply appropriately in contextual situations.

## STANDARD A2: FUNCTION

Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.

### A2.1 Definitions, Representations, and Attributes of Functions

**A2.1.1** Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.

**A2.1.2** Read, interpret, and use function notation, and evaluate a function at a value in its domain.

**A2.1.3** Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.

**A2.1.4** Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined (e.g., absolute value and greatest integer functions).

**A2.1.5** Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions (e.g., $f(0) = 5$, and $f(n) = f(n-1) + 2$ ).

**A2.1.6** Identify the zeroes of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as $x$ approaches postive or negative infinity, from the symbolic and graphical representations.

**A2.1.7** Identify and interpret the key features of a function from its graph, (e.g. intercept(s), asymptote(s), maximum and minimum value(s), symmetry, rate of change, and periodicity).

### A2.2 Operations, Transformations, and Inverses

**A2.2.1** Combine functions by addition, subtraction, multiplication, and division.

**A2.2.2** Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the $x$- and $y$-axes) to basic functions, and represent symbolically.

**A2.2.3** Determine whether a given function has an inverse and find the inverse if it exists.

## A2.3 Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

**A2.3.1** Identify a function as a member of a family of functions based on its symbolic, or graphical representation; recognize that different classes of functions have different rates of change.

**A2.3.2** Describe the tabular pattern associated with constant rate of change (linear); or variable rates of change.

**A2.3.3** Write the general symbolic forms that characterize each family of functions. (e.g., $f(x) = A_0a^x$; $f(x) = A\sin Bx$ )

## A2.4 Lines and Linear Functions

**A2.4.1** Write the symbolic forms of linear functions (standard [i.e., $Ax + By = C$], point-slope, and slope-intercept) given appropriate information, and convert between forms.

**A2.4.2** Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information.

**A2.4.3** Relate the coefficients in a linear function to the slope and $x$- and $y$-intercepts of its graph.

**A2.4.4** Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the fact that non-vertical parallel lines have equal slopes, that non-vertical perpendicular lines have slopes that multiply to give -1.

## A2.5 Exponential and Logarithmic Functions

**A2.5.1** Write the symbolic form and sketch the graph of an exponential function given appropriate information. (e.g., given an initial value of 4 and a rate of growth of 1.5, write $f(x) = 4\ (1.5)^x$).

**A2.5.2** Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., $f(x) = 10^x$, $f(x) = \log x$, $f(x) = e^x$, $f(x) = ln\ x$.)

**A2.5.3** Apply properties of exponential and logarithmic functions (e.g., $a^{x+y} = a^x a^y$; $\log(ab) = \log a + \log b$).

**A2.5.4** Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and how base affects the rate of growth or decay.

**A2.5.5** Understand and use the fact that an exponential function (defined on the real numbers for a positive base other than 1) has an inverse, the logarithmic function.

**A2.5.6** Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

## A2.6 Quadratic Functions

**A2.6.1** Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).

**A2.6.2** Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.

**A2.6.3** Convert quadratic functions from standard to vertex form by completing the square.

**A2.6.4** Relate the number of real solutions for a quadratic equation to its graph.

**A2.6.5** Analyze the graphs of quadratic functions, noting symmetry and zeroes.

**A2.6.6** Understand how different symbolic representations of functions make different purposes (e.g., $f(x) = (x-3)^2-16$ shows the minimum value of the function and $f(x) = (x - 7)(x + 1)$ shows its zeroes).

## A2.7 Power Functions (including roots, cubics, quartics, etc.)

**A2.7.1** Write the symbolic form and sketch the graph of power functions.

**A2.7.2** Recognize characteristics of direct and inverse variation functions (e.g., $y = kx^n$ and $y = kx^{-n}$, $n > 0$).

**A2.7.3** Analyze the graphs of power functions, noting reflectional or rotational symmetry.

### A2.8 Polynomial Functions

**A2.8.1** Write the symbolic form and sketch the graph of polynomial functions.

**A2.8.2** Understand the effects of degree, leading coefficient, and multiplicity of real zeroes on the graph polynomial functions of degree > 2.

**A2.8.3** Determine the maximum possible number of zeroes of a polynomial function, and understand the relationship between the $x$-intercepts of the graph and the factored form of the function.

### A2.9 Rational Functions

**A2.9.1** Write the symbolic form and sketch the graph of simple rational functions.

**A2.9.2** Analyze graphs of simple rational functions and understand the relationship between the zeroes of the numerator and denominator and the function's intercepts, asymptotes, and domain (e.g., $\frac{2x + 1}{x - 1}$ ; $\frac{x}{x^2 - 4}$).

