Comparing Mathematics Assessment Frameworks between the National Assessment of Educational Progress and the Michigan Curriculum Framework Grade Level Content Expectations

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ABSTRACT

BACKGROUND
This paper examines a comparison between the 2005 National Assessment of Educational Progress Mathematics Framework and the 2004 Michigan Curriculum Framework (MCF) Grade Level Content Expectations (GLCE).

METHODS
We used Michigan GLCE documentation and grouped them according to a match between NAEP content objectives using the NAEP Assessment Framework.

RESULTS
The content standards have an overlap of 81.5% at the Grade 4 level and a 78% overlap at the Grade 8 level. On the Grade 4 comparison, there were 12 NAEP objectives that were not covered by any GLCE (roughly 18.5%). On the Grade 8 comparison, there were 23 NAEP content standards that were not covered by any GLCE (roughly 22%).

DISCUSSION
While the results between MEAP and NAEP assessment are not directly comparable, there may be information from NAEP state sample results that can inform instructional policy in Michigan. The GLCE's have not been in place long enough to see any instructional impact on NAEP results but the areas of relatively low performance and the emphasis placed on these same areas by the GLCE's should have some positive impact in NAEP performance.

The first question that comes to mind is why do this comparison? The primary reason is to inform instructional programming issues using strengths and weaknesses on the 2005 NAEP Mathematics assessment in Michigan and using the Grade Level Content Expectations (GLCE) assessment framework as a contrast. Another reason is to provide a way to examine Michigan's framework with another Mathematics framework for the purpose of a) a better understanding Michigan GLCE's and, b) providing information that might inform discussion if the GLCE's were to be modified in the future (Currently there are no plans to consider changing the GLCE's.)

The Michigan Educational Assessment Program (http://www.mi.gov/meap) began in 1969, about the same time as the first NAEP assessment. Both programs have achieved longevity and both have worked hard to keep up with changes in law and policy. Both programs have worked through the many psychometric issues of criterion referenced testing.

GLCE's are developed at each grade level from grades Kindergarten to grade 8 in Mathematics and English Language Arts. The GLCE concept was developed to comply with No Child Left Behind ACT (NCLB) and was implemented in the 2004-05 school year. The statewide MEAP assessment for Fall 2005 was the first administration based on GLCE's. The GLCE content standards are aligned to the Michigan Curriculum Framework (developed in the mid 90's) and are developed by committees composed of educators and teachers from Michigan schools. The GLCE's consist of 5 Strands; Numbers and Operations, Algebra, Measurement, Geometry, Data & Probability - these content strands map directly with one to one correspondence with the NAEP content areas. The GLCE strands are further divided into sub-areas called domains. For example, at the fourth grade level, the Strand: Number and Operations has 12 domains with a total of 37content expectations.

The current MEAP assessment program uses an assessment framework based on the GLCE's. The framework further classifies the GLCE's into core, extended, and future. Core GLCE's are assessed with three items. Extended and future GLCE's are assessed with one item. Linking items based on GLCE's from one grade below are also part of the framework. A scaled score is computed using core, extended and linking items. MEAP provides every student testing at grades 3 through 8 and high school. Individual scores and performance levels are reported and are used for instructional improvement. MEAP is used as the outcome measures to fulfill No Child Left Behind Act requirements.

The NAEP program (http://www.nationsreportcard.gov) uses an assessment framework developed through a consensus process governed by the National Assessment Governing Board (NAGB – http://www.nagb.org). NAGB awarded a contract to the Council of Chief State School Officers (CCSSO) in September 2000 to update the mathematics assessment framework for 2005. According to the document, *NAEP 2005 Mathematics Assessment Framework*, the CCSSO used a consensus framework and established a Steering Committee representative of national policy organizations, mathematics associations, research mathematicians, business and industry, and educators to develop policy recommendations. A Planning Committee of mathematics educators, mathematicians, curriculum supervisors, and teachers was established to draft the content framework. A Technical Advisory Committee was tasked with reviewing policy and content from the perspective of whether the standards would continue to allow the NAEP assessment to continue short-term mathematics trend lines begun in 1990. NAEP uses a different assessment approach than MEAP. NAEP uses a student sampling method and within students, an item sampling method with the unit of measurement being the state or national sampling representation. Individual student data is not retained. In comparison to MEAP, NAEP uses more constructed responses on a given test.

