ALGEBRA II

1 Credit
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Welcome
This guide was developed to assist teachers in successfully implementing the Michigan Merit Curriculum. The identified content expectations and guidelines provide a useful framework for designing curriculum, assessments and relevant learning experiences for students. Through the collaborative efforts of Governor Jennifer M. Granholm, the State Board of Education, and the State Legislature, these landmark state graduation requirements are being implemented to give Michigan students the knowledge and skills to succeed in the 21st Century and drive Michigan’s economic success in the global economy. Working together, teachers can explore varied pathways to help students demonstrate proficiency in meeting the content expectations and guidelines.

Curriculum Unit Design
One of the ultimate goals of teaching is for students to acquire transferable knowledge. To accomplish this, learning needs to result in a deep understanding of content and mastery level of skills. As educational designers, teachers must use both the art and the science of teaching. In planning coherent, rigorous instructional units of study, it is best to begin with the end in mind.

Engaging and effective units include:

• appropriate content expectations
• students setting goals and monitoring own progress
• a focus on big ideas that have great transfer value
• focus and essential questions that stimulate inquiry and connections
• identified valid and relevant skills and processes
• purposeful real-world applications
• relevant and worthy learning experiences
• varied flexible instruction for diverse learners
• research-based instructional strategies
• explicit and systematic instruction
• adequate teacher modeling and guided practice
• substantial time to review or apply new knowledge
• opportunities for revision of work based on feedback
• student evaluation of the unit
• culminating celebrations
Relevance
Instruction that is clearly relevant to today’s rapidly changing world is at the forefront of unit design. Content knowledge cannot by itself lead all students to academic achievement. Classes and projects that spark student interest and provide a rationale for why the content is worth learning, enable students to make connections between what they read and learn in school, their lives, and their futures. An engaging and effective curriculum provides opportunities for exploration and exposure to new ideas. Real-world learning experiences provide students with opportunities to transfer and apply knowledge in new diverse situations.

Student Assessment
The assessment process can be a powerful tool for learning when students are actively involved in the process. Both assessment of learning and assessment for learning, are essential. Reliable formative and summative assessments provide teachers with information they need to make informed instructional decisions that are more responsive to students’ needs. Engagement empowers students to take ownership of their learning and builds confidence over time.

**Sound assessments:**
- align with learning goals
- vary in type and format
- use authentic performance tasks
- use criteria scoring tools such as rubrics or exemplars
- allow teachers and students to track growth over time
- validate the acquisition of transferable knowledge
- give insight into students’ thinking processes
- cause students to use higher level thinking skills
- address guiding questions and identified skills and processes
- provide informative feedback for teachers and students
- ask students to reflect on their learning
High School Content Expectation Codes
To allow for ease in referencing expectations, each mathematics expectation has been coded by strand, standard, topic, and expectation. For example:

- **A1.2.3**: 3rd expectation in the 2nd topic of Standard A1

### Organizational Structure

<table>
<thead>
<tr>
<th>STRAND 1</th>
<th>STANDARDS (and number of core expectations in each standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitative Literacy and Logic (L)</td>
<td><strong>L1</strong>: Reasoning About Numbers, Systems and Quantitative Situations (13)</td>
</tr>
<tr>
<td></td>
<td><strong>L2</strong>: Calculation, Algorithms, and Estimation (13)</td>
</tr>
<tr>
<td></td>
<td><strong>L3</strong>: Mathematical Reasoning, Logic, and Proof (10)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRAND 2</th>
<th>STANDARDS (and number of core expectations in each standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra and Functions (A)</td>
<td><strong>A1</strong>: Expressions, Equations, and Inequalities (16)</td>
</tr>
<tr>
<td></td>
<td><strong>A2</strong>: Functions (16)</td>
</tr>
<tr>
<td></td>
<td><strong>A3</strong>: Families of Functions (27)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRAND 3</th>
<th>STANDARDS (and number of core expectations in each standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry and Trigonometry (G)</td>
<td><strong>G1</strong>: Figures and Their Properties (29)</td>
</tr>
<tr>
<td></td>
<td><strong>G2</strong>: Relationships Between Figures (10)</td>
</tr>
<tr>
<td></td>
<td><strong>G3</strong>: Transformations of Figures in the Plane (5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRAND 4</th>
<th>STANDARDS (and number of core expectations in each standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics and Probability (S)</td>
<td><strong>S1</strong>: Univariate Data—Examining Distributions (9)</td>
</tr>
<tr>
<td></td>
<td><strong>S2</strong>: Bivariate Data—Examining Relationships (6)</td>
</tr>
<tr>
<td></td>
<td><strong>S3</strong>: Samples, Surveys, and Experiments (3)</td>
</tr>
<tr>
<td></td>
<td><strong>S4</strong>: Probability Models and Probability Calculation (4)</td>
</tr>
</tbody>
</table>
Organization of this Document

In the Mathematics credit requirement documents, the expectations are organized by strand and standard underneath topic headings. The organization in no way implies an instructional sequence. Curriculum personnel or teachers are encouraged to organize these topics and expectations in a manner that encourages connections between strands and among topics with a strand.