### A2.10 Trigonometric Functions

**A2.10.1** Write the symbolic form and sketch the graph of trigonometric functions.

**A2.10.2** Use the unit circle to define sine and cosine; use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.

**A2.10.3** Use the relationship between degree and radian measures to solve problems.

**A2.10.4** Use the unit circle to determine the exact values of sine and cosine, for 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $\pi$, and their integer multiples, and apply in various contexts.

**A2.10.5** Graph the sine, cosine, and tangent functions; analyze graphs by noting domain, range, period, amplitude, location of maxima and minima, and asymptotes.

**A2.10.6** Graph transformations of basic trigonometric functions (involving changes in period, amplitude, phase, and midline) and understand the relationship between constants in the formula and the transformed graph.

## STANDARD A3: MATHEMATICAL MODELING

Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so. They solve applied problems involving linear programming.

### A3.1 Models of Real-world Situations Using Classes of Functions.

*Example: An initial population of 300 people grows at 2% per year. What will the population be in 10 years?*

**A3.1.1** Identify the family of function best suited for modeling a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. *In the example above, recognize that the appropriate general function is exponential* ($P = P_0 a^t$)

**A3.1.2** Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. *In the example above, substitute the given values* $P_0 = 300$ and $a = 1.02$ to obtain $P = 300(1.02)^t$.

**A3.1.3** Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. *In the example above, the exact solution is 365.698, but for this problem an appropriate approximation is 365.*

**A3.1.4** Recognize that the accuracy of predictions from mathematical models is limited by the accuracy of the model.

**A3.1.5** Use methods of linear programming to represent and solve simple real-life problems.

# STRAND 3: GEOMETRY AND TRIGONOMETRY (G)

In Grades K–8, students learn to recognize and draw figures such as triangles, rectangles, circles, rectangular solids, cylinders, and spheres. They also examine similarities among and differences between geometric shapes; study relevant properties of shapes such as angle measure, area, and volume; develop basic concepts of congruence, similarity, and symmetry; and use these properties to solve problems related to the real world.

In Grades 9–12, students build upon the geometry studied in earlier grades by solving more complex problems about familiar figures. They also study other 2- and 3-dimensional figures, including regular polyhedra; and they examine relationships between geometric figures, particularly congruence and similarity. They study transformations of figures, e.g., reflections and their applications to tessellations of the plane; and trigonometry. Theorem and proof should be emphasized. To prove that conjectures are true, students are expected to use various forms of deductive reasoning described in the Quantitative Literacy strand, including both direct and indirect proof presented in narrative form. Students' spatial visualization skills will be developed through study of the relationships between two- and three-dimensional shapes.

## STANDARD G1: FIGURES AND THEIR PROPERTIES

Students represent basic geometric figures, polygons, discrete geometric figures and conic sections. They apply their definitions and properties in solving problems and justifying arguments, including constructions and representations in the coordinate plane. Students represent three-dimensional figures, understand the concepts of volume and surface area and use them to solve problems. They know and apply properties of common three-dimensional figures.

### G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry

**G1.1.1** Solve multi-step problems and construct proofs involving vertical angles, linear pairs of angles, supplementary angles, complementary angles, and right angles.

**G1.1.2** Solve multi-step problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.

**G1.1.3** Perform and justify constructions, including midpoint of a line segment and bisector of an angle.

**G1.1.4** Given a line and a point, construct a line through the point that is parallel to the original line; given a line and a point, construct a line through the point that is perpendicular to the original line; justify the steps of the constructions.

**G.1.1.5** Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.

**G.1.1.6** Recognize the existence of non-Euclidean geometry, where the parallel postulate does not hold.

### G1.2 Triangles and Their Properties

**G1.2.1** Prove that the angle sum of a triangle is 180°, and that the exterior angle of a triangle is the sum of the two remote interior angles.

**G1.2.2** Construct and justify arguments and solve multi-step problems involving angle measure, side length, perimeter, and area of all types of triangles.

**G1.2.3** Know, use, and prove the Pythagorean Theorem and its converse.

**G1.2.4** Know, justify, and use the relationships among the side lengths and the angles of 30°- 60°- 90° triangles and 45°- 45°- 90° triangles.

**G1.2.5** Know, justify, and use the properties of medians, altitudes, and perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle; construct these lines and find the equations of these lines in triangles given in the coordinate plane.

## G1.3 Triangles and Trigonometry

**G1.3.1** Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides; solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.

**G1.3.2** Know and use the Law of Sines and the Law of Cosines and use them to solve problems; find the area of a triangle with sides $a$ and $b$ and included angle $\theta$ using the formula Area = (1/2) $a\ b$ sin $\theta$.

