TECHNICAL REPORT NUMBER ONE

# STATE OF MICHIGAN

DEPARTMENT OF CONSERVATION
P. J. HOFFMASTER, Director

GEOLOGICAL SURVEY DIVISION
G. E. EDDY, State Geologist

# GROUND WATER HYDRAULICS AS A GEOPHYSICAL AID

1948

BY
JOHN G. FERRIS

PREPARED IN COOPERATION WITH
THE UNITED STATES DEPARTMENT OF THE INTERIOR
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Ground-water hydraulics as a geophysical aid[1]

By John G. Ferris[2]

The publication of the non-equilibrium formula in 1935 in a paper by Theis /3 marked the opening of a new era in the analysis and understanding of the hydraulics of percolating ground waters. Through the past decade, an ever-increasing number of engineers and geologists have become familiar with the application of this formula to practical problems of ground-water flow and have tested it in the field, against precise observations, under controlled conditions. Although the highly idealized aquifer assumed for the derivation of this formula is not of widespread occurrence in the field, we gain increasing confidence in the use of the Theis method as our backlog of proven data accumulates until we now look askance at test data which do not conform to this theory. In many cases, careful study of these anomalous data will reveal the means for estimating the degree or manner in which an observed aquifer diverges from the idealized aquifer.

The Theis equation for the discharge of a well in an areally infinite aquifer may be written

$$s = \frac{114.6\ Q}{T} \int_u^\infty \frac{e^{-u}}{u}\ du = \frac{114.6\ Q}{T} \cdot W(u) \qquad (1)$$

where $W(u) = \int_u^\infty \frac{e^{-u}}{u}\ du$ and $u = \frac{1.87\ r^2 S}{Tt}$ (2)

/1 Published by permission of the Director of the U. S. Geological Survey.

/2 District Engineer, Ground-Water Division, U. S. Geological Survey.

/3 Theis, C. V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans. 1935, pp. 519-524.

s = drawdown or recovery at any point in the cone of influence, in feet

Q = discharge of pumped well, in gallons per minute

T = coefficient of transmissibility, in gallons per day per foot

S = coefficient of storage, as a ratio or decimal

r = distance of observation point from pumped well, in feet

t = time since pumping started, in days

It is assumed in the derivation of this formula that (1) the aquifer is homogeneous and isotropic; (2) the discharging well completely penetrates the formation and is of infinitesimal diameter; (3) the coefficient of transmissibility is constant at all places and at all times; (4) the aquifer is bounded by impermeable strata above and below; (5) the coefficient of storage is constant and water is released from storage instantaneously with a decline in head; (6) flow is laminar and radial or uni-dimensional; and (7) the aquifer is of infinite areal extent.

Although most aquifers are more or less heterogeneous in character, it seems probable that hydraulic interconnection throughout the aquifer will result in continual adjustment of flow between regions of different permeability and trend toward a steady-state cone of depression, which reflects the average and over-all transmissibility of the aquifer. The general and widespread agreement between field data and theory serves to confirm this proposition of aquifer behavior. Research in progress by the U. S. Geological Survey includes the collection of experimental evidence from groups of observation wells which are finished at different depths within an aquifer, to determine the effect of vertical variation in permeability.

The distances from observation well to discharging well are generally so great in relation to the small diameter of the pumping wells that the condition of

infinitesimal diameter for the discharging well is usually satisfied. The case of partial penetration has been analyzed by Jacob [4] who demonstrated that proper spacing of observation wells can minimize the effect of vertical-flow components.

Variations in the coefficient of transmissibility caused by a decrease in the thickness of the aquifer, as a result of extensive dewatering, may be compensated by a method developed by Jacob [4]. Changes in aquifer thickness which result from slopes of the upper or lower confining bed will cause variations in the transmissibility and may void the use of the non-equilibrium method. Although this case has not been analyzed to date, it seems probable that appropriate correction could be made through the use of a variable transmissibility which is proportioned to the variation in aquifer thickness, or by the use of a system of partial image wells.

If the overlying and underlying confining beds or aquicludes bounding the aquifer are not impermeable, correction for leakage from these beds may be made by a method recently derived by Jacob [5].

The assumption of instantaneous release of water from storage with a decline in head is generally satisfied in artesian aquifers. In the case of water-table aquifers, the slow draining of interstitial water is not in accord with this assumption. However, after a sufficient period of time the observational data will satisfy the non-equilibrium formula, and accordingly the Theis method may by applied to the water-table aquifer if proper allowance is made for the drainage-lag effect.

[4] Jacob, C. E., Notes on determining permeability by pumping tests under water-table conditions, U. S. Geological Survey mimeographed report, June 1944.

[5] Jacob, C. E., Radial flow in leaky artesian aquifer: Am. Geophys. Union Trans., pp. 198-208, Apr. 1946.

The condition of laminar flow is generally satisfied with the possible exceptions of limited regions in the immediate vicinity of the face of a discharging well or in large solution cavities or channels. The limiting assumption of one dimensional flow imposes the greatest restriction on the formula, but this condition may be fulfilled by proper choice of observation wells and pumping test sites. If observations are based on wells located at distances from the discharging well where the flow lines are parallel and within the region where the flow is steady, the vertical flow components may be discounted for most problems which involve partially penetrating wells or surface sources and sinks of limited penetration.

The remaining condition, that of infinite areal extent, is fulfilled for most practical purposes in the case of a few major aquifers of sedimentary rocks, such as the Dakota sandstone described by Meinzer [6]. In most areas, however, the existence of boundaries of the formation or of folds and faults, or dissection by surface streams, serves to limit the continuity of consolidated strata to distances measurable in miles or tens of miles. In the unconsolidated materials and particularly in the glaciated areas the prerequisite of infinite areal extent is seldom satisfied by most aquifers. As in the case of the previous assumptions, it is necessary to make appropriate adjustment for the effect of geologic boundaries before the formula can be applied to areally limited aquifers. The mathematical treatment of an abrupt discontinuity in a conductor was analyzed by Lord Kelvin in his work on electrical theory, by use of the method of images. So far as known, the first published application of the image method to problems of ground-water flow was made by Muskat [7] in 1937.

[6] Meinzer, O. E., and Hard, H. A., The artesian-water supply of the Dakota sandstone in North Dakota, with special reference to the Edgeley quadrangle: U. S. Geol. Survey Water-Supply Paper 520, pp. 73-95, 1925.

[7] Muskat, Morris, The flow of homogeneous fluids through porous media, McGraw-Hill Book Co., Inc., 1937.

To demonstrate the use of the image method in problems of ground-water hydraulics, an idealized section of an aquifer with a single boundary is shown by figure 1. The sand and gravel aquifer, which occupies the left side of the diagram, is bounded to the right and is underlain by an impervious shale. The vertical dashed line marks the position of the idealized and abrupt boundary, which is equivalent to the actual sloping boundary. The substitution of a vertical line for the actual sloping boundary introduces little or no error for wells some distance from the boundary, but we may expect some deviation between the computed and observed behavior of wells close to the boundary.

Inasmuch as the shale forming the boundary is impervious and yields no water to the aquifer, the physical condition to be satisfied is that there shall be no flow across the boundary. This boundary limit can be duplicated by replacing the real aquifer with an imaginary aquifer of infinite extent and adding an imaginary discharging well, which is an image of the real well and is located at an equal distance from but on the opposite side of the boundary. As shown by the central diagram in figure 1, the image well produces a drawdown of water level at the boundary which is equal to the effect of the real well. The net result is the development of a water-table divide, everywhere along the boundary. As no flow can occur across a divide, the substitute image solution satisfies the physical boundary condition and is therefore a solution of the original problem. The drawdown cone for the real aquifer is the composite obtained by adding the real components of the drawdown cones for the real well and the image well and the trace of this resultant cone is shown in the lower part of figure 1.

