

## CHAPTER 1 PROGRAM AND INSTRUCTION

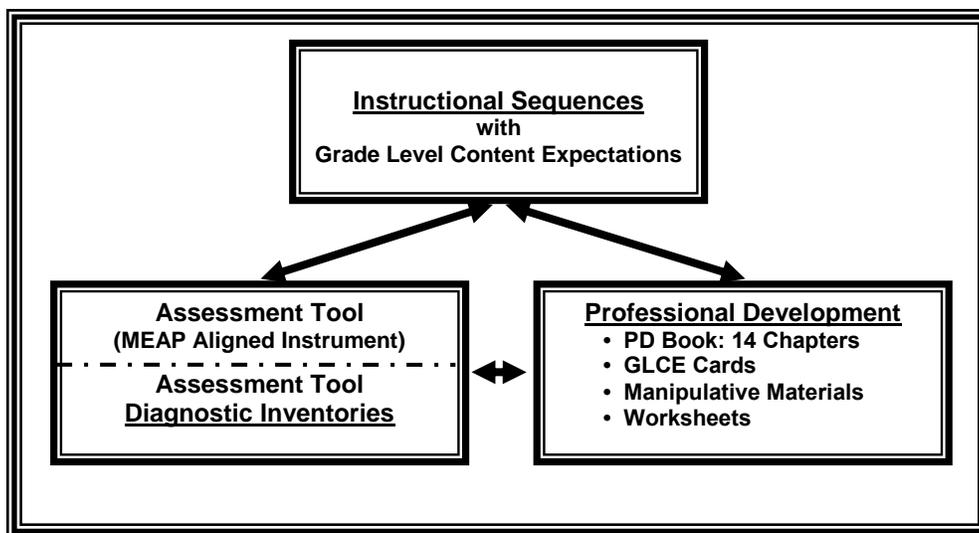
The Michigan Mathematics Program Improvement (MMPI) project, formerly the State Improvement Grant in Mathematics (SIG-Mathematics) is designed to assist educators to improve their mathematics program and to accomplish adequate yearly progress (AYP). The project has started with the number strand of the state grade level content expectations because it is the dominant strand. Attention has also been paid to MEAP tests because they are the major factor in determining adequate yearly progress. The project has completed a three day geometry sequence and is developing a three day algebra sequence. Project materials include workshop activities, instructional sequences, diagnostics inventories, intervention lessons and models for instructional planning. MDE curriculum and assessment documents for both general education and special education are also included.

### Project Focus

The MMPI project is focused to provide professional service for both special education and general education teachers of special education students who are slated to take the MEAP test (Grades 3-8). The project will offer continuing support to participants through continued access to materials and inservice as they are developed. The focus is intentionally narrow to address AYP issues but the project is relevant to a much broader audience.

### Program Components

There are three interrelated program components: instructional sequences, diagnostic inventories, and instructional planning.



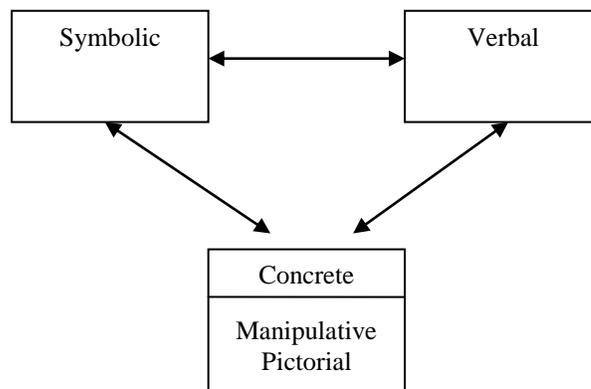
- The instructional sequences are a series of topics in an order that provide instructional coherence to the Michigan Grade Level Content Expectations (GLCEs). The GLCEs are statements of what the state will assess and are not structured for instructional purposes. Each of the GLCEs are imbedded in and aligned to the topics of the instructional sequences.
- Diagnostic inventories are designed to identify student knowledge and understanding relative to the instructional sequences. There are currently seven inventories: whole number place

value, addition, subtraction, multiplication, division; fraction (basic); and, decimal (basic). A cross-grade assessment has been developed to determine grade level performance of students not at grade level.

- Using student performance information on the inventories, content expectations can be identified for that student for the purpose of instructional planning, specifically that student's individualized educational plan (IEP).

### **Instructional Components**

The MMPI project emphasizes connections among three types of representations. The instructional components highlight the various representations used to show the number system and its operations. These representations are symbolic, verbal, and concrete (both pictorial and manipulative). The ability to move fluently from one representation to another is a measure of a student's depth of understanding. Students' understanding of mathematics has been described by some as "a mile wide but only an inch deep." This model is designed to develop deep understanding.



The project also utilizes four models for the concrete representation. Three of the representations have been selected to illustrate "tradability or bustability" and "proportionality or non-proportionality." The fourth representation is linear.

