Improving Mathematical Problem Solving in Grades 4 Through 8

NCEE 2012-4055
U.S. DEPARTMENT OF EDUCATION

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The goal of this practice guide is to offer educators specific, evidence-based recommendations that address the challenge of improving mathematical problem solving in grades 4 through 8. The guide provides practical, clear information on critical topics related to improving mathematical problem solving and is based on the best available evidence as judged by the authors.

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IES Practice Guide

Improving Mathematical Problem Solving in Grades 4 Through 8

May 2012

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This report was prepared for the National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences under Contract ED-07-CO-0062 by the What Works Clearinghouse, which is operated by Mathematica Policy Research.

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Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide* (NCEE 2012-4055). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications_reviews.aspx#pubsearch/.

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Review of Recommendations

Recommendation 1.

Prepare problems and use them in whole-class instruction.

1. Include both routine and non-routine problems in problem-solving activities.
2. Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.
3. Consider students' knowledge of mathematical content when planning lessons.

Recommendation 2.

Assist students in monitoring and reflecting on the problem-solving process.

1. Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.
2. Model how to monitor and reflect on the problem-solving process.
3. Use student thinking about a problem to develop students' ability to monitor and reflect.

Recommendation 3.

Teach students how to use visual representations.

1. Select visual representations that are appropriate for students and the problems they are solving.
2. Use think-alouds and discussions to teach students how to represent problems visually.
3. Show students how to convert the visually represented information into mathematical notation.

Recommendation 4.

Expose students to multiple problem-solving strategies.

1. Provide instruction in multiple strategies.
2. Provide opportunities for students to compare multiple strategies in worked examples.
3. Ask students to generate and share multiple strategies for solving a problem.

Recommendation 5.

Help students recognize and articulate mathematical concepts and notation.

1. Describe relevant mathematical concepts and notation, and relate them to the problem-solving activity.
2. Ask students to explain each step used to solve a problem in a worked example.
3. Help students make sense of algebraic notation.

Acknowledgments

The panel appreciates the efforts of Madhavi Jayanthi, Rebecca Newman-Gonchar, and Kelly Haymond from the Instructional Research Group, and of Joshua Furgeson and Robin Patfield from Mathematica Policy Research, who participated in the panel meetings, described the research findings, and drafted the guide. We also thank Russell Gersten, Alison Wellington, and Annalisa Mastri for helpful feedback and reviews of earlier versions of the guide.

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Institute of Education Sciences Levels of Evidence for Practice Guides

This section provides information about the role of evidence in Institute of Education Sciences' (IES) What Works Clearinghouse (WWC) practice guides. It describes how practice guide panels determine the level of evidence for each recommendation and explains the criteria for each of the three levels of evidence (strong evidence, moderate evidence, and minimal evidence).

The level of evidence assigned to each recommendation in this practice guide represents the panel's judgment of the quality of the existing research to support a claim that, when these practices were implemented in past research, positive effects were observed on student outcomes. After careful review of the studies supporting each recommendation, panelists determine the level of evidence for each recommendation using the criteria in Table 1. The panel first considers the relevance of individual studies to the recommendation and then discusses the entire evidence base, taking the following into consideration:

- UIF OVNCFS PG TUVEJFT
- UIF EFTJHO PG UIF TUVEJFT
- UIF RVBMJUZ PG UIF TUVEJFT
- XIFUIFS UIF TUVEJFT SFQSFTFOU UIF SBOHF of participants and settings on which the recommendation is focused
- XIFUIFS OEJOHT GSPN UIF TUVEJFT DBO CF attributed to the recommended practice
- XIFUIFS OEJOHT JO UIF TUVEJFT BSF DPOTJT-tently positive

A rating of *strong evidence* refers to consistent evidence that the recommended strategies, programs, or practices improve student outcomes for a wide population of students.[1] In other words, there is strong causal and generalizable evidence.