The 2005 NAEP Mathematics Framework refers to the GLCE strands as "content areas," which also consist of the same 5 areas; Numbers and Operations, Algebra, Measurement, Geometry, Data & Probability. NAEP further breaks the content areas into subscales. For example, in Grade 4 - Numbers and Operations, the content area is broken down into 5 subscales; Number Sense, Estimation, Number Operations, Ratios and Proportional Reasoning and Properties of Numbers and Operations. The final level is a series of objectives that make up the subscales.

Comparing NAEP and MEAP content standards cannot be used to provide any means of comparability of outcome measures. Comparison of the two frameworks will elucidate further reasons why the two are not directly comparable. A lengthy discussion of problems in comparing outcomes between NAEP and state wide assessment results can be found in Feuer (1999). Some of the many reasons for not being able to directly compare scores between NAEP and the state-wide MEAP assessment will be included later in this paper. However, NAEP remains the only assessment method to provide a common assessment sample across all states and the results can be useful for comparison among state programs and for informing state policy.

Methods

For the purpose of this study, we compared the GLCE's with the NAEP objectives using the NAEP content area and subscales as a category organizer. These results appear in Tables 1A and 2A in Appendix A.

The Grade 4 NAEP content area subscale objectives were mapped to the Grades K-4 GLCE's. The Grade 8 NAEP content area subscale objectives were mapped to the Grades 5-8 GLCE's. The relationship was generally many (GLCE's) to one (NAEP objective).

Once the cross-matrix mapping was established, we developed a summary table of NAEP Content area > subscale > objective by Michigan GLCE's. In the calculation of the percent of overlapping between the two frameworks, we used a ratio of the total number of subscale expectations on NAEP / a minimum of at least one GLCE per subscale (more than one GLCE in the same subscale still counted as one).

Results

Grade 4 Results.

Table One represents a summary report of Table 1A in Appendix A. The percent of coverage between NAEP and the GLCE was 79%. The emphasis between NAEP objectives and the GLCE's varied greatly. There were 14 occasions where there were NAEP objectives with no corresponding GLCE. The mapping does not work in the other direction however, there were no GLCE's that failed to map to a NAEP objective. The highest ratio of GLCE's to NAEP objectives were two objectives that tied with 11 GLCE's per objective. These NAEP objectives were; "1) Add and subtract: whole numbers; or fractions with like denominators; or decimals through hundredths," and "2) Interpret whole number operations and the relationships between them." In fact both of these objectives fall under the NAEP subscale of Number operations which had the highest ratio of GLCE expectations to NAEP objectives of 39/6 in the entire Grade 4 framework.

Table One. Grade 4 Summary of NAEP / GLCE Comparison and NAEP Results

	NAEP	MI-GLCE	Objective Gap	MI SS Avg	US SS Avg	MI White SS Avg	MI Black SS Avg	MI White/Black Gap
Numbers and operations				235	235	243	219	24
Number sense	6	24						
Estimation	3	4						
Number operations	6	39						
Ratios and proportional reasoning	1	0	1					
Properties of numbers and operations	4	5	1					
Measurement				237	236	244	217	27
Measuring physical attributes	6	8						
Systems of Measurement	4	18	1					
Geometry				237	236	244	222	22
Dimension and Shape	4	13						
Transformation of Shapes and Preservation of Properties	4	5	1					
Relationships between geometric figures	4	8						
Position and Direction	2	1						
Mathematical Reasoning	1	1						
Data Analysis and Probability				242	241	247	227	20
Data Representation	3	9						
Characteristics of Data Sets	2	4						
Experiments and Samples	0	0						
Probability	4	0	4					
Algebra				244	243	250	229	21
Patterns and Relationships	5	6	3					
Algebraic Representation	3	5	1					
Variables, Expressions and Operations	2	10						
Equations and Inequalities	1	4						
Total	**65**	**164**	**12**	238	237	245	213	32
Percent of coverage between NAEP and MI-GLCE			81.5%					

The numbers and operations content area is heavily loaded with GLCE's in comparison to other portions of the test, yet Michigan students score about the same as the National Average. However, the Numbers and Operations and the Measurement subscales account for the largest ethnic gap differences between White and Black students (other minority students had insufficient sample size to compute averages).