The case of two impermeable boundaries, although somewhat more complex, can be solved in a very similar manner. The setup of the primary images for this case is shown in figure 2, the drawdown cones being shown as solid lines in the region

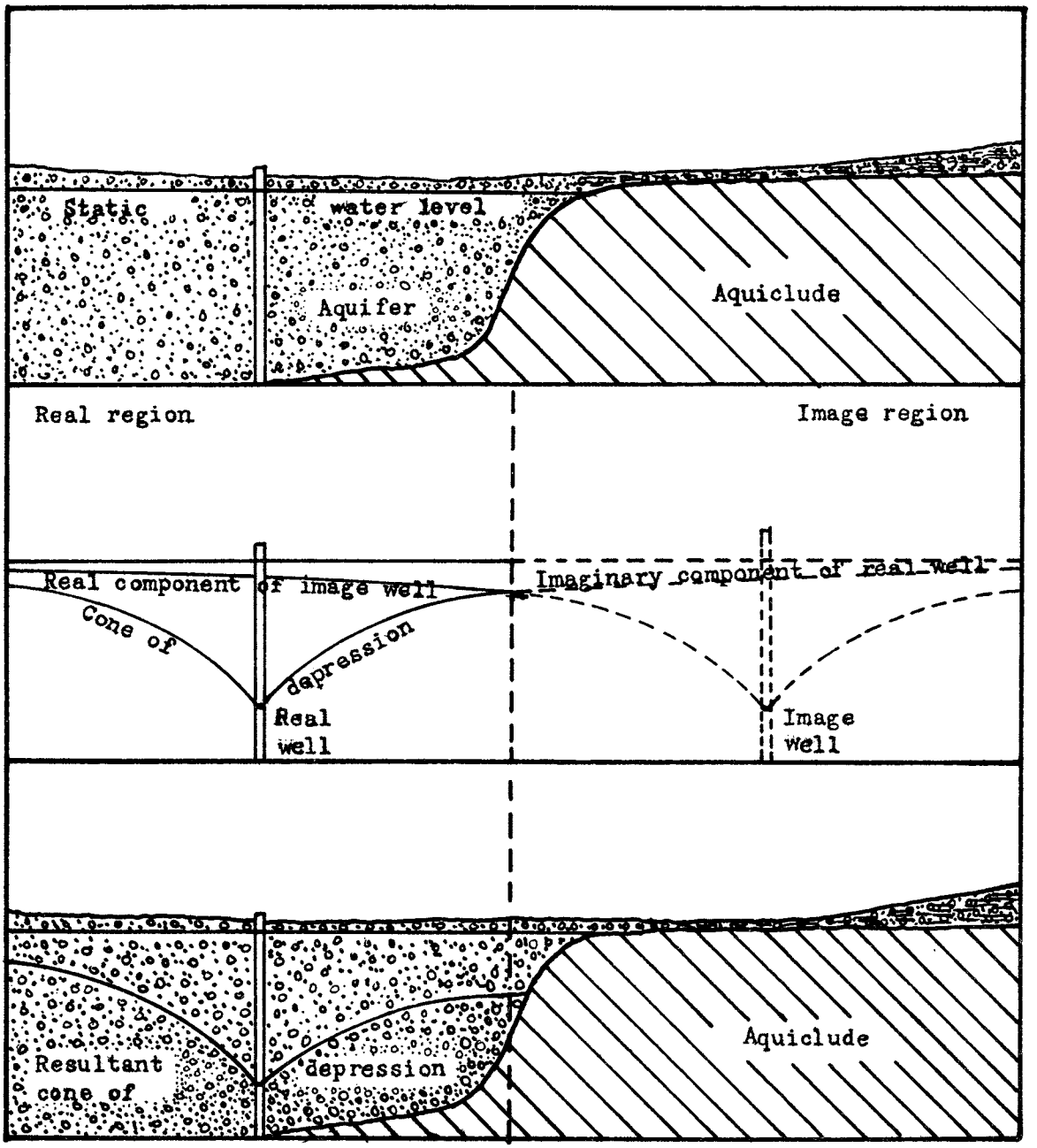



Fig. 1 Idealized section of aquifer bounded by impermeable formation and setup of hypothetical well system used for solution of flow problems under this condition.

of real values and as dashed lines in the region of imaginary values. Note that although these primary images balance the drawdown effect of the real well at their respective boundaries, each image produces an unbalanced drawdown at the farther boundary. Inasmuch as these unbalanced drawdowns at the boundaries theoretically would produce a gradient and consequent flow across the boundary, it is necessary to add a secondary set of image wells, at the appropriate distances, to compensate for the effect of the primary images. Each well of the secondary image set in turn disturbs the balance at the farther boundary and all successive sets of images to infinity leave residuals at the boundaries. In practice it is necessary only to add image pairs until the residual effects are negligible in comparison to the total effect.

In demonstrating the use of the image-well method in conjunction with the Theis formula for the solution of ground-water problems in bounded aquifers, it is more convenient to work with Jacob's /8 approximation of the non-equilibrium formula. The exponential integral in equation (1) can be evaluated by the following series

$$\int_u^\infty \frac{e^{-u}du}{u} = W(u) = -0.5772 - \log_e u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \cdots \qquad (3)$$

From equation (2) it may be noted that u will be small for sufficiently small values of $r^2/t$. Under this condition the terms beyond $\log_e u$, in the above series, may be omitted with little error. There follows, then, the modified equation as given by Jacob.

$$s = \frac{114.6\ Q}{T} \left[\log_e (1/u) - 0.5772\right] \qquad (4)$$

$$s = \frac{114.6\ Q}{T} \left[\log_e (Tt/1.87r^2S) - 0.5772\right] \qquad (5)$$

/8 Jacob, C. E., Drawdown test to determine effective radius of artesian wells, Am. Soc. Civil Engrs. Proc., vol. 72, No. 5, pp. 629-646, May 1946.

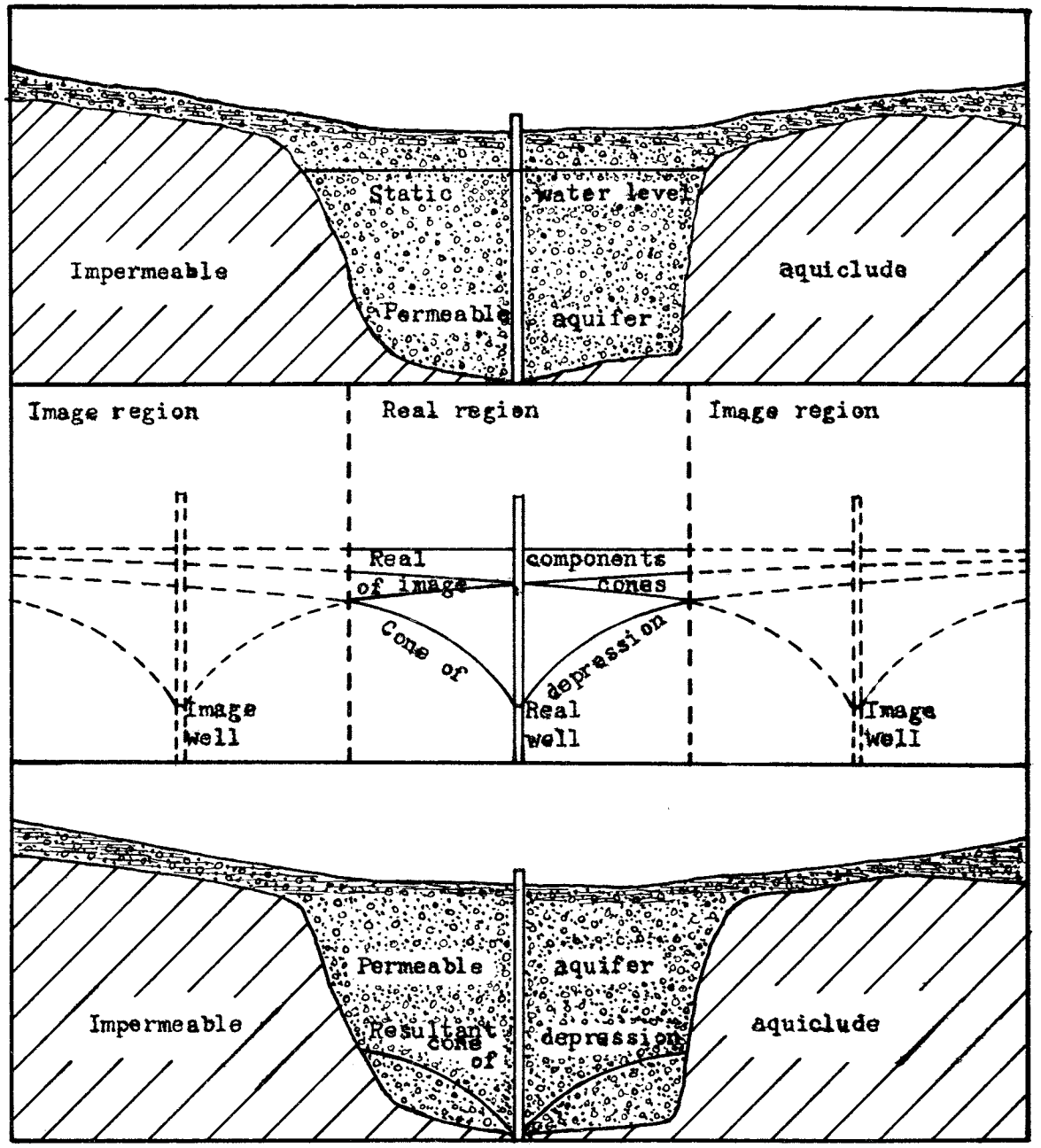



Fig. 2 Idealized section of under flow channel bounded by impermeable formation and setup of hypothetical well system used for solution of flow problems under this condition.