A rating of *moderate evidence* refers either to evidence from studies that allow strong causal conclusions but cannot be generalized with assurance to the population on which a recommendation is focused (perhaps because the findings have not been widely replicated) or to evidence from studies that are generalizable but have some causal ambiguity. It also might be that the studies that exist do not specifically examine the outcomes of interest in the practice guide, although they may be related.

A rating of *minimal evidence* suggests that the panel cannot point to a body of research that demonstrates the practice's positive effect on student achievement. In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental fashion;[2] in other cases, it means that researchers have not yet studied this practice, or that there is weak or conflicting evidence of effectiveness. A minimal evidence rating does not indicate that the recommendation is any less important than other recommendations with a strong evidence or moderate evidence rating.

In developing the levels of evidence, the panel considers each of the criteria in Table 1. The level of evidence rating is determined as the lowest rating achieved for any individual criterion. Thus, for a recommendation to get a strong rating, the research must be rated as strong on each criterion. If at least one criterion receives a rating of moderate and none receive a rating of minimal, then the level of evidence is determined to be moderate. If one or more criteria receive a rating of minimal, then the level of evidence is determined to be minimal.

Table 1. Institute of Education Sciences levels of evidence for practice guides

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Validity	High internal validity (high-quality causal designs). Studies must meet WWC standards with or without reservations.[3] **AND** High external validity (requires multiple studies with high-quality caVTal designs that represent the population on which the recommendation is focused). Studies must meet WWC standards with or without reservations.	High internal validity but moderate external validity (i.e., studies that support strong causal conclusions but generalization is uncertain). **OR** High external validity but moderate internal validity (i.e., studies that support the generality of a relation but the causality is uncertain).[4]	The research may include evidence from studies that do not meet the criteria for moderate or strong evidence (e.g., case studies, qualitative research).
Effects on relevant outcomes	Consistent positive effects without contradictory evidence (i.e., no statistically significant negative effects) in studies with high internal validity.	A preponderance of evidence of positive effects. Contradictory evidence (i.e., statistically significant negative effects) must be discussed by the panel and considered with regard to relevance to the scope of the guide and intensity of the recommendation as a component of the intervention evaluated.	There may be weak or contradictory evidence of effects.
Relevance to scope	Direct relevance to scope (i.e., ecological validity)—relevant context (e.g., classroom vs. laboratory), sample (e.g., age and characteristics), and outcomes evaluated.	Relevance to scope (ecological validity) may vary, including relevant context (e.g., classroom vs. laboratory), sample (e.g., age and characteristics), and outcomes evaluated. At least some research is directly relevant to scope (but the research that is relevant to scope does not qualify as strong with respect to validity).	The research may be out of the scope of the practice guide.
Relationship between research and recommendations	Direct test of the recommendation in the studies or the recommendation is a major component of the intervention tested in the studies.	Intensity of the recommendation as a component of the interventions evaluated in the studies may vary.	Studies for which the intensity of the recommendation as a component of the interventions evaluated in the studies is low; and/or the recommendation reflects expert opinion based on reasonable extrapolations from research.

(continued)

Table 1. Institute of Education Sciences levels of evidence for practice guides *(continued)*

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Panel confidence	Panel has a high degree of confidence that this practice is effective.	The panel determines that the research does not rise to the level of strong but is more compelling than a minimal level of evidence. Panel may not be confident about whether the research has effectively controlled for other explanations or whether the practice would be effective in most or all contexts.	In the panel's opinion, the recommendation must be addressed as part of the practice guide; however, the panel cannot point to a body of research that rises to the level of moderate or strong.
Role of expert opinion	Not applicable	Not applicable	Expert opinion based on defensible interpretations of theory (theories). (In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental fashion; in other cases, it means that researchers have not yet studied this practice.)
When assessment is the focus of the recommendation	For assessments, meets the standards of *The Standards for Educational and Psychological Testing*.[5]	For assessments, evidence of reliability that meets *The Standards for Educational and Psychological Testing* but with evidence of validity from samples not adequately representative of the population on which the recommendation is focused.	Not applicable