Table Two. NAEP objectives that do not match GLCE's.

NAEP subscale	NAEP Objectives that do not match any GLCE
1. Ratios and Proportional Reasoning	Use simple ratios to describe problem situations.
2. Properties of Numbers and Operations	Explain or justify a mathematical concept or relationship (e.g. explain why 15 is an odd number or why 7-3 is not the same as 3-7).
3. Systems of Measurement	Determine appropriate size of unit of measurement in problem situation involving such attributes as length, time, capacity, or weight.
4. Transformation of Shapes and Preservation of Properties	Match or draw congruent figures in a given collection.
5. Probability	Use informal probabilistic thinking to describe events (i.e. likely and unlikely, certain and impossible).
6. Probability	Determine a simple probability from a context that includes a picture.
7. Probability	List all possible outcomes of a given situation or event.
8. Probability	Represent the probability of a given outcome using a picture or other graphic.
9. Patterns, Relations, and Functions	Given a description, extend or find a missing term in a pattern or sequence.
10. Patterns, Relations, and Functions	Create a different representation of a pattern or sequence given a verbal description.
11. Patterns, Relations, and Functions	Recognize or describe a relationship in which quantities change proportionally.
12. Algebraic Representations	Verify a conclusion using algebraic properties

Two areas of content stand out as underrepresented in comparison to NAEP and the GLCE's. There are no Probability GLCE's in the grades K-4 GLCE's. There is also significant under representation of the Patterns, Relations, and Functions.

Grade 8 Results.

Table Three represents a summary report of Table 2A in Appendix A. Table Three refers to the overall results of grade 8 NAEP and GLCE comparisons. The percent of coverage between NAEP and the GLCE was 78%. The emphasis between NAEP objectives and the GLCE's varied greatly. There were 23 occasions where there were NAEP objectives with no corresponding GLCE. The mapping does not work in the other direction however, there were no GLCE's that failed to map to a NAEP objective. The highest ratio of GLCE's to NAEP objectives was "Perform computations with rational numbers," mapping to 20 GLCE's to one NAEP objective. This objective falls under the NAEP subscale of Number operations which had the highest ratio of GLCE expectations to NAEP objectives of 101/31 in the entire Grade 8 framework.

Table Three. Grade 8 Summary of NAEP / GLCE Comparison and NAEP Results

	NAEP	MI-GLCE	Objective Gap	MI SS Avg	US SS Avg	MI – White SS Avg	MI – Black SS Avg	Mi White/ Black Gap
Numbers and operations				275	276	282	249	33
Number sense	12	36						
Estimation	4	9						
Number operations	5	30	1					
Ratios and proportional reasoning	4	13						
Properties of numbers and operations	6	13						
Measurement				275	274	287	236	51
Measuring physical attributes	7	27	2					
Systems of Measure	6	12	1					
Geometry				274	275	282	248	34
Dimension and Shape	6	6	2					
Transformation of Shapes and Preservation of Properties	5	10						
Relationships between geometric figures	5	10	1					
Position and Direction	4	0	4					
Mathematical Reasoning	1	1						
Data Analysis and Probability				284	280	292	255	37
Data Representation	5	4	2					
Characteristics of Data Sets	5	8	1					
Experiments and Samples	3	1	2					
Probability	7	6	2					
Algebra				279	281	286	254	32
Patterns and Relationships	5	0	5					
Algebraic Representation	6	16						
Variables, Expressions and Operations	2	11						
Equations and Inequalities	5	15						
Total	**103**	**228**	**23**	284	280	292	255	37
Percent of coverage between NAEP and GLCE			77.7%					

While the average Michigan student score continues to be about the same as the National Average. The White/Black Average Scale Score gap is considerably larger than at Grade 4. The most alarming difference is an average of 51 scale score points on Measurement. There appears to be a significant amount of GLCE coverage although there are 3 NAEP objectives not covered by GLCE's.

Table Four. Grade 8 NAEP objectives that do not match GLCE's.