A plot of equation (4) in dimensionless form is shown as figure 3. The intercept of the straight-line plot on the $s = 0$ axis yields the basis for determining S, the coefficient of storage.

For when $s = 0$, from equation (5)

$$\log_e (Tt/1.87r^2S) = 0.5772$$

$$Tt/1.87r^2S = e^{0.5772}$$

$$S = Tt/1.87r^2e^{0.5772} = 0.3\ Tt/r^2 \tag{6}$$

For two observation wells at distances $r_1$ and $r_2$ or for one observation well at distances $r_1$ and $r_2$ from two discharging wells there follows from equation (6)

$$S = 0.3\ Tt_1/r_1^2 = 0.3\ Tt_2/r_2^2$$

$$t_1/r_1^2 = t_2/r_2^2 \tag{7}$$

The determination of the time intercept by extrapolating the semi-log plot to the zero-drawdown axis may involve appreciable error if the observational data are dispersed and if the slope of the semi-log plot is small, because the intercept is poorly defined for small slopes. In addition, the intercept generally occurs at very small values of time and consequently small deviations in the intercept locus result in large variations in the time quantity.

The following method was devised to avoid the disadvantages of the intercept method. Assume two observation wells at distances $r_1$ and $r_2$ from the discharging well and from equation (5) the drawdown in each well is given as

$$s_1 = 114.6\ Q/T \left[\log_e(Tt_1/1.87\ r_1^2\ S) - 0.5772\right]$$

$$s_2 = 114.6\ Q/T \left[\log_e(Tt_2/1.87\ r_2^2\ S) - 0.5772\right]$$

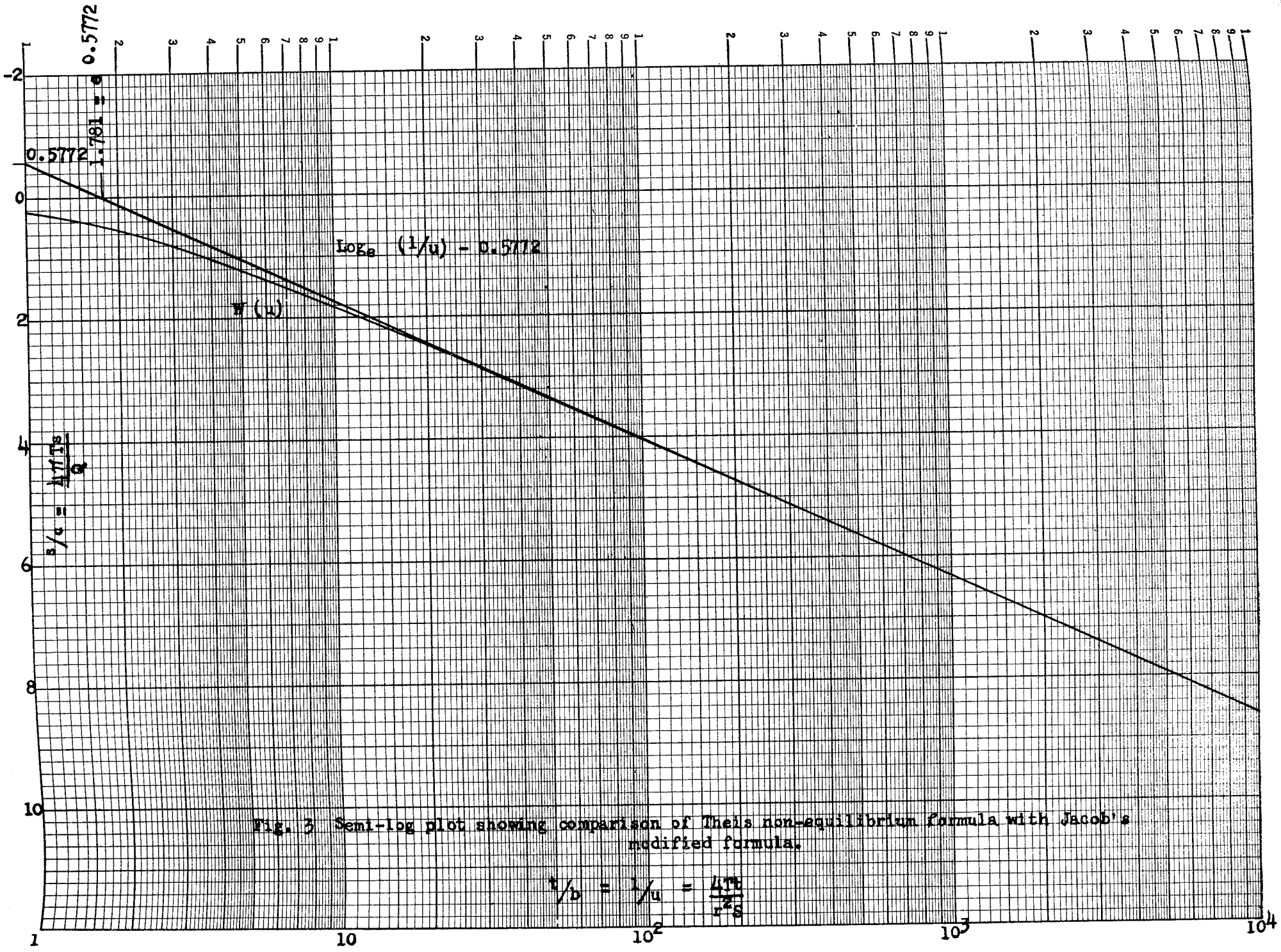



Fig. 3 Semi-log plot showing comparison of Theis non-equilibrium formula with Jacob's modified formula.

$$t/b = 1/u = \frac{4Tt}{r^2 S}$$

Note from the semi-log plot for one well, the value of the time for a particular value of s and from the plot of the second well, select the time value at the same value of s. When $s_1 = s_2$ there follows

$$\frac{114.6\ Q}{T}\left[\log_e \frac{Tt_1}{1.87\ r_1^2\ S} - 0.5772\right] = \frac{114.6\ Q}{T}\left[\log_e \frac{Tt_2}{1.87\ r_2^2\ S} - 0.5772\right]$$

whence

$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} \qquad (8)$$

It is evident from the above relation that the time of occurrence of equal drawdown in each observation well is related to its distance from the discharging well in the same manner as the time intercepts on the axis of zero drawdown.

From the form of equations (7) and (8) it is noted that for a given aquifer the time intercepts at zero drawdown or at equal drawdown vary directly as the square of the distance from the pumped well to the observation well and are independent of the rate of pumping. This statement may be expressed in the form

$$t_1/r_1^2 = t_2/r_2^2 = t_3/r_3^2 = \ldots\ldots\ldots\ldots\ldots\ t_n/r_n^2 \qquad (9)$$

The form of equation (9) is equivalent and the principle stated is analogous to the law of times defined by Ingersoll and Zobel/9.