### **Representations**

- Bustable and proportional model (straws)
- Tradable and proportional model (base ten blocks)
- Tradable and non-proportional model (money), and
- Linear model (number line)

The instructional representations are used consistently throughout the project to illustrate coherence. We describe deep knowledge as fluency among these representations. Activities are sequenced to illustrate development from concept and representation to complex computation as follows:

- Given a contextual situation, make and draw a representation of that situation
- Given a mathematical statement, identify a related representation
- Given a representation, write a related mathematical statement
- Compute basic facts
- Compute basic problems
- Compute intermediate problems
- Compute complex problems

### **Chapter Format**

Each of the content chapters has the following format:

- Overview of the chapter
- The sequence of topics for each of whole numbers, decimals, fractions, rational number and integers. A list of the topics follows this section. Activities for each topic include an analysis of aligned grade level content expectations, relevant MEAP item analysis data, selected MEAP release items, and a variety of instructional strategies.
- Diagnostic inventories are designed from visual representations to procedural skills as outlined below:
  - Given a contextual situation, draw a representation of that situation
  - Given a mathematical statement, identify a related representation
  - Given a representation, write a related mathematical statement
  - Compute basic facts
  - Compute basic problems
  - Compute intermediate problems
  - Compute complex problems

### **Instructional Sequence Topics**

Whole Number Instructional Sequence:

- WN1 Quantification of set of objects to 4.
- WN2 Counting to quantify sets of objects.
- WN3 Counting forward and backward.
- WN4 Associate numerals with counting.
- WN5 Relative value of numbers/position on the number line.
- WN6 All combinations for each numeral.
- WN7 Place value.
- WN8 Meaning of addition and subtraction.
- WN9 Addition and subtraction fact families and relationships.

- WN10 Addition and subtraction
- WN11 Multiple digit addition and subtraction
- WN12 Meaning of multiplication and division.
- WN13 Multiplication and division fact families.
- WN14 The relationship between multiplication and division
- WN15 Multiplying by multiples of powers of ten
- WN16 Distributive multiplication.
- WN17 Multiplying by multi-digits numbers
- WN18 Multiples and factors
- WN19 Division as a fraction
- WN20 Equivalent division problems
- WN21 Division algorithm with no remainder
- WN22 Division algorithm with remainder
- WN23 Division algorithm with remainders as fractions remainders
- WN24 Division algorithm with decimal remainders
- WN25 Use Estimation and Approximation

Decimal Instructional Sequence:

- D1 Meaning
- D2 Counting and writing
- D3 Size comparison and the numberline
- D4 Rounding decimals
- D5 Relationship to money
- D6 Addition and subtraction
- D7 Multiplication of decimals
- D8 Division of decimals
- D9 Fractions converted to decimals
- D10 Decimals converted to fractions
- D11 Percent to decimal conversion
- D12 Percent computation
- D13 Percent computation with easy fractions

Fractional Instructional Sequence:

- F1 Meaning of fractions.
- F2 Vocabulary related to fractions.
- F3 Ordering fractions (common denominators and less than one) and placing them on the number line.
- F4 Ordering fractions (common denominators and greater than one) by placing them on the number line.
- F5 Learn the set model for fractions.
- F6 Equivalent fractions.
- F7 Fractions on the number line.
- F8 Fractions  $>1$  as mixed numbers.
- F9 Order fractions without the number line.
- F10 Add and subtract fractions.
- F11 Multiply a fraction by a whole number.
- F12 Relationship of dividing by a whole number and multiplying by a fraction.
- F13 Multiply fractions less than one.
- F14 Multiply any two fractions.
- F15 Division of fractions.

Rational Number and Integer Sequence:

- R1 Factorization
- R2 Rational Numbers
- R3 Rate, Ratio, and Proportion
- R4 Exponents, Roots, and Scientific Notation
- I1 Understand and Compute with Integers

**The Grade Level Content Expectations by Grade (March 2005)**

The table shows the number of GLCEs by grade and by strand and also illustrates the dominance of the number strand.

GLCEs	K	1	2	3	4	5	6	7	8	Total
<b>Number and Operations</b>	10	16	22	21	36	24	20	8	11	168
<b>Algebra</b>							14	13	13	40
<b>Measurement</b>	5	8	11	13	11	10	3			61
<b>Geometry</b>	3	6	6	7	5	7	5	6	10	55
<b>Data and Probability</b>		3	3	3	3	4	2	4	6	28
<b>Total</b>	18	33	42	44	55	45	44	31	40	352

**No Child Left Behind % Proficient**

The table shows the percent of students who must achieve at level one or two on the MEAP tests for the years 2002 through 2014 to meet AYP.