NAEP subscale	NAEP Objectives that do not match any GLCE
1 Number operations	Describe the effect of multiplying and dividing by numbers including the effect of multiplying by zero or; a number less than zero, or; a number between zero and one; one or ; a number greater than one.
2. Properties of number and operations	Describe odd and even integers and how they behave under different operations.
3. Measuring physical attributes	Estimate the size of an object with respect to a given measurement attribute.
4. Measuring physical attributes	Solve problems involving indirect measurement such as finding the height of a building by comparing its shadow with the height and shadow of a known object.
5. Systems of Measurement	Determine appropriate size of unit of angle, or volume measurement in problem situation involving such attributes as length, area, or volume.
6. Dimension and Shape	Identify a geometric object given a written description of its properties.
	Identify, define, or describe geometric shapes in the plane and in three-dimensional space given a visual representation.
7. Relationships between geometric figures	Represent problem situations with simple geometric models to solve mathematical or real-world problems.
8. Position and Direction	Describe relative positions of points and lines using the geometric ideas of midpoint, points on common line through a common point, parallelism, or perpendicularity.
9. Position and Direction	Describe the intersection or two or more geometric figures in the plane.
10. Position and Direction	Visualize or describe the cross section of a solid.
11. Position and Direction	Represent geometric figures using rectangular coordinates on a plane.
12. Data Representation	Read or interpret data, including interpolating or extrapolating from data.
13. Data Representation	Compare and contrast the effectiveness of different representations of the same data.
14. Characteristics of Data Sets	Identify outliers and determine their effect on mean, median, mode or range.
15. Experiments and Samples	Distinguish between a random and nonrandom sample.
16. Experiments and Samples	Evaluate the design of an experiment.
17. Probability	Estimate the probability of simple and compound events through experimentation or simulation.
18. Probability	Determine the sample space for a given situation
19. Patterns, Relations, and Functions	Recognize, describe or extend numerical and geometric patterns using tables, graphs, words, or symbols.
20. Patterns, Relations, and Functions	Generalize a pattern appearing in a numerical sequence or table or graph using words or symbols.
21. Patterns, Relations, and Functions	Analyze or create patterns, sequences, or linear functions given a rule.
22. Patterns, Relations, and Functions	Identify functions as linear or nonlinear or contrast distinguishing properties of functions from tables, graphs, or equations.
23. Patterns, Relations, and Functions	Interpret the meaning of slope or intercepts in linear functions.

The number of NAEP objectives not covered by by GLCE's is 23 on the Grade 8 comparison. The NAEP subscale and all subsequent objectives in Patterns and Relationships are entirely missing from the GLCE coverage.

Discussion

NAEP is a voluntary assessment program and it is not incumbent on Michigan to adopt any NAEP objectives that are not currently covered by Michigan curricula. Comparisons in the next two NAEP state

samples in Mathematics (2007 and 2009) will be interesting since at the time of the 2005 cycle, the GLCE's had been in place for less than one year. It will be interesting to see what 3 or 5 years mean for the effects on student performance of the relatively new GLCE's. It is also important to keep in mind that the GLCE's were not intended to be the entire curriculum of a local educational agency. It would appear that given how Michigan compares well to the National Average subscale score performance on areas that the GLCE's do not cover or emphasize - there is some instruction (also perhaps some innate mathematical understanding) going on in those curriculum areas not covered by MDE GLCE's.

Both assessment frameworks, MEAP and NAEP appear to address about 80% of the same content. However it is evident that the MEAP assessments, which are mapped to the GLCE's will have a very different emphasis based on measuring outcomes from each GLCE. In addition, the MEAP test uses fewer constructed response items, and assesses every student with scores reported at the student level as opposed to a stratified random sample of students taking a sample matrix of assessment items anonymously for NAEP. In addition, there is absolutely no relationship between how cut scores for each performance level are set between the MEAP system and NAEP nor any type of equating or linking of scaled scores. While both tests measure very similar content, there are too many differences in content emphasis and testing methods to allow any kind of comparability between scores.

Differences between White and Black ethnic groups in performance on both the 4^{th} and 8^{th} grade NAEP tests are a cause for concern. The most significant discrepancy is Grade 8 Measurement. MDE and Michigan LEA's should look carefully at MEAP results for the same pattern. If a similar disparity in MEAP profiles are found, this might be worthy of making it a high priority area of emphasis to improve minority achievement.

The next time the GLCE's are open for revision, the curriculum committees may wish to use the discrepancies found in this study as a guide for discussing GLCE reform.