In applying equation (5) to observations of the rate of drawdown of water level in a single observation well, we have at time $t_1$ :

$$s_1 = 114.6\ Q/T\left[\log_e (Tt_1/1.87\ r^2S) - 0.5772\right]$$

at time $t_2$ :

$$s_2 = 114.6\ Q/T\left[\log_e (Tt_2/1.87\ r^2S) - 0.5772\right]$$

/9 Ingersoll, L. R., and Zobel, O. J., Introduction to the mathematical theory of heat conduction, Ginn and Co., p. 78, 1913.

Then $s_2 - s_1 = 114.6\ Q/T \left[\log_e t_2 / t_1\right]$ (10)

Converting to logarithms to the base 10

$$s_2 - s_1 = 264\ Q/T \left[\log_{10} t_2 / t_1\right] \quad (11)$$

A semi-log plot of the drawdown or recovery of water level for each observation well is made by plotting the drawdown s versus the logarithm of the time, t. From this plot an arbitrary choice of $t_1$ and $t_2$ can be made and the corresponding $s_1$ and $s_2$ values are noted. For convenience $t_1$ and $t_2$ may be chosen one log cycle apart, then $\log_{10}\left[t_2 / t_1\right] = 1$ and

$$s_2 - s_1 = \Delta s = 264\ Q/T$$

$$\text{or} \qquad T = 264\ Q/\Delta s \quad (12)$$

