# Michigan Merit Curriculum

## Course/Credit Requirements

# STATISTICS & PROBABILITY

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MICHIGAN
Department of
Education

**1 Credit (optional)**

# Welcome

This guide was developed to assist teachers in successfully implementing the Michigan Merit Curriculum. The identified content expectations and guidelines provide a successful framework for designing curriculum, assessments and relevant learning experiences for students. Through the collaborative efforts of Governor Jennifer M. Granholm, the State Board of Education, and the State Legislature, these landmark state graduation requirements are being implemented to give Michigan students the knowledge and skills to succeed in the 21st Century and drive Michigan's economic success in the global economy. Working together, teachers can explore varied pathways to help students demonstrate proficiency in meeting the content expectations and guidelines.

# Curriculum Unit Design

One of the ultimate goals of teaching is for students to acquire transferable knowledge. To accomplish this, learning needs to result in a deep understanding of content and mastery level of skills. As educational designers, teachers must use both the art and the science of teaching. In planning coherent, rigorous instructional units of study, it is best to *begin with the end in mind*.

*Engaging and effective units include*

- appropriate content expectations
- students setting goals and monitoring own progress
- a focus on big ideas that have great transfer value
- focus and essential questions that stimulate inquiry and connections
- identified valid and relevant skills and processes
- purposeful real-world applications
- relevant and worthy learning experiences
- varied flexible instruction for diverse learners
- research-based instructional strategies
- explicit and systematic instruction
- adequate teacher modeling and guided practice
- substantial time to review or apply new knowledge
- opportunities for revision of work based on feedback
- student evaluation of the unit
- culminating celebrations

## Relevance

Instruction that is clearly relevant to today's rapidly changing world is at the forefront of unit design. Content knowledge cannot by itself lead all students to academic achievement. Classes and projects that spark student interest and provide a rationale for why the content is worth learning enable students to make connections between what they read and learn in school, their lives, and their futures. An engaging and effective curriculum provides opportunities for exploration and exposure to new ideas. Real-world learning experiences provide students with opportunities to transfer and apply knowledge in new, diverse situations.

## Student Assessment

The assessment process can be a powerful tool for learning when students are actively involved in the process. Both assessment *of* learning and assessment *for* learning are essential. Reliable formative and summative assessments provide teachers with information they need to make informed instructional decisions that are more responsive to students' needs. Engagement empowers students to take ownership of their learning and builds confidence over time.

*Sound assessments:*

- align with learning goals
- vary in type and format
- use authentic performance tasks
- use criteria-scoring tools such as rubrics or exemplars
- allow teachers and students to track growth over time
- validate the acquisition of transferable knowledge
- give insight into students' thinking processes
- cause students to use higher level thinking skills
- address guiding questions and identified skills and processes
- provide informative feedback for teachers and students
- ask students to reflect on their learning

# High School Content Expectation Codes

To allow for ease in referencing expectations, each mathematics expectation has been coded by strand, standard, topic, and expectation. For example:

**S1.2**
- **A:** Algebra & Functions strand
- **A1:** Standard 1 of the Algebra and Functions strand
- **A1.2:** Topic 2 in Standard A1
- **A.1.2.3:** 3rd expectation in the 2nd topic of Standard A1

| STRAND 1<br>**Quantitative Literacy<br>and Logic (L)** | STRAND 2<br>**Algebra and Functions<br>(A)** |
|---|---|
| **STANDARDS** (and number of core expectations in each standard) ||
| **L1:** Reasoning About Numbers, Systems, and Quantitative Situations (9)<br><br>**L2:** Calculation, Algorithms, and Estimation (9)<br><br>**L3:** Measurement and Precision (5)<br><br>**L4:** Mathematical Reasoning, Logic, and Proof (10) | **A1:** Expressions, Equations, and Inequalities (16)<br><br>**A2:** Function (39)<br><br>**A3:** Mathematical Modeling (3) |
| *Recommended Quantitative Literacy and Logic Expectations (3)* | *Recommended Algebra and Functions Expectations (5)* |

| STRAND 3<br>**Geometry and Trigonometry<br>(G)** | STRAND 4<br>**Statistics and Probability<br>(S)** |
|---|---|
| **STANDARDS** (and number of core expectations in each standard) ||
| **G1:** Figures and Their Properties (29)<br><br>**G2:** Relationships Between Figures (10)<br><br>**G3:** Transformations of Figures in the Plane (5) | **S1:** Univariate Data— Examining Distributions (9)<br><br>**S2:** Bivariate Data— Examining Relationships (6)<br><br>**S3:** Samples, Surveys, and Experiments (3)<br><br>**S4:** Probability Models and Probability Calculation (4) |
| *Recommended Geometry and Trigonometry Expectations (3)* | *Recommended Statistics and Probability Expectations (6)* |

# Recommended Course Credit

The standards in this section, while not required for all students, are strongly recommended. These standards describe material to be taken after completion of the required High School Content Expectations. The knowledge and skills described in these standards will prepare students for postsecondary training, including apprenticeships and introductory college courses.