References

Feuer, M.J., Holland, P.W., Green, B.F., Berethal, M.W., & Hemphill, F.C. (Eds.). (1999). *Uncommon Measures: Equivalence and Linking Among Educational Tests.* Washington D.C.: National Academy Press.

Appendix A.

Table 1A. Grade 4 Cross Matrix Table Between NAEP objectives and GLCE expectations.

NUMBER AND OPERATIONS – Grade 4

Number sense

NAEP	GLCE
Identify the place value and actual value of digits in number.	**N.ME.03.02** Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402; identify the place value of a digit in a number, e.g., in 3,241, 2 is in the hundreds place.
Represent numbers using models such as base 10 representations, number lines, and two-dimensional models.	**N.ME.02.05** Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials. **N.MR.02.07** Find the distance between numbers on the number line, e.g., how far is 79 from 26? **N.MR.02.09** Given a contextual situation that involves addition and subtraction for numbers up to two digits: model using objects or pictures, explain in words, record using numbers and symbols; solve. **N.ME.02.20** Place 0 and halves, e.g., on the number line; relate to a ruler. **N.ME.03.17** Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models. **N.ME.03.18** Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8.
Compose or decompose whole quantities by place value.	**N.ME.01.07** Compose and decompose numbers to 30, including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones. **N.ME.02.05** Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.
Write or rename whole numbers (e.g. 10: 5+5, 12-2, 2x5).	**N.ME.01.08** List number facts (partners inside of numbers) for 2 through 10; e.g., 8 = 7 + 1 = 6 + 2 = 5 + 3 = 4 + 4; 10 = 8 + 2 = 2 + 8. **N.FL.02.06** Decompose 100 into addition pairs, e.g., 99 + 1, 98 + 2 …
Connect model, number word, or number using various models and representations for whole numbers, fractions and decimals.	**N.ME.02.22** Recognize that fractions such as are equal to the whole (one). **N.ME.03.21** Understand the meaning of $0.50 and $0.25 related to money, e.g., $1.00 shared by two people means $1.00 ÷ 2 = dollar = $0.50. **N.ME.04.15** Read and interpret decimals up to two decimal places; relate to money and place value decomposition. **N.ME.04.16** Know that terminating decimals represent fractions whose denominators are 10, 10 x 10, 10 x 10 x 10, etc. e.g., powers of 10. **N.ME.04.17** Locate tenths and hundredths on a number line. **N.ME.04.18** Read, write, interpret, and compare decimals up to two decimal places.

NAEP	GLCE
Order and compare whole numbers, decimals, and fractions.	**N.ME.01.03** Order numbers to 110; compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21,16, 35, 8. **N.ME.01.04** Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100. **N.MR.01.09** Compare two or more sets in terms of the difference in number of elements.**N.ME.04.17** Locate tenths and hundredths on a number line. **N.ME.02.21** For unit fractions from 1/12 to 1/2, unde rstand the inverse relationship between the size of a unit fraction and the size of the denominator; compare unit fractions from 1/12 to 1/2. **N.ME.03.03** Compare and order numbers up to 10,000. **N.ME.03.18** Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8. **N.ME.04.18** Read, write, interpret, and compare decimals up to two decimal places.

Estimation

NAEP	GLCE
Use benchmarks for whole numbers, decimals, and fractions in contexts (e.g. ½ and 0.5 may be used as benchmarks for fractions and decimals between 0 and 1.00).	**N.MR.04.19** Write tenths and hundredths in decimal and fraction forms, and know the decimal equivalents for halves and fourths.
Make estimates appropriate to a given situation with whole numbers, fractions, and decimals by: Knowing when to estimate. Selecting the appropriate type of estimate including, overestimate, underestimate, and range of estimate. Selecting the appropriate method of estimation.	**N.FL.02.11** Estimate and calculate the sum of two numbers with three digits that do not require regrouping. **N.FL.03.07** Estimate the sum and difference of two numbers with three digits (sums up to 1000), and judge reasonableness of estimates.
Verify solutions and determine the reasonableness of results.	**N.FL.03.07** Estimate the sum and difference of two numbers with three digits (sums up to 1000), and judge reasonableness of estimates.