**G1.3.3** Determine the exact values of sine, cosine, and tangent for 0°, 30°, 45°, 60°, and their integer multiples, and apply in various contexts.

## G1.4 Quadrilaterals and Their Properties

**G1.4.1** Solve multi-step problems involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.

**G1.4.2** Prove properties of quadrilaterals (e.g., diagonals of a rhombus are perpendicular) using Euclidean methods and analytic geometry (e.g., coordinate representations).

**G1.4.3** Describe and justify hierarchical relationships among quadrilaterals, (e.g. every rectangle is a parallelogram).

**G1.4.4** Prove theorems about the interior and exterior angle sums of a quadrilateral (e.g., prove that the sum of the exterior angles of a convex polygon is 360°).

## G1.5 Other Polygons and Their Properties

**G1.5.1** Find angle measures or the area of a polygon (e.g., regular octagon, non-regular pentagon) using dissection or circumscription methods.

**G1.5.2** Know, justify, and use formulas for the perimeter and area of a regular $n$-gon and formulas to find interior and exterior angles of a regular $n$-gon.

## G1.6 Circles and Their Properties

**G1.6.1** Solve multi-step problems involving circumference and area of circles.

**G1.6.2** Solve problems and justify arguments about chords (e.g., if line through the center of a circle is perpendicular to a chord, it bisects the chord) and tangents of circles (e.g., a line tangent to a circle is perpendicular to the radius drawn to the point of contact).

**G1.6.3** Solve problems and justify arguments about central angles and inscribed angles and triangles in circles.

**G1.6.4** Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.

## G1.7 Conic Sections and Their Properties

**G.1.7.1** Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

**G1.7.2** Identify and distinguish among parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

**G1.7.3** Graph ellipses and hyperbolas with axes parallel to the x- and y-axes.

## G1.8 Three- Dimensional Figures

**G1.8.1** Solve multi-step problems and provide proofs involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

**G1.8.2** Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.

## STANDARD G2: RELATIONSHIPS BETWEEN FIGURES

Students use and justify relationships between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and provide proofs about congruence and similarity.

### G2.1 Relationships Between Area and Volume Formulas

**G2.1.1** Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.

**G2.1.2** Know and demonstrate the relationships between the area formulas of various quadrilaterals (e.g., explain how to find the area of a trapezoid based on the areas of parallelograms and triangles).

**G2.1.3** Know and demonstrate the relationship between the volume formulas for pyramids and prisms and for cones and cylinders.

### G2.2 Relationships Between Two-dimensional and Three-dimensional Representations

**G2.2.1** Identify or sketch a possible 3-dimensional figure, given 2-dimensional views (e.g., nets, multiple views); create a 2-dimensional representation of a 3-dimensional figure.

**G2.2.2** Identify or sketch cross-sections of 3-dimensional figures or solids formed by revoling 2-dimensional figures around lines.

### G2.3 Congruence and Similarity

**G2.3.1** Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and for right triangles, the hypotenuse-leg criterion.

**G2.3.2** Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.

**G2.3.3** Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.

**G2.3.4** Use theorems about similar triangles to solve problems with and without use of coordinates.

**G2.3.5** Know and apply the theorem stating that the effect of a scale factor $k$ on length, area, and volume is to multiply each by $k$, $k^2$, and $k^3$, respectively.

## STANDARD G3: TRANSFORMATIONS OF FIGURES IN THE PLANE

Students will solve problems about distance-preserving transformations and shape-preserving transformations, including design or analysis of tessellations of the plane.

### G3.1 Distance-preserving Transformations: Isometries

**G3.1.1** Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.

**G3.1.2** Given two figures that are images of each other under an isometry, describe the isometry completely as a composite of reflections, rotations, or translations.

**G3.1.3** Find the image of a figure under the composition of two or more isometries, and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.

### G3.2 Shape-preserving Transformations: Dilations and Isometries

**G3.2.1** Define a dilation (size change) and find the image of a figure under a dilation, given the center of the dilation and the scale factor.

**G3.2.2** Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.

**G3.2.3** Find the image of a figure under a composite of a dilation and an isometry, with and without using coordinates.

# STRAND 4: STATISTICS AND PROBABILITY (S)

In Kindergarten through Grade 8, students develop the ability to read, analyze, and construct a repertoire of statistical graphs. Students also examine the fundamentals of experimental and theoretical probability in informal ways. The Basic Counting Principle and tree diagrams serve as tools to solve simple counting problems in these grades.

During high school, students build on that basic foundation. They develop the data interpretation and decision-making skills that will serve them in their further study of mathematics as well as in their coursework in the physical, biological, and social sciences. These skills are also highly valuable outside of school, both in the workplace and in day-to-day life.