Year	Mathematics		
	Elementary	Middle	High
<b>2002</b>	47%	31%	31%
<b>2003</b>	47%	31%	31%
<b>2004</b>	47%	31%	31%
<b>2005</b>	56%	43%	44%
<b>2006</b>	56%	43%	44%
<b>2007</b>	56%	43%	44%
<b>2008</b>	64%	54%	56%
<b>2009</b>	64%	54%	56%
<b>2010</b>	64%	54%	56%
<b>2011</b>	73%	66%	67%
<b>2012</b>	82%	77%	78%
<b>2013</b>	91%	89%	89%
<b>2014</b>	100%	100%	100%

**Fall 2005-7 MEAP Mathematics Data**

Grade	Year	# Included	Level 4				
			4	3	2	1	1&2
3	2005	117,848	1%	12%	39%	48%	87%
	2006	117,088	1%	11%	37%	51%	88%
	2007	115,560	0%	10%	41%	49%	90%
4	2005	118,193	4%	14%	45%	36%	82%
	2006	117,078	3%	13%	50%	34%	85%
	2007	115,702	2%	12%	45%	41%	86%
5	2005	120,726	5%	21%	42%	32%	73%
	2006	117,827	4%	20%	40%	36%	76%
	2007	116,046	4%	21%	35%	39%	74%
6	2005	124,297	10%	25%	36%	29%	65%
	2006	121,506	8%	23%	35%	34%	69%
	2007	117,925	8%	20%	29%	44%	73%
7	2005	128,830	11%	30%	33%	27%	60%
	2006	124,763	7%	29%	30%	34%	64%
	2007	121,763	4%	24%	32%	41%	73%
8	2005	129,646	14%	23%	33%	31%	63%
	2006	126,803	7%	25%	39%	29%	68%
	2007	122,797	9%	19%	30%	41%	71%

**Your MEAP Mathematics Data**

Grade	Year	# Included	Level 4				
			4	3	2	1	1&2
3	2005						
	2006						
	2007						
4	2005						
	2006						
	2007						
5	2005						
	2006						
	2007						
6	2005						
	2006						
	2007						
7	2005						
	2006						
	2007						
8	2005						
	2006						
	2007						

**District Curriculum Guide**

If not already in place, districts should consider creating a curriculum guide to explicitly state the following:

- Content: What students should know and be able to do
- Instruction: How to best engage students in learning content
- Performance expectation: What we expect students to do – based upon incremental improvement
- Assessment: A measure of meeting performance expectations

**Private Universe**

Discussion Questions:

1. How our natural curiosity leads to misconceptions
2. How misconceptions block learning
3. Why misconceptions must be dealt with before learning can take place
4. Why we may fail to communicate even when we think we've made our point

**Two Challenges from this Project**

1. To focus on the role of a teacher working with underachieving students
  - a. Instructional experiences of a remedial nature rather than simply good instruction (because remediation requires the displacement of misconceptions and or incorrect procedures)
2. To assist colleagues working with underachieving students
  - a. Instructional models and strategies
  - b. Diagnostic and grade performance tools
  - c. Pretest
  - d. Instruct
  - e. Post-test-short term and Post-test-long term
  - f. Instructional planning models

**Materials List**

Base10 Class Set	2/team	GLCEs-by Strand (11x17)
Base 10 Flats [10]	2/team	Place value number sets
Base 10 Rods [50]	4/team	Hundreds charts
Base 10 Units [100]	3/team	Number lines
Base 10 O/H	2/team	Evaluations
Fraction Circles	2/team	
Fraction Circles O/H	2/team	
Dimes [100]	2/team	
Pennies [100]	2/team	
Coffee stirrers & small rubber bands		
Paper strips (2.125x14)		
Dollar bills (\$1, \$10, \$100, \$1000, \$10,000, and \$100,000)		
Sheet protector		

**Web Page :**

All files used in the workshop, including power point presentations, are made available for participants on the project's website [www.michiganmathematics.org](http://www.michiganmathematics.org) .

Chapter 1: Program and Instruction with References (June 2008) (11 pages, 8.5x11, PDF, 240 KB)

**Power Point Presentations:**

- MMPI Project Overview (Power Point, 210 KB)
- Resources and Recommendations (Power Point, 22 MB)
- MEAP Item Analysis and P-Value Data (Power Point, 1.1 MB)

**Resource Books:**

- Decimals: A Place Value Approach. Linda Patriarca, Marilyn Scheffel, and Sheila Hedeman. 1998, Dale Seymour Publications
- Elementary and Middle School Mathematics: Teaching Developmentally, 5<sup>th</sup> Edition. John Van de Walle, 2004, Pearson Education, Inc.
- Adding It Up: Helping Children Learn Mathematics. Jane Swafford, Bradford Findell, Jeremy Kilpatrick Editors; Mathematics Learning Study Committee. National Research Council ISBN: 0-309-50524-0, 480 pages, 7 x 10, (2001). This PDF is available from the National Academies Press at: <http://www.nap.edu/catalog/9822.html>
- How Students Learn: Mathematics in the Classroom  
M. Suzanne Donovan and John D. Bransford, editors, Committee on How People Learn: A Targeted Report for Teachers. National Research Council ISBN: 0-309-54802-0, 272 pages, 7 x 10, (2005). This PDF is available from the National Academies Press at: <http://www.nap.edu/catalog/11101.html>

**Documents from the Michigan Department of Education:**

Review the following documents from the state Department of education:

- The Michigan Grade Level Content Expectations (GLCEs) (both Grade format and Strand Formats) (December 2005)
- MEAP classification of the GLCEs (that identify which expectations are core, extended core, and future core) (December 2005)

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Chapter 7

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