Number Operations

NAEP	GLCE
Add and subtract: whole numbers; or fractions with like denominators; or decimals through hundredths.	**N.FL.01.12** Know all the addition facts up to 10 + 10, and solve the related subtraction problems fluently. **N.FL.01.14** Add three one-digit numbers. **N.FL.01.16** Compute sums and differences up to two-digit numbers using number facts and strategies, but no formal algorithm. **N.FL.02.10** Add fluently two numbers up to two digits each, using strategies including formal algorithms; subtract fluently two numbers up to two digits each. **N.FL.02.11** Estimate and calculate the sum of two numbers with three digits that do not require regrouping. **N.FL.02.12** Calculate mentally sums and differences involving: three-digit numbers and ones; three-digit numbers and tens; three-digit numbers and hundreds. **N.FL.03.06** Add and subtract fluently two numbers: up to and including two-digit numbers with regrouping and up to four-digit numbers without regrouping. **N.FL.03.07** Estimate the sum and difference of two numbers with three digits (sums up to 1000), and judge reasonableness of estimates. **N.FL.03.08** Use mental strategies to fluently add and subtract two-digit numbers. **N.ME.03.19** Understand that any fraction can be written as a sum of unit fractions, e.g.. **N.MR.03.20** Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining or taking away segments on the number line.
Multiply whole numbers: no larger than two-digit x two-digit with paper and pencil computation; or larger numbers with use of calculator.	**N.FL.02.17** Develop strategies for fluently multiplying numbers up to 5 x 5. **N.FL.03.11** Find products fluently up to 10 x 10; find related quotients using multiplication and division relationships. **N.FL.03.13** Mentally calculate simple products and quotients: up to a three-digit number by a one-digit number involving multiples of 10, e.g., 500 x 6, or 4008. **N.FL.04.10** Multiply fluently any whole number by a one-digit number, and a three-digit number by a two-digit number; for two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm.
Divide whole numbers: up to a three-digits by a one-digit with paper and pencil; or up to five-digits by two-digits with use of calculator.	**N.MR.03.12** Find solutions to open sentences, such as 7 x = 42 or 12 ÷ = 4, using the inverse relationship between multiplication and division. **N.FL.03.13** Mentally calculate simple products and quotients: up to a three-digit number by a one-digit number involving multiples of 10, e.g., 500 x 6, or 4008. **N.MR.03.14** Solve simple division problems involving remainders, viewing remainder as the "number left over" (less than the divisor), e.g., 4 children per group; we have 25 children; there are 6 groups with 1 child left over; interpret based on problem context. **N.FL.04.11** Divide numbers up to four-digits by one-digit numbers and by 10. **N.FL.04.12** Find unknowns in equations such as
Describe the effect of operations on size (whole numbers).	**N.MR.01.10** Model addition and subtraction for numbers less than 20 for a given contextual situation using objects or pictures; explain in words; record using numbers and symbols; solve. **N.MR.02.14** Represent multiplication using area and array models.

NAEP	GLCE
Interpret whole number operations and the relationships between them.	**N.MR.01.11** Understand the inverse relationship between addition and subtraction, e.g., subtraction "undoes" addition: if 3 + 5 = 8, we know that 8 – 3 = 5 and 8 – 5 = 3; recognize that some problems involving combining, "taking away", or comparing can be solved by either operation. **N.MR.02.08** Find missing values in open sentences, e.g., 42 + = 57; use relationship between addition and subtraction. **N.MR.02.13** Understand multiplication as the result of counting the total number of objects in a set of equal groups, e.g., 3 x 5 gives the number of objects in 3 groups of 5 objects, or 3 x 5 = 5 + 5 + 5 = 15. **N.MR.02.15** Understand division (÷) as another way of expressing multiplication, using fact families within the 5 x 5 multiplication table; emphasize that division "undoes" multiplication, e.g., 2 x 3 = 6 can be rewritten as 6 ÷ 2 = 3 or 6 ÷ 3 = 2. **N.ME.03.04** Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication. **N.MR.03.09** Use multiplication and division fact families to understand the inverse relationship of these two operations, e.g., because 3 x 8 = 24, we know that 24 ÷ 8 = 3 or 24 ÷ 3 = 8; express a multiplication statement as an equivalent division statement. **N.MR.03.10** Recognize situations that can be solved using multiplication and division including finding "How many groups?" and "How many in a group?" and write mathematical statements for those situations. **N.FL.03.11** Find products fluently up to 10 x 10; find related quotients using multiplication and division relationships. **N.MR.03.12** Find solutions to open sentences, such as 7 x = 42 or 12 ÷ = 4, using the inverse relationship between multiplication and division. **N.MR.04.13** Use the relationship between multiplication and division to simplify computations and check results. **N.FL.04.12** Find unknowns in equations such as
Solve application problems involving numbers and operations.	**N.MR.03.15** Given problems that use any one of the four operations with appropriate numbers, represent with objects, words, (including "product" and "quotient"), and mathematical statements; solve. **N.FL.04.14** Solve applied problems involving whole number multiplication and division.