# Organization of this Document

In the Mathematics credit requirement documents, the expectations are organized by strand and standard. The organization in no way implies an instructional sequence. Curriculum personnel or teachers are encouraged to organize these topics and expectations in a manner that encourages connections between strands.

# Introduction to Statistics and Probability

Students encounter variability in their lives, in their science and social studies coursework, and in the news media. Study of the topics in statistics and probability gives students methods for summarizing data, introduces students to mathematical models for random phenomena, and provides the tools for decision making under uncertainty.

The concepts and techniques of statistics and probability should be introduced and used in the context of specific studies. Sometimes these are called "real-world" applications, although it is often useful to clean up data before presenting it to students so the essential concepts are not obscured by the complexities of the data. Wherever possible, technology (calculators or statistical software) should be used for computations and graphing. Simulation can give students deeper understanding of many probability and inferential concepts and should be used often.

# Statistics and Probability Goal Statement

*A course in statistics and probability beyond the core curriculum is built around four themes:*

- **Data exploration and study design.** Students explore and compare univariate and bivariate data sets using both graphical and numerical summary measures and learn principles of data collection and study design that aim to minimize bias and variability of resulting data.

- **Probability models and their application.** Students use discrete probability distributions as models for random systems and use conditional probability and Bayes' Theorem to solve applied problems. The normal distribution is studied and sampling distributions for means and proportions are developed based on the normal distribution.

- **Statistical inference.** Students learn the logic and terminology of confidence interval estimation and significance testing. They learn to apply these techniques to questions involving means and proportions in one and two sample settings, to categorical data, and to the simple linear regression model.

- **Model assessment.** Students learn to assess the validity of assumptions of statistical and probabilistic models and to assess the effect of departures from model assumptions.

## Statistics and Probability Standards Outline

STANDARD S1: EXPLORING UNIVARIATE AND BIVARIATE DATA

STANDARD S2: SAMPLING AND STUDY DESIGN

STANDARD S3: PROBABILITY MODELS

STANDARD S4: SAMPLING DISTRIBUTIONS

STANDARD S5: POINT AND INTERVAL ESTIMATION

STANDARD S6: SIGNIFICANCE TESTING

STANDARD S7: INFERENCE FOR REGRESSION

STANDARD S8: ASSESSING ASSUMPTIONS OF STATISTICAL MODELS

# CONTENT EXPECTATIONS FOR STATISTICS AND PROBABILITY

## S1    Exploring univariate and bivariate data

**S1.1**    Given the mean, variance, and standard deviation of a data set, compute the mean, variance, and standard deviation if the data undergo a linear transformation.

**S1.2**    Compute and interpret the within-group and between-group variation when comparing two or more data sets.

**S1.3**    Compute residuals and use plots of residuals to assess the adequacy of a simple linear regression model.

**S1.4**    Identify influential points in a bivariate data set and predict and verify the effect of their removal on the least-squares line.

**S1.5**    When applicable, use logarithmic and power transformations to achieve linearity and use the transformed data to make predictions.

**S1.6**    Explore categorical data via contingency tables, computing and interpreting marginal, joint, and conditional relative frequencies and examining measures of association.

## S2    Sampling and study design

**S2.1**    Describe the strengths and weaknesses of sampling methods, including simple random sampling, stratified random sampling, convenience sampling, voluntary response, and cluster sampling. Recognize potential difficulties in implementing each method.

**S2.2**    Critically assess the validity of conclusions drawn from surveys such as political polls, recognizing possible biases such as size bias and non-response bias and understanding the role of question formulation.

**S2.3** Know and recognize in context the concepts of treatment group, control group, and experimental unit and demonstrate the importance of double-blind protocol, random assignment, experimental unit, and replication.

**S2.4** Understand and describe how to implement completely randomized and randomized block designs (including matched-pair designs), recognizing when and how blocking can lower variability.

## S3 Probability models

**S3.1** Know the subjective and relative frequency interpretations of probabilities, including an informal understanding of the law of large numbers.