From the form of equation (12) it is noted that for a given aquifer the rate of drawdown per log cycle, or the slope of the semi-log plot, $\Delta s$, is dependent only on Q, the rate of pumping.

If a geologic boundary is also a hydraulic boundary, as is often the case, then the above methods may be used to determine its location. For purpose of illustration, a brief summary is presented of the results obtained from a pumping test conducted in an underflow conduit near Flint, Michigan. A plot of the test well set-up is shown in figure 4. The discharging well was pumped for several days at a constant rate and periodic observations were made of the water-level decline in the observation wells. The semi-log plots of the observed water-level data are shown as figures 5 to 10, inclusive, and it may be noted that each plot shows a tri-partite sequence. The slope of each subsequent limb of the plot is related to the original limb by an integral multiple. The second limb declines at twice the rate of the first limb, as would be expected from the addition of

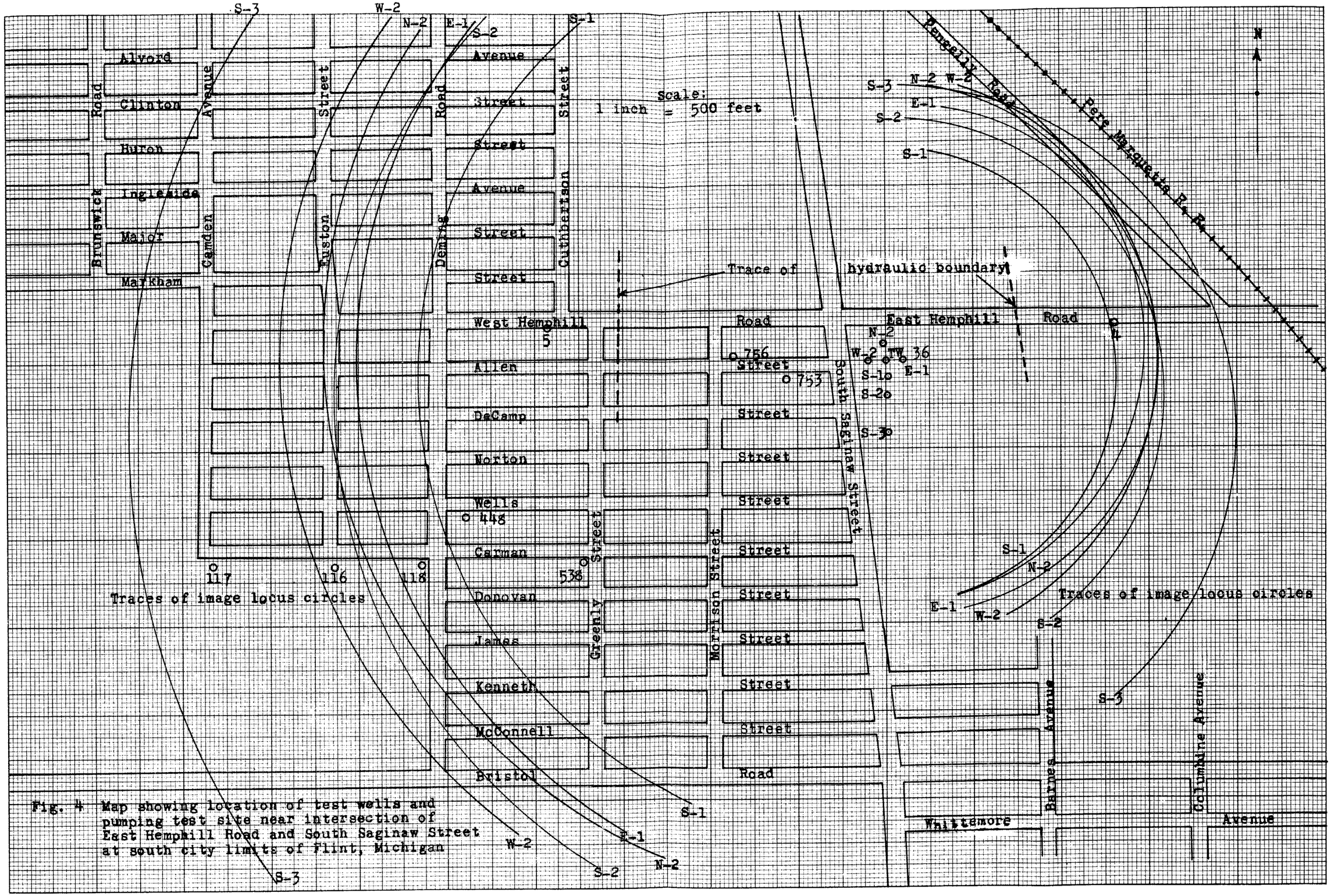
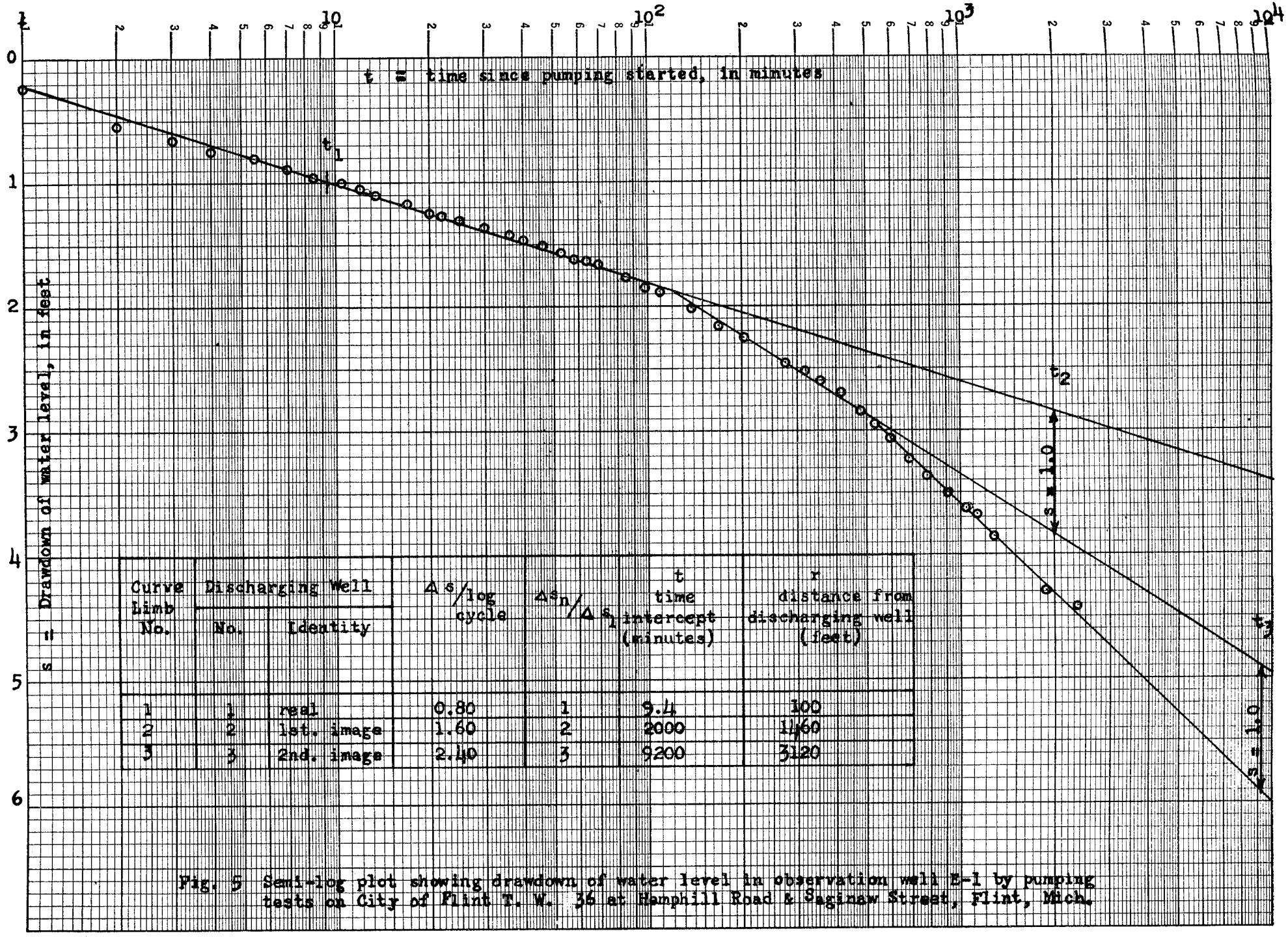
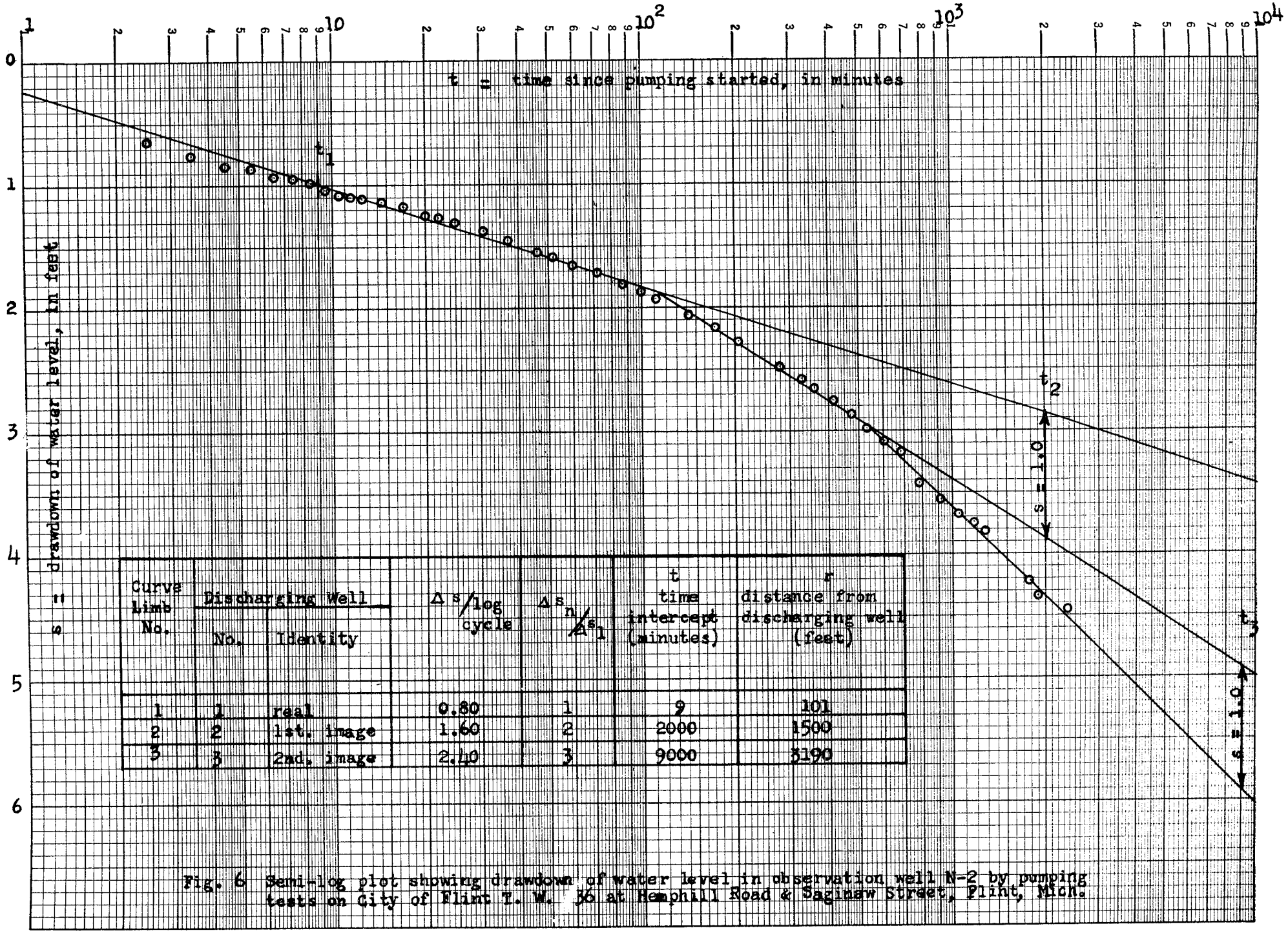