The panel relied on WWC evidence standards to assess the quality of evidence supporting educational programs and practices. WWC evaluates evidence for the causal validity of instructional programs and practices according to WWC standards. Information about these standards is available at http://ies.ed.gov/ncee/wwc/DocumentSum.aspx?sid=19. Eligible studies that meet WWC evidence standards or meet evidence standards with reservations are indicated by **bold text** in the endnotes and references pages.

Introduction to the Improving Mathematical Problem Solving in Grades 4 Through 8 Practice Guide

This section outlines the importance of improving mathematical problem solving for students in grades 4 through 8 and explains key parameters considered by the panel in developing the practice guide. It also summarizes the recommendations for readers and concludes with a discussion of the research supporting the practice guide.

Students who develop proficiency in mathematical problem solving early are better prepared for advanced mathematics and other complex problem-solving tasks.[6] Unfortunately, when compared with students in other countries, students in the U.S. are less prepared to solve mathematical problems.[7] For example, recent Trends in International Mathematics and Science Study (TIMSS) data suggest that, when compared to other industrialized countries such as the Netherlands, China, and Latvia, U.S. 4th-graders rank tenth and 8th-graders rank seventh out of 41 countries in problem solving.[8]

Problem solving involves reasoning and analysis, argument construction, and the development of innovative strategies. These abilities are used not only in advanced mathematics topics—such as algebra, geometry and calculus—but also throughout the entire mathematics curriculum beginning in kindergarten, as well as in subjects such as science. Moreover, these skills have a direct impact on students' achievement scores, as many state and national standardized assessments and college entrance exams include problem solving.[9]

Traditional textbooks[10] often do not provide students rich experiences in problem solving.[11] Textbooks are dominated by sets of problems that are not cognitively demanding, particularly when assigned as independent seatwork or homework, and teachers often review the answers quickly without discussing what strategies students used to solve the problems or whether the solutions can be justified.[12] The lack of guidance in textbooks is not surprising, given that state and district standards are often less clear in their guidelines for process skills, such as problem solving, than they are in their wording of grade-level content standards.[13]

The goal of this practice guide is to give teachers and administrators recommendations for improving mathematical problem-solving skills, regardless of which curriculum is used. The guide offers five recommendations that provide teachers with a coherent approach for regularly incorporating problem solving into their classroom instruction to achieve this end. It presents evidence-based suggestions for putting each recommendation into practice and describes roadblocks that may be encountered, as well as possible solutions.

Scope of the practice guide

Audience and grade level. The need for effective problem-solving instruction is particularly critical in grades 4 through 8, when the mathematics concepts taught become more complicated and when various forms of assessments—from class tests to state and national assessments—begin incorporating problem-solving activities. In this guide, the panel provides teachers with five recommendations for instructional practices that improve students' problem-solving ability. Math coaches and other administrators also may find this guide helpful as they prepare teachers to use these practices in their classrooms. Curriculum developers may find the guide useful in making design decisions, and researchers may find opportunities to extend or explore variations in the evidence base.

Content. The literature reviewed for this guide was restricted to mathematical problem-solving topics typically taught in grades 4 through 8. The panelists reviewed a number of definitions of *problem solving* as part of the process of creating this guide, but a single, prevalent definition of *problem solving* was not identified. This is understandable,

given the different contexts in which the term *problem solving* is used in mathematics. Some definitions are exceedingly broad and applied to a general level of problem solving that goes beyond mathematics into everyday human affairs. For example, *problem solving* is often defined as the "movement from a given state to a goal state with no obvious way or method for getting from one to the other."[14] This kind of definition underscores the non-routine nature of problem solving and the fact that it is not the execution of memorized rules or shortcuts, such as using key words, to solve math word problems.