Introduction to Algebra II

The increasing use of quantitative methods in all disciplines has made algebra the fundamental tool for mathematical applications. Algebraic thinking is learned most effectively when it is studied in the context of applications, both mathematical and real-world. These applications reveal the power of algebra to model real problems and to generalize new situations. Algebra is not only a theoretical tool for analyzing and describing mathematical relationships, but it is also a powerful tool for the mathematical modeling and solving of real-world problems. These problems can be found all around us: the workplace, the sciences, technology, engineering, and mathematics.

Algebra II Goal Statement

The goal of Algebra II is to build upon the concepts taught in Algebra I and Geometry while adding new concepts to the students’ repertoire of mathematics. In Algebra I, students studied the concept of functions in various forms such as linear, quadratic, polynomial, and exponential. Algebra II continues the study of exponential and logarithmic functions and further enlarges the catalog of function families to include rational and trigonometric functions. In addition to extending the algebra strand, Algebra II will extend the numeric and logarithmic ideas of accuracy, error, sequences, and iteration. The topic of conic sections fuses algebra with geometry. Students will also extend their knowledge of univariate and bivariate statistical applications.
It is also the goal of this model to help students see the connections in the mathematics that they have already learned. For example, students will not only gain an in-depth understanding of circular trigonometry, but will also understand its connections to triangular trigonometry.

Throughout Algebra I & II, students will experience mathematics generally, and algebra in particular, not only as the theoretical study of mathematical patterns and relationships, but also as a language that allows us to make sense of mathematical symbols. Finally, students will develop an understanding that algebraic thinking is an accessible and powerful tool that can be used to model and solve real-world problems.
Algebra II Content Expectations Outline

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

L1.2 Representations and Relationships
L1.3 Counting and Probabilistic Reasoning

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

L2.1 Calculation Using Real and Complex Numbers
L2.2 Sequences and Iteration
L2.3 Measurement Unit, Calculations and Scales
L2.4 Understanding Error

STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)
A1.2 Solutions of Equations and Inequalities (linear, exponential, logarithmic, quadratic, power, polynomial, rational, and trigonometric)

STANDARD A2: FUNCTIONS

A2.1 Definitions, Representations, and Attributes of Functions
A2.2 Operations and Transformations with Functions
A2.3 Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A2.4 Models of Real-World Situations Using Families of Functions

**STANDARD A3: Families of Functions**

A3.2 Exponential and Logarithmic Functions

A3.6 Rational Functions

A3.7 Trigonometric Functions

**STANDARD G1: FIGURES AND THEIR PROPERTIES**

G1.7 Conic Sections and Their Properties

**STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS**

S1.1 Producing and Interpreting Plots

S1.2 Measures of Center and Variation

S1.3 The Normal Distribution

**STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS**

S3.1 Data Collection and Analysis

**STANDARD S4: PROBABILITY MODELS AND CALCULATIONS**

S4.1 Probability

S4.2 Application and Representation
STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE LITERACY

L1.2 Representations and Relationships
  L1.2.1 Use mathematical symbols to represent quantitative relationships and situations.

L1.3 Counting and Probabilistic Reasoning
  L1.3.1 Describe, explain, and apply various counting techniques; relate combinations to Pascal’s triangle; know when to use each technique.
  L1.3.2 Define and interpret commonly used expressions of probability.
  L1.3.3 Recognize and explain common probability misconceptions such as “hot streaks” and “being due.”

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

L2.1 Calculation Using Real and Complex Numbers
  L2.1.3 Explain the exponential relationship between a number and its base 10 logarithm, and use it to relate rules of logarithms to those of exponents in expressions involving numbers.
  L2.1.5 Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.

L2.2 Sequences and Iteration
  L2.2.1 Find the $n$th term in arithmetic, geometric, or other simple sequences.
  L2.2.2 Compute sums of finite arithmetic and geometric sequences.
L2.2.3 Use iterative processes in such examples as computing compound interest or applying approximation procedures.

L2.3 Measurement Units, Calculations, and Scales

L2.3.2 Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements; solve applied problems.

L2.4 Understanding Error

L2.4.1 Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.

L2.4.2 Describe and explain round-off error, rounding, and truncating.

L2.4.3 Know the meaning of and interpret statistical significance, margin of error, and confidence level.

STANDARD A1: EXPRESSIONS, EQUATIONS AND INEQUALITIES

A1.1 Construction, Interpretation, and Manipulation of Expressions

A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.
A1.1.4  Add, subtract, multiply, and simplify polynomials and rational expressions.

A1.1.5  Divide a polynomial by a monomial.

A1.1.6  Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms, including the inverse relationship between exponents and logarithms.

### A1.2  Solutions of Equations and Inequalities

A1.2.2  Associate a given equation with a function whose zeros are the solutions of the equation.

A1.2.5  Solve polynomial equations and equations involving rational expressions and justify steps in the solution.

A1.2.7  Solve exponential and logarithmic equations and justify steps in the solution.

A1.2.8  Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.

A1.2.9  Know common formulas and apply appropriately in contextual situations.