Fig. 4 Map showing location of test wells and pumping test site near intersection of East Hemphill Road and South Saginaw Street at south city limits of Flint, Michigan



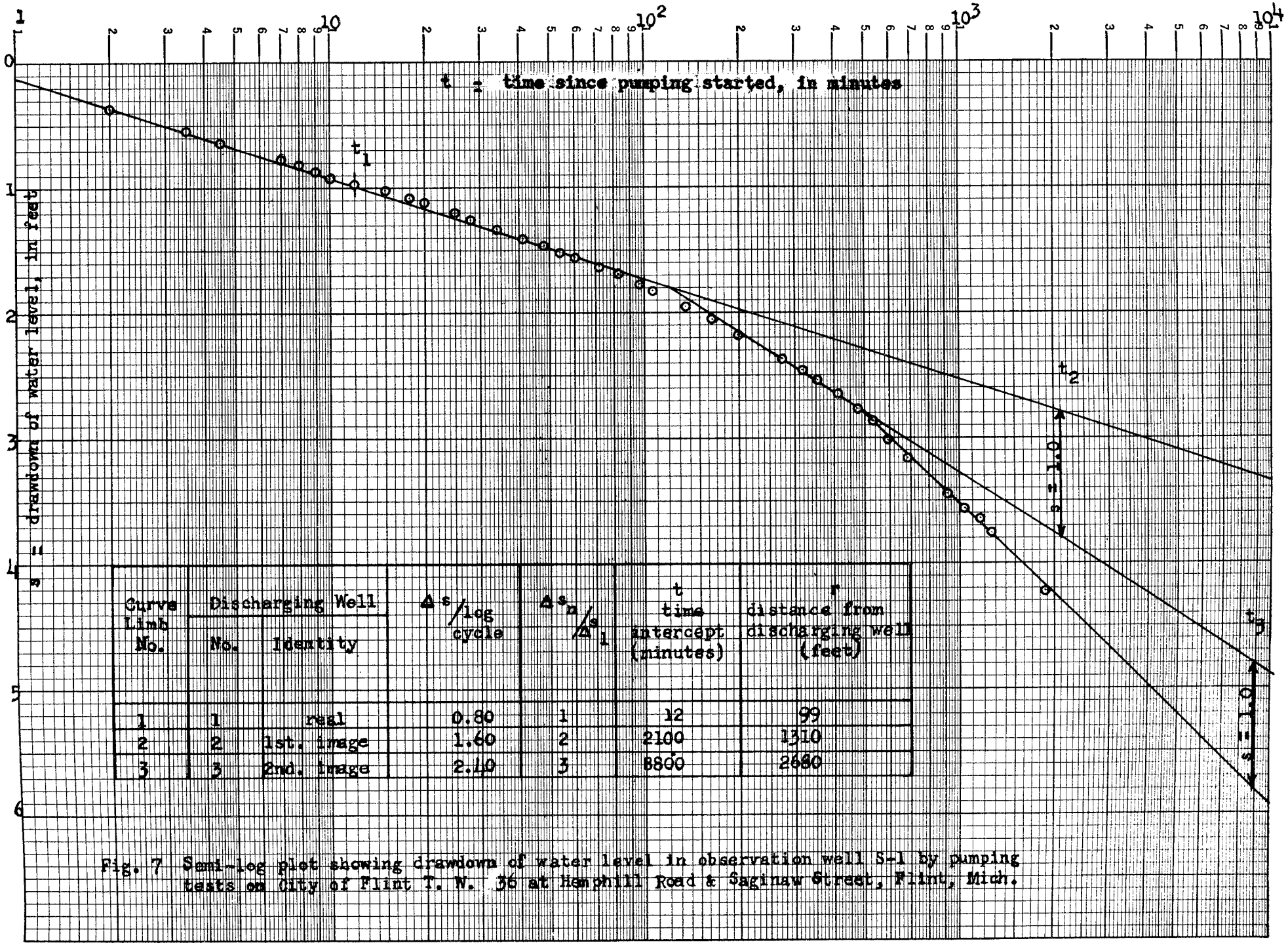

| Curve Limb No. | Discharging Well No. | Discharging Well Identity | $\Delta s$/log cycle | $\Delta s_n/\Delta s_1$ | t time intercept (minutes) | r distance from discharging well (feet) |
|---|---|---|---|---|---|---|
| 1 | 1 | real | 0.80 | 1 | 9.4 | 100 |
| 2 | 2 | 1st. image | 1.60 | 2 | 2000 | 1460 |
| 3 | 3 | 2nd. image | 2.40 | 3 | 9200 | 3120 |

Fig. 5 Semi-log plot showing drawdown of water level in observation well E-1 by pumping tests on City of Flint T. W. 36 at Hamphill Road & Saginaw Street, Flint, Mich.



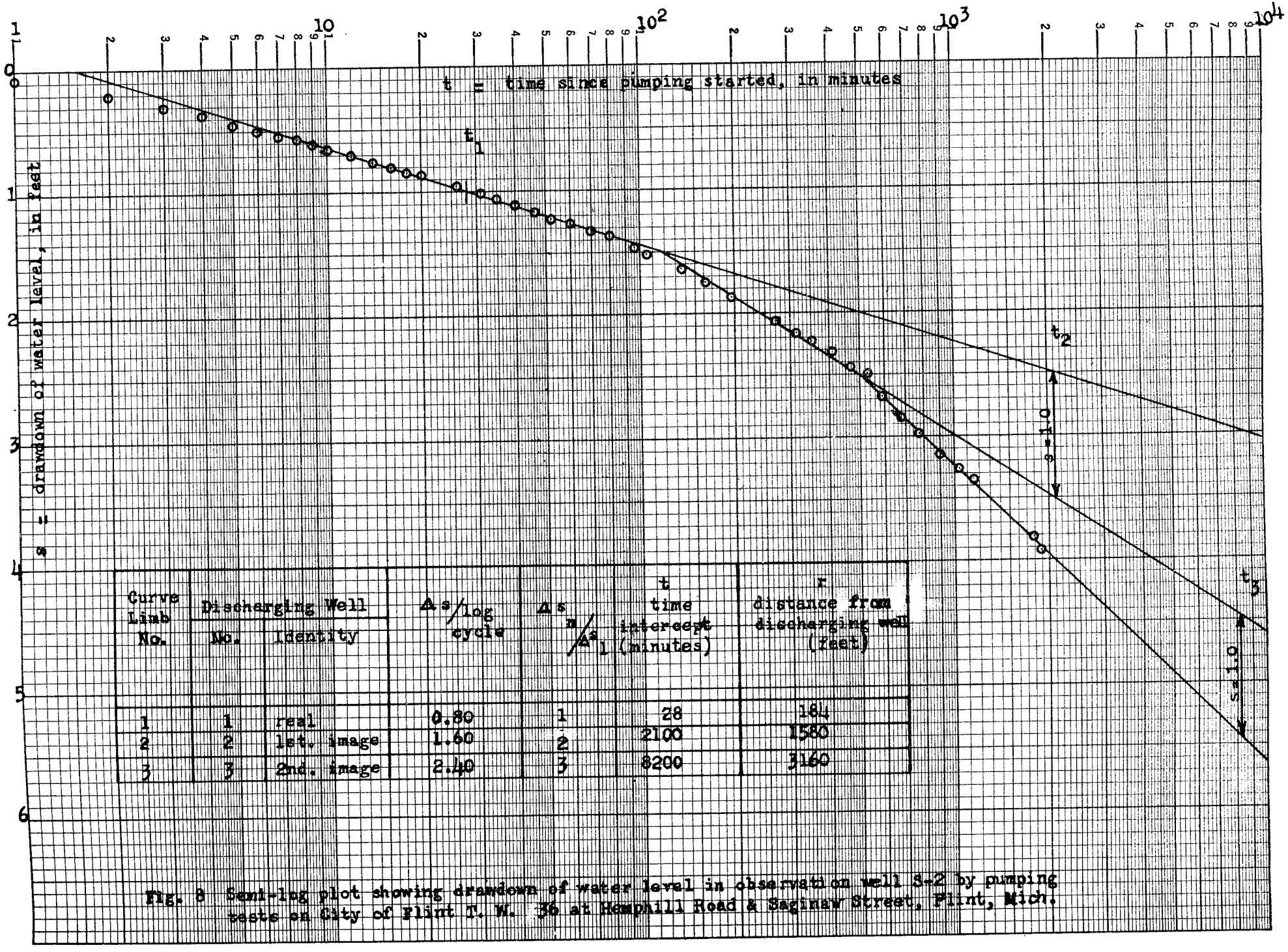

| Curve Limb No. | Discharging Well No. | Discharging Well Identity | $\Delta s$/log cycle | $\Delta s_n / \Delta s_1$ | t time intercept (minutes) | r distance from discharging well (feet) |
|---|---|---|---|---|---|---|
| 1 | 1 | real | 0.80 | 1 | 9 | 101 |
| 2 | 2 | 1st. image | 1.60 | 2 | 2000 | 1500 |
| 3 | 3 | 2nd. image | 2.40 | 3 | 9000 | 3190 |

Fig. 6 Semi-log plot showing drawdown of water level in observation well N-2 by pumping tests on City of Flint T. W. #6 at Hemphill Road & Saginaw Street, Flint, Mich.