More contemporary definitions of *problem solving* focus on communication, reasoning, and multiple solutions. In addition to the non-routine nature of the process, this kind of mathematical problem solving is portrayed as the opportunity to engage in mathematics and derive a reasonable way or ways to solve the problem.[15] In light of the long-standing historical variations and disputes over definitions of *problem solving*, the panel ultimately decided that it was not in their purview to resolve this issue. The panel defined the characteristics of problem solving that applied to this guide as follows:

- ' JSTU TUVEFOUT DBO NFBSO BU FN BUJDBM problem solving; it is neither an innate talent nor happenstance that creates skilled problem solvers.
- 4FDPOE N BU FN BUJDBMQSPCMFN TPMWJOH JT relative to the individual. What is challenging or non-routine for one student may be comparatively straightforward for a more advanced student.
- 5I JSE N BU FN BUJDBMQSPCMFN TPMWJOH OFFE not be treated like just another topic in the pacing guide; instead, it can serve to support and enrich the learning of mathematics concepts and notation.
- ' PVSUI PGUFO N PSF UI BO PPF TUSBUFHZ can be used to solve a problem. Learning multiple strategies may help students see different ideas and approaches for solving problems and may enable students to think more flexibly when presented with a problem that does not have an obvious solution.

Problem solving includes more than working *word problems*. While word problems have been the mainstay of mathematics textbooks for decades, they are only one type of math problem. Other types of math problems appropriate to grades 4 through 8, such as algebraic and visual-spatial problems (e.g., "How many squares are there on a checkerboard?"), are addressed in this guide. The panel excluded whole number addition and subtraction, which are typically taught in kindergarten through grade 3, as well as advanced algebra and advanced geometry, which are typically taught in high school.

When developing recommendations, the panel incorporated several effective instructional practices, including explicit teacher modeling and instruction, guided questions, and efforts to engage students in conversations about their thinking and problem solving. The panel believes it is important to include the variety of ways problem solving can be taught.

There are several limitations to the scope of this guide. The literature reviewed for this guide was limited to studies pertaining to mathematical problem solving; therefore, it did not include cognitive or psychological dimensions of problem solving that fell outside of this topic area.[16] While the panel considered studies that included students with disabilities and students who were learning English, this guide does not address specific instructional practices for these groups. Instead, this guide is intended for use by all teachers, including general education, special education teachers, and teachers of English learners, of mathematics in grades 4 through 8.

Summary of the recommendations

The five recommendations in this guide can be used independently or in combination to help teachers engage students in problem solving on a regular basis. To facilitate using the recommendations in combination, the panel provided a discussion of how the

recommendations can be combined in the lesson-planning process. This discussion is presented in the conclusion section of the guide.

Recommendation 1 explains how teachers should incorporate problem-solving activities into daily instruction, instead of saving them for independent seatwork or homework. The panel stresses that teachers must consider their unit goals and their students' background and interests when preparing problem-solving lessons.

Recommendation 2 underscores the importance of thinking through or reflecting on the problem-solving process. Thinking through the answers to questions such as "What is the question asking me to do?" and "Why did these steps in solving the problem work or not work?" will help students master multistep or complex problems.

Recommendations 3, 4, and 5 focus on specific ways to teach problem solving.

Recommendation 3 covers instruction in visual representations, such as tables, graphs, and diagrams. Well-chosen visual representations help students focus on what is central to many mathematical problems: the relationship between quantities.

Recommendation 4 encourages teachers to teach multiple strategies that can be used to solve a problem. Sharing, comparing, and discussing strategies afford students the opportunity to communicate their thinking and, by listening to others, become increasingly flexible in the way they approach and solve problems. Too often students become wedded to just one approach and then flounder when it does not work on a different or more challenging problem.