A1.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals.
STANDARD A2: FUNCTIONS

A2.1 Definitions, Representations, and Attributes of Functions

A2.1.1 Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function, and identify its domain and range.

A2.1.2 Read, interpret, and use function notation, and evaluate a function at a value in its domain.

A2.1.3 Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.

A2.1.6 Identify the zeros of a function, the intervals where the values of a function are positive or negative, and describe the behavior of a function as x approaches positive or negative infinity, given the symbolic and graphical representations.

A2.1.7 Identify and interpret the key features of a function from its graph or its formula(s).

A2.2 Operations and Transformations with Functions

A2.2.1 Combine functions by addition, subtraction, multiplication, and division.

A2.2.2 Apply given transformations to parent functions, and represent symbolically.

A2.2.3 Recognize whether a function (given in tabular or graphical form) has an inverse, and recognize simple inverse pairs.
A2.3 Representations of Functions

A2.3.1 Identify a function as a member of a family of functions based on its symbolic or graphical representation; recognize that different families of functions have different asymptotic behavior.

A2.3.3 Write the general symbolic forms that characterize each family of functions.

A2.4 Models of Real-World Situations Using Families of Functions

A2.4.1 Identify the family of functions best suited for modeling a given real-world situation.

A2.4.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers.

A2.4.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled.

STANDARD A3: FAMILIES OF FUNCTIONS

A3.2 Exponential and Logarithmic Functions

A3.2.2 Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions.

A3.2.3 Apply properties of exponential and logarithmic functions.
A3.6 Rational Functions

A3.6.1 Write the symbolic form and sketch the graph of simple rational functions.

A3.6.2 Analyze graphs of simple rational functions and understand the relationship between the zeros of the numerator and denominator, and the function’s intercepts, asymptotes, and domain.

A3.7 Trigonometric Functions

A3.7.1 Use the unit circle to define sine and cosine; approximate values of sine and cosine; use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.

A3.7.2 Use the relationship between degree and radian measures to solve problems.

A3.7.3 Use the unit circle to determine the exact values of sine and cosine, for integer multiples of $\pi/6$ and $\pi/4$.

A3.7.4 Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.

A3.7.5 Graph transformations of basic trigonometric functions (involving changes in period, amplitude, phase, and midline) and understand the relationship between constants in the formula and the transformed graph.
STANDARD G1: FIGURES AND THEIR PROPERTIES

G1.7 Conic Sections and Their Properties

G.1.7.1 Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

G1.7.2 Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

G1.7.3 Graph ellipses and hyperbolas with axes parallel to the $x$- and $y$-axes, given equations.

STANDARD S1: UNIVARIATE DATA-EXAMINING DISTRIBUTIONS

S1.1 Producing and Interpreting Plots

S1.1.1 Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.

S1.1.2 Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.
S1.2 Measures of Center and Variation

S1.2.1 Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.

S1.2.2 Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.

S1.2.3 Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

S1.3 The Normal Distribution

S1.3.1 Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.

S1.3.2 Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.

S1.3.3 Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.

S1.3.4 Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.
STANDARD S3: SAMPLES, SURVEYS AND EXPERIMENTS

S3.1 Data Collection and Analysis

S3.1.1 Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.

S3.1.2 Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.

S.3.1.3 Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

Standard S4: PROBABILITY MODELS AND CALCULATIONS

S4.1 Probability

S4.1.1 Understand and construct sample spaces in simple situations

S.4.1.2 Define mutually exclusive events, independent events, dependent events, compound events, complementary events, and conditional probabilities; and use the definitions to compute probabilities.
S4.2 Application and Representation

S4.2.1 Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.

S4.2.2 Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology to make informed decisions.
**Successful Post-Secondary Engagement**

**Components of Mathematical Proficiency**

**Conceptual Understanding**
- Comprehension of mathematical concepts, operations, and relations.

**Procedural Fluency**
- Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

**Strategic Competence**
- Ability to formulate, represent, and solve mathematical problems.

**Adaptive Reasoning**
- Capacity for logical thought, reflection, explanation, and justification.

**Productive Disposition**
- Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

*Adding it Up, National Research Council, 2001*

**State/National Landscape**
- Expert perspective
- Education reform environment
- Research-based practices
- Work force requirements

**State/Federal Expectations**
- No Child Left Behind (NCLB)
- National Governors’ Association (NGA)
- Legislation/Policy

**MI High School Math Content Standards and Expectations**

1. Quantitative Literacy & Logic
2. Algebra & Functions
3. Geometry & Trigonometry
4. Statistics & Probability

**Models for District Alignment / Mapping**
- District curriculum documents
- Documents from other districts/states
- Backmapping to standards and expectations

**Other Documents/Programs**
- State Standards—teaching and learning content (all areas), assessment
- Grade Level Content Expectations
- State Assessments
- Career/Technical Education
  - Department of Labor and Economic Growth

**K-8 Educational Experience**

**Content Knowledge**
- English Language Arts
- Mathematics
- Science
- Social Studies
- Other

**Learning Processes**
- Strategies & Skills
- Reasoning
- Analytical Thinking
- Constructing New Meaning
- Communication