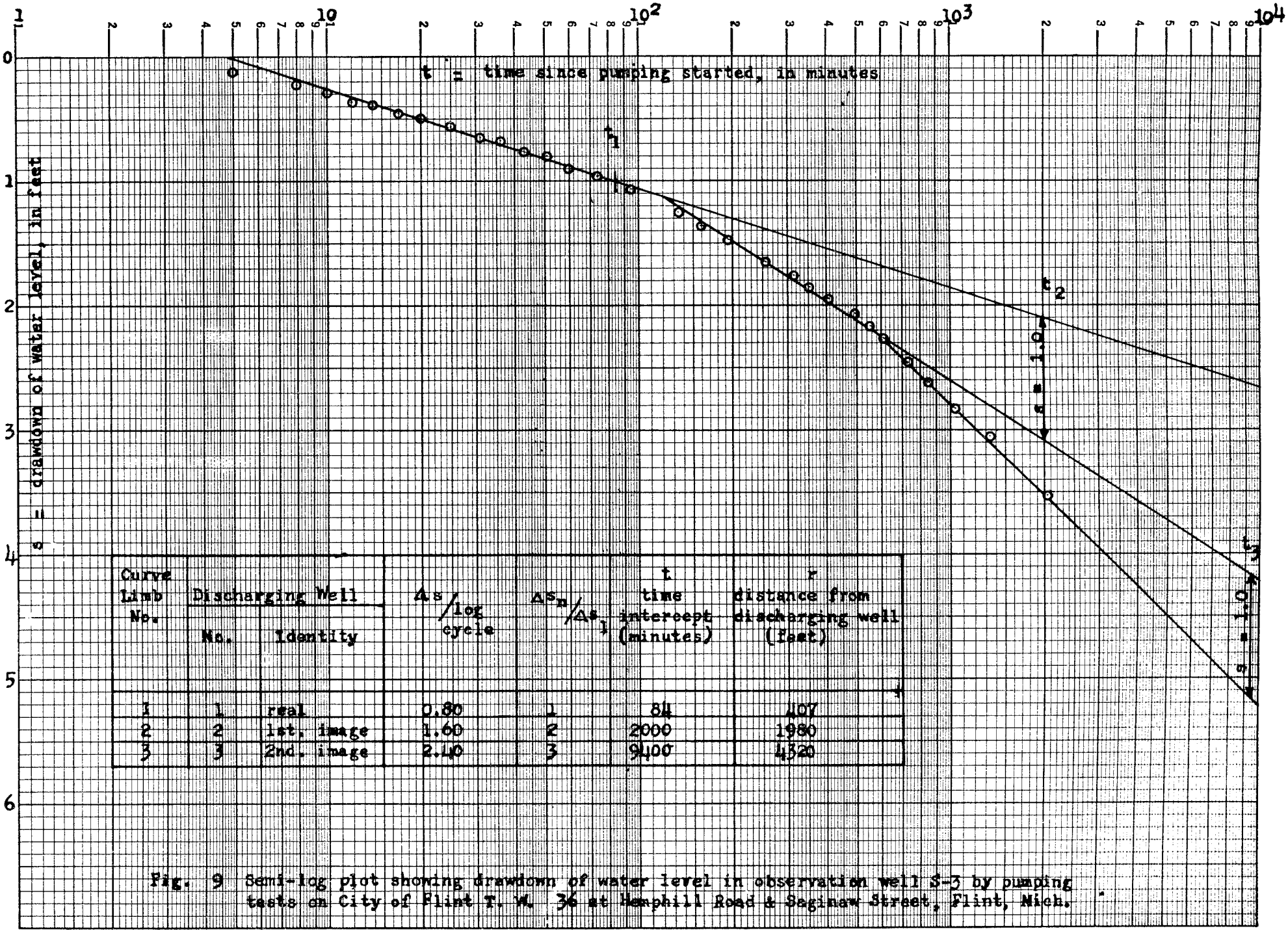

| Curve Limb No. | Discharging Well | | $\Delta s$ / log cycle | $\Delta s_n / \Delta s_1$ | t time intercept (minutes) | r distance from discharging well (feet) |
|---|---|---|---|---|---|---|
| | No. | Identity | | | | |
| 1 | 1 | real | 0.80 | 1 | 12 | 99 |
| 2 | 2 | 1st. image | 1.60 | 2 | 2100 | 1310 |
| 3 | 3 | 2nd. image | 2.40 | 3 | 8800 | 2680 |

Fig. 7 Semi-log plot showing drawdown of water level in observation well S-1 by pumping tests on City of Flint T. W. 36 at Hemphill Road & Saginaw Street, Flint, Mich.



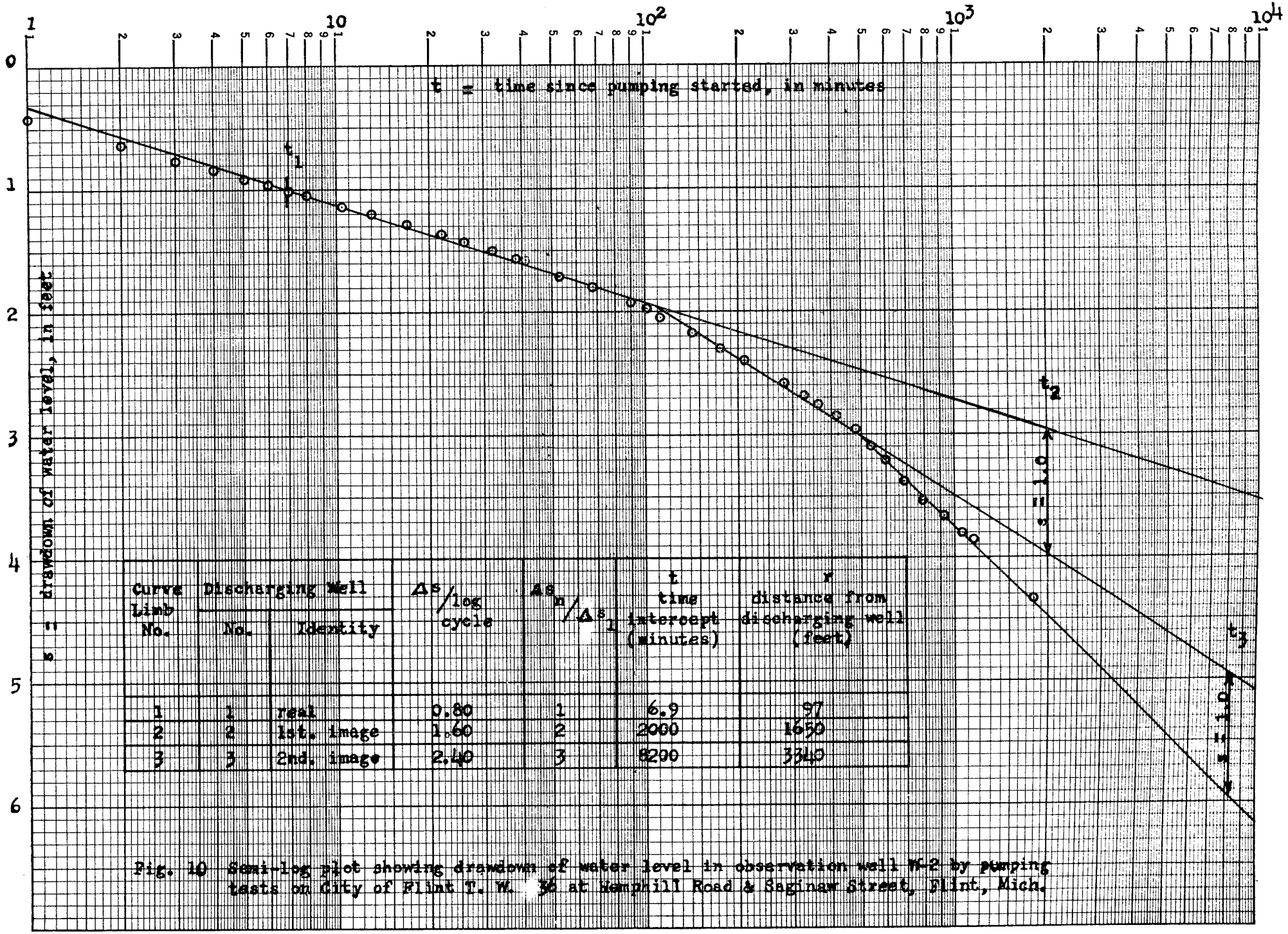

| Curve Limb No. | Discharging Well No. | Discharging Well Identity | $\Delta s$/log cycle | $\Delta s_n / \Delta s_1$ | t time intercept (minutes) | r distance from discharging well (feet) |
|---|---|---|---|---|---|---|
| 1 | 1 | real | 0.80 | 1 | 28 | 184 |
| 2 | 2 | 1st. image | 1.60 | 2 | 2100 | 1580 |
| 3 | 3 | 2nd. image | 2.40 | 3 | 8200 | 3160 |

Fig. 8 Semi-log plot showing drawdown of water level in observation well S-2 by pumping tests on City of Flint T. W. 36 at Hemphill Road & Saginaw Street, Flint, Mich.




| Curve Limb No. | Discharging Well | | $\Delta s$ / log cycle | $\Delta s_n / \Delta s_1$ | t time intercept (minutes) | r distance from discharging well (feet) |
|---|---|---|---|---|---|---|
| | No. | Identity | | | | |
| 1 | 1 | real | 0.80 | 1 | 84 | 407 |
| 2 | 2 | 1st. image | 1.60 | 2 | 2000 | 1980 |
| 3 | 3 | 2nd. image | 2.40 | 3 | 9400 | 4320 |

Fig. 9 Semi-log plot showing drawdown of water level in observation well S-3 by pumping tests on City of Flint T. W. 36 at Hemphill Road & Saginaw Street, Flint, Mich.




| Curve Limb No. | Discharging Well No. | Discharging Well Identity | $\Delta s$ / log cycle | $\Delta s_n / \Delta s_1$ | t time intercept (minutes) | x distance from discharging well (feet) |
|---|---|---|---|---|---|---|
| 1 | 1 | real | 0.80 | 1 | 6.9 | 97 |
| 2 | 2 | 1st. image | 1.60 | 2 | 2000 | 1650 |
| 3 | 3 | 2nd. image | 2.40 | 3 | 8200 | 3340 |

Fig. 10 Semi-log plot showing drawdown of water level in observation well W-2 by pumping tests on City of Flint T. W. 36 at Hemphill Road & Saginaw Street, Flint, Mich.

one complete image of the discharging well. In turn, the slope of the third limb is three times as great as the slope of the first limb, because the addition of the second image well raises the total withdrawal rate to three times its original value.