Students learn important skills related to the collection, display, and interpretation of both univariate and bivariate data. They understand basic sampling methods and apply principles of effective data analysis and data presentation. Statistical process control is included because of its importance in the manufacturing workplace.

In probability, students utilize simulation strategies to estimate and probability models to calculate probabilities and make decisions. The normal distribution and its properties are studied. Students then use their understanding of probability to make decisions, solve problems, and determine whether or not statements about probabilities of events are reasonable. Students use technology when appropriate, including spreadsheets. This strong background in statistics and probability will enable students to be savvy decision-makers and smart information-consumers and producers who have a full range of tools in order to make wise choices.

## STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS

Students plot and analyze univariate data by considering shape of distributions and analyzing outliers; they find and interpret commonly-used measures of center and variation; and they explain and use properties of the normal distribution.

### S1.1 Producing and Interpreting Plots

**S1.1.1** Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data.

**S1.1.2** Given a distribution, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation).

### S1.2 Measures of Center and Variation

**S1.2.1** Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.

**S1.2.2** Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.

**S1.2.3** Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

**S1.2.4** Describe how additional data points, such as outliers, affect measures of center and variation.

**S1.2.5** Compare data sets and interpret differences based on graphs and summary statistics.

### S1.3 The Normal Distribution

**S1.3.1** Understand the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.

**S1.3.2** Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.

**S1.3.3** Know the basic characteristics of the normal distribution, including that the data are distributed symmetrically about the mean, and that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively.

**S1.3.4** Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informal decisions, and informal interval estimates, from data.

## STANDARD S2: BIVARIATE DATA – EXAMINING RELATIONSHIPS

Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.

### S2.1 Scatterplots and Correlation

**S2.1.1** Construct a scatterplot for a bivariate data set with appropriate labels and scales.

**S2.1.2** Describe shapes of scatterplots; identify clusters and outliers; recognize no correlation, weak correlation, and strong correlation in a scatterplot.

**S2.1.3** Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.

**S.2.1.4** Differentiate between correlation and causation; know that a strong correlation does not imply a cause-and-effect relationship; recognize the role of lurking variables in correlation.

### S2.2 Linear Regression

**S2.2.1** For bivariate data which appear to form a linear pattern, find the line of best fit by estimating visually and by calculating the equation of the regression line; interpret the slope and $y$-intercept of equation for a regression line

**S2.2.2** Use an estimated line of best fit and the equation of the regression line to make appropriate predictions.

## STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS

Students understand and apply sampling and various sampling methods, examine and design surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias, and understand and apply principles of good experiment design.

### S3.1 Data Collection and Analysis

**S3.1.1** Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.

**S3.1.2** Identify possible sources of bias in data collection and sampling methods; describe how such bias can be reduced and controlled; and explain the impact of such bias on conclusions made from analysis of the data.

**S3.1.3** Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

**S3.1.4** Design simple experiments or investigations to collect data to answer questions of interest.

### S3.2 Statistical Process Control

**S3.2.1** Explain the basic ideas of statistical process control, including recording data from a process over time.

**S3.2.2** Read and interpret basic control charts; detect patterns and departures from patterns.

## STANDARD S4: PROBABILITY MODELS AND PROBABILITY CALCULATION

Students understand probability and find probabilities in various situations, including those involving compound events, using simulation, diagrams, tables, geometric models and counting strategies; they apply the concepts of probability to make decisions.

### S4.1 Probability

**S4.1.1** Compare probabilities of two or more events; recognize equally likely events.

**S4.1.2** Compare theoretical probabilities with the empirical results of simple experiments, and know that if an event is independently repeated a large number of times, the empirical probability will approach the theoretical probability.

**S4.1.3** Understand and construct sample spaces in simple situations (e.g., tossing two coins, rolling two number cubes and summing the results).

**S4.1.4** Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.

**S4.1.5** Know the definitions of mutually exclusive events, independent events, dependent events, compound events, and complementary events, and use the definitions to compute possibilities.

### S4.2 Application of Probabilities

**S4.2.1** Determine probabilities and conditional probabilities of events using permutations and combinations, geometric models, Venn diagrams, tree diagrams, and two-way contingency tables.

**S4.2.2** Determine the odds in favor of or against an event given data, a list of outcomes, or the event's probability; determine the probability of an event given the odds in favor of or against the occurrence of the event.

**S4.2.3** Apply probability concepts to practical situations (in such settings as finance, health, ecology, or epidemiology) to make informed decisions.


**Michigan Department of Education**
Office of School Improvement
Dr. Yvonne Caamal Canul, Director
(517) 241-3147  www.michigan.gov/mde