Recommendation 5 encourages teachers to help students recognize and articulate mathematical concepts and notation during problem-solving activities. The key here is for teachers to remember that students' problem solving will improve when students understand the formal mathematics at the heart of each problem.

Of the five recommendations the panel shares in this guide, the panel chose to present the recommendation (Recommendation 1) that provides guidance for preparing problem-solving activities first. Even though the level of evidence supporting this recommendation is not strong, the panel believes teachers should plan before undertaking these activities. The first two recommendations can be used regularly when preparing and implementing problem-solving lessons; in contrast, the panel does not think recommendations 3 through 5 must be used in *every* lesson. Instead, teachers should choose the recommendations that align best with their goals for a given lesson and its problems. For example, there are occasions when visual representations are not used as part of problem-solving instruction, such as when students solve an equation by considering which values of the variable will make both sides equal.

Use of research

The evidence used to create and support the recommendations in this practice guide ranges from rigorous experimental studies to expert reviews of practices and strategies in mathematics education; however, the evidence ratings are based solely on high-quality group-design studies (randomized controlled trials and rigorous quasi-experimental designs) that meet What Works Clearinghouse (WWC) standards. Single-case design studies that meet WWC pilot standards for well-designed single-case design research are also described, but do not affect the level of evidence rating. The panel paid particular attention to a set of high-quality experimental and quasi-experimental studies that meets the WWC criteria, including both national and international studies of strategies for teaching problem solving to students in grades 4 through 8.[17] This body of research included strategies and curricular materials developed by researchers or ones commonly being used by teachers in classrooms. The panel also considered studies recommended by panel members that included students in grades 3 and 9.

Studies of problem-solving interventions in the past 20 years have yielded few causal evaluations of the effectiveness of the variety of approaches used in the field. For example, as much as the panel believes that teaching students to persist in solving challenging problems is important to solving math problems, it could not find causal research that isolated the impact of persistence. The panel also wanted to include studies of teachers using their students' culture to enhance problem-solving instruction; however, panelists could not find enough research that met WWC standards and isolated this practice. The panel was able to include suggestions for teaching the language of mathematics and for adapting problems so that contexts are more relevant to students—but these suggestions are supported by limited evidence.

The research base for this guide was identified through a comprehensive search for studies evaluating instructional practices for improving students' mathematical problem solving. An initial search for literature related to problem-solving instruction in the past 20 years yielded more than 3,700 citations; the panel recommended an additional 69 citations. Peer reviewers suggested several additional studies. Of these studies, only 38 met the causal validity standards of the WWC and were related to the panel's recommendations.[18]

The supporting research provides a strong level of evidence for two of the recommendations, a moderate level of evidence for another two of the recommendations, and a minimal level of evidence for one recommendation. Despite the varying levels of evidence, the panel believes all five recommendations are important for promoting effective problem-solving skills in students. The panel further believes that even though the level of evidence for Recommendation 1 is minimal, the practice holds promise for improving students' mathematical problem solving. Very few studies examine the effects of teacher planning on student achievement; therefore, few studies are available to support this recommendation. Nonetheless, the panel believes that the practice of intentionally preparing problem-solving lessons can lead to improvement in students' problem-solving abilities.

Table 2 shows each recommendation and the strength of the evidence that supports it as determined by the panel. Following the recommendations and suggestions for carrying out the recommendations, Appendix D presents more information on the research evidence that supports each recommendation. It also provides details on how studies were assessed as showing positive, negative, or no effects.

Table 2. Recommendations and corresponding levels of evidence

	Levels of Evidence		
Recommendation	**Strong Evidence**	**Moderate Evidence**	**Minimal Evidence**
1. Prepare problems and use them in whole-class instruction.			◆
2. Assist students in monitoring and reflecting on the problem-solving process.	◆		
3. Teach students how to use visual representations.	◆		
4. Expose students to multiple problem-solving strategies.		◆	
5. Help students recognize and articulate mathematical concepts and notation.		◆	