**S3.2** Use basic probability rules such as the addition rule, law of total probability, and complement rule to compute probabilities in a variety of models.

**S3.3** Use Bayes' Theorem to solve conditional probability problems, with emphasis on the interpretation of results.

**S3.4** Know the definition of random variable and be able to derive a discrete probability distribution based on the probability model of the original sample space.

**S3.5** Compute the expected value and standard deviation of discrete random variables and know the effect of a linear transformation of a random variable on its mean and standard deviation.

**S3.6** Apply standard discrete distributions, including the binomial, geometric, and hypergeometric.

**S3.7** Know the definition of independence of two discrete random variables and use the joint distribution to determine whether two discrete random variables are independent.

**S3.8**    Use tables and technology to determine probabilities and percentiles of normal distributions.

**S3.9**    Use simulation methods to answer questions about probability models that are too complex for analytical treatment at this level, e.g., interacting particle system models.

## S4    Sampling Distributions

**S4.1**    Given the mean and standard deviation of each random variable in a set of random variables, compute the mean of the sum and, assuming independence, compute the variance and standard deviation of the sum.

**S4.2**    Know an informal statement of the Central Limit Theorem and understand its relevance to sampling distributions.

**S4.3**    Assuming a normal model or the applicability of the Central Limit Theorem, compute probabilities for the sample mean, including probabilities that are needed to compute $p$-values.

**S4.4**    Apply the (large sample) distribution of the sample proportion to compute probabilities for the sample proportion and know and use rules of thumb for the applicability of the large sample distribution.

**S4.5**    Assuming a normal model or the applicability of the Central Limit Theorem, derive a P% confidence interval for the mean under the assumption that the population standard deviation is known.

**S4.6**    Compute control limits for commonly used control charts and use these to assess whether a process is out of control.

## S5    Point and interval estimation

**S5.1**    Compute bias, variance, and mean squared error of estimators of the mean and proportion.

**S5.2**    Know the logic of confidence intervals, the meaning of confidence level, and the effect of changing sample size, confidence level, and variability on the width of the interval.

**S5.3**    Compute and interpret confidence intervals for one mean and for the difference between two means (in both the paired and unpaired setting) when the standard deviation is unknown, using the t distribution.

**S5.4**    Compute and interpret (large sample) confidence intervals for one proportion and the difference between two proportions using the normal distribution.

**S5.5**    Compute the sample size required for a fixed confidence level and interval width for confidence intervals for means and proportions.

## S6    Significance testing

**S6.1**    Know the terminology and logic of significance testing, including null and alternative hypotheses, $p$-value, Type I and Type II errors, and power.

**S6.2**    Assuming a normal model and known standard deviation, carry out a significance test for a single mean, with emphasis on understanding the computation and interpretation of the p-value, and compute the power curve of a test.

**S6.3**    Carry out (large sample) significance tests for one proportion and the difference of two proportions, with emphasis on proper interpretation of results.

**S6.4**    Carry out significance tests for one mean and the difference of two means (paired and unpaired) using the $t$ distribution, with emphasis on proper interpretation of results.

**S6.5**    Carry out chi-squared significance tests of homogeneity, independence, and goodness-of-fit, with emphasis on proper interpretation of results.

**S6.6**    Assuming a normal model and known standard deviation, compute the sample size necessary to achieve a pre-specified power at a pre-specified value of the population mean.

**S6.7**    Demonstrate, in the context of specific studies, the understanding that a result can be statistically significant while of insignificant practical importance and that a failure to reject a null hypothesis may be due to low power and does not necessarily imply the null hypothesis is true.

## S7    Inference for regression

**S7.1**    Know the statistical model for regression, including linearity, normality of errors, and constancy of error variance.

**S7.2**    Compute and interpret a confidence interval for the slope of a regression line using the $t$ distribution.

**S7.3**    Test hypotheses about the slope of a regression line, with emphasis on interpretation of results.

## S8    Assessing assumptions of statistical models

**S8.1**    Demonstrate knowledge of the assumptions required for all of the inferential procedures (confidence intervals and significance tests).

**S8.2**    In the context of specific studies, recognize aspects of study design that either support or offer evidence against required assumptions.

**S8.3**    Demonstrate knowledge of the possible effects of incorrect assumptions (i.e., improperly specified models) on inferential procedures and of the robustness of inferential procedures to departures from specified assumptions.

**S8.4** Show in context an understanding that statistical models are approximations to reality and that care should be exercised in assigning too much precision to measures such as confidence levels or $p$-values.

# NOTES

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