Each limb on the semi-log plots is extrapolated to permit measurement of the departure between the several limbs. The time values used for computing the image distances are determined from the graph by noting the time required for the first limb to reach a given drawdown and the total time elapsed when the departure between limbs reaches the same drawdown value. These intercepts are noted on each graph and the corresponding image calculations are shown.

As previously mentioned, the use of Jacob's approximate method should be restricted to small values of u which occur when the distance r is small and the time t is large. Generally, image distances are relatively large compared to the distance from an observation well to a pumping well. If more than one image is involved, it may be found that the distance from an observation well to each of several image wells may be of the same general order. For this condition the semi-log plot of drawdown versus time will record an initial straight-line segment under the influence of the pumping well. When the effect of the first image well reaches the observation well the observed data will follow a curved path through the region where the values of u are large. As t becomes sufficiently large relative to the value of $r^2$ the value of u for the image well becomes small until the observed data once again follows a straight-line path. Thus, if only a single image well is involved, a two-limbed plot will result.

If more than one image well occurs and if the image wells are at comparable distances then the effect of second, third, or higher-order image wells may reach an observation well before sufficient time has elapsed for u to become small

enough to warrant the application of this method to the effect of the first image well. Under these circumstances the observed data trends along a curved path. This was the case in the Flint pumping test which is used herewith for illustration.

It is possible to secure approximate values for the location of the image wells by drawing tangents to the observed-data plot at the appropriate slope value. The intersection of these tangents gives a first approximation for the image distance. Inasmuch as the observed data does not reach the full slope value for any given image before the effect of a higher-order image occurs, it may be shown that the time intercept determined by the tangent method is too small or the image distance is short of the correct value.

The image distances obtained by this first approximation may be increased a small and arbitrary amount and used as the basis for computing the drawdown at each observation well. Compare the computed curve with the observed data and estimate what additional change in the image distance will be required to effect agreement. Through successive trials in this manner the location of the image wells and boundaries may be determined. To avoid undue complication in the presentation of this method, the above refinement has been omitted and thus any data computed on the basis of the preliminary image locations shown by figure 4 will not check the observed-data plots.

The image distance from each semi-log plot is circumscribed on figure 4, the center of the circle being located at the appropriate observation well. The intersection of the arcs theoretically pinpoints the locus of the image well.

However, the deviations of the real aquifer from the vertically-bounded aquifer which is assumed in using the method of images results in a dispersion

of the arcs and their intersections. As shown by figure 4, most of the arc intersections for the east image well occur within a narrow band which intersects East Hemphill Road southwest of Pengelly Road. The concentration of arc intercepts for the east image infers that the east boundary is quite abrupt. With only a few arc intersections and with the wide dispersion of arcs the locus of the west image is placed at the center of the arc band, where the arcs are most closely grouped. The dispersion of the arcs for the west image well indicate that the west boundary of the aquifer is much less abrupt than the east boundary.

By construction, the hydraulic boundary is located at the midpoint of a line drawn from the discharging well to the respective image well. The trace of the computed boundary is shown on figures 4 and 11. In comparing the hydraulic boundaries with the geologic section shown in figure 11, it should be noted that the computed boundaries define the limits of the idealized rectangular aquifer which duplicates the hydraulic behavior of the real aquifer.

Although the methods presented above are limited in their application by the many idealized assumptions which are necessary to their derivation, it seems probable that these methods can assist in the interpretation of local geology from pumping-test data. These data serve also to emphasize the point that the hydraulic evidence from pumping tests will agree with and support the geologic evidence, if interpretation of the pumping-test data is correct. Conversely, it may be stated that any disagreement between the hydraulic and geologic evidence in a given problem is untenable and points to incorrect interpretation of either or both sets of data.

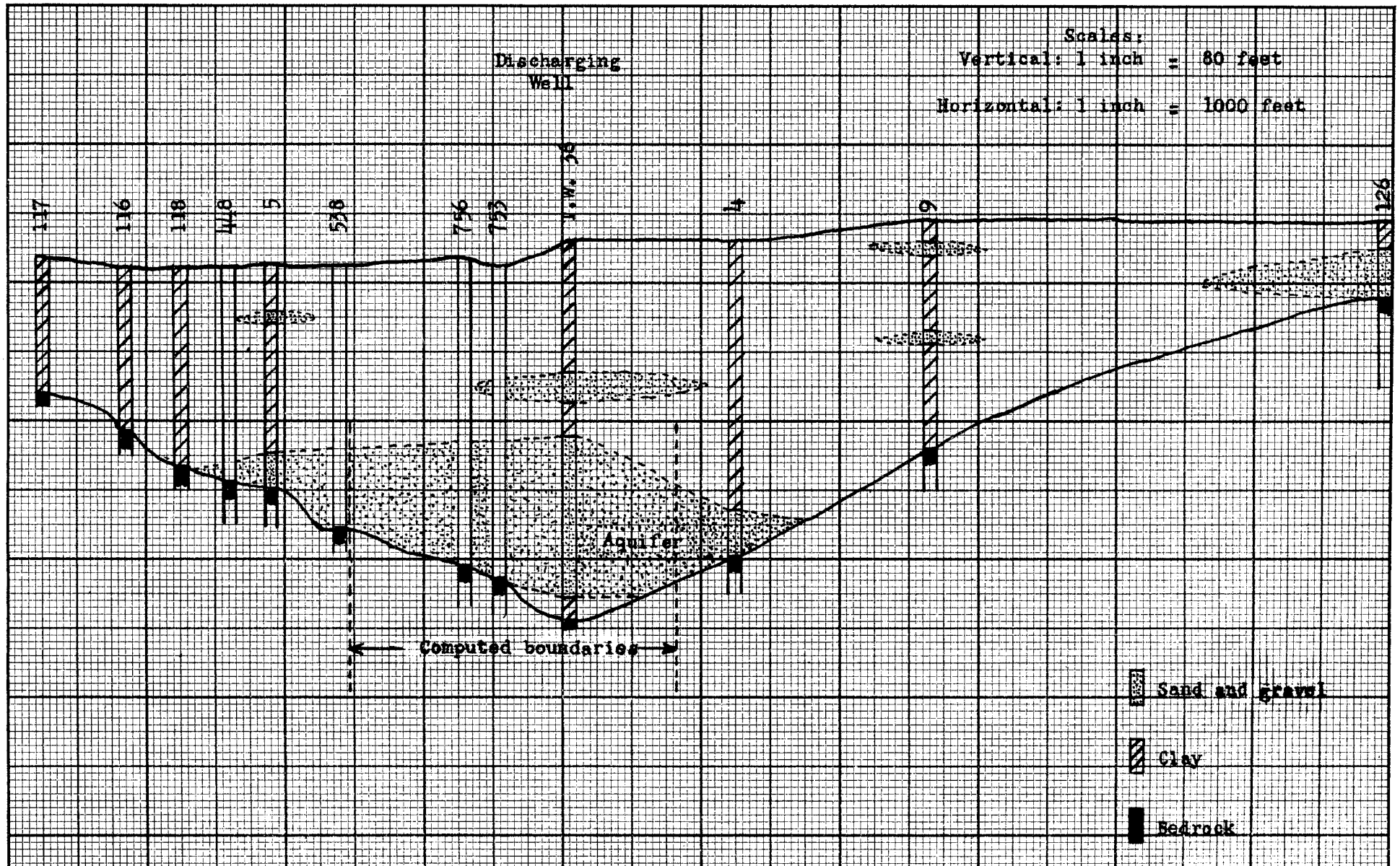



Fig. 11 Generalized geologic cross section along Hemphill Road through site of pumping test on the City of Flint test well 36.