Ratios and Proportional Reasoning

NAEP	GLCE
Use simple ratios to describe problem situations.	

Properties of number and operations

NAEP	GLCE
Identify odd and even numbers.	**N.ME.03.05** Know that even numbers end in 0, 2, 4, 6, or 8; name a whole number quantity that can be shared in two equal groups or grouped into pairs with no remainders; recognize even numbers as multiples of 2. Know that odd numbers end in 1, 3, 5, 7, or 9, and work with patterns involving even and odd numbers.
Identify the factors of whole numbers.	**N.ME.04.04** Find all factors of a whole number up to 50, and list factor pairs.

NAEP	GLCE
Apply basic properties of operations.	**N.MR.02.15** Understand division (÷) as another way of expressing multiplication, using fact families within the 5 x 5 multiplication table; emphasize that division "undoes" multiplication, e.g., 2 x 3 = 6 can be rewritten as 6 ÷ 2 = 3 or 6 ÷ 3 = 2. **N.FL.03.11** Find products fluently up to 10 x 10; find related quotients using multiplication and division relationships. **N.FL.04.10** Multiply fluently any whole number by a one-digit number, and a three-digit number by a two-digit number; for two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm.
Explain or justify a mathematical concept or relationship (e.g. explain why 15 is an odd number or why 7-3 is not the same as 3-7).	

MEASUREMENT: Grade 4

Measuring physical attributes

NAEP	GLCE
Identify the attribute that is appropriate to measure in a given situation.	**M.UN.03.07** Distinguish between units of length and area and choose a unit appropriate in the context. **M.UN.04.01** Measure using common tools and select appropriate units of measure.
Compare objects with respect to a given attribute, such as length, area, capacity, time and temperature.	**M.UN.01.02** Compare measured lengths using the words shorter, shortest, longer, longest, taller, tallest, etc. **M.PS.02.02** Compare lengths; add and subtract lengths (no conversion of units).
Estimate the size of an object with respect to a given measurement attribute.	**M.TE.03.09** Estimate the perimeter of a square and rectangle in inches and centimeters; estimate the area of a square and rectangle in square inches and square centimeters.
Select and use appropriate measurement instruments such as ruler, meter stick, protractor, thermometer, clock; calendar, or other scaled instruments.	**M.UN.02.09** Read temperature using the scale on a thermometer in degrees Fahrenheit.
Solve problems involving perimeter of plane figures.	**M.PS.03.13** Solve contextual problems about perimeters of rectangles and areas of rectangular regions.
Solve problems involving area of rectangles.	**M.PS.03.13** Solve contextual problems about perimeters of rectangles and areas of rectangular regions.

Systems of Measurement

NAEP	GLCE
Select or use appropriate type of unit for the attribute being measured such as length, time, or temperature.	**M.UN.01.03** Tell time on a twelve-hour clock face to the hour and half hour. **M.UN.01.04** Identify the different denominations of coins and bills. **M.UN.02.01** Measure lengths in meters, centimeters, inches, feet, and yards approximating to the nearest whole unit and using abbreviations: cm, m, in, ft, yd.

NAEP	GLCE
Solve problems involving conversions within the same measurement system such as conversions involving inches and feet or hours and minutes.	**M.TE.04.05** Carry out the following conversions from one unit of measure to a larger or smaller unit of measure: meters to centimeters, kilograms to grams, liters to milliliters, hours to minutes, minutes to seconds, years to months, weeks to days, feet to inches, ounces to pounds (using numbers that involve only simple calculations.)
Determine appropriate size of unit of measurement in problem situation involving such attributes as length, time, capacity, or weight.	
Determine situations in which a highly accurate measurement is important.	**M.PS.04.02** Give answers to a reasonable degree of precision in the context of a given problem.

GEOMETRY: Grade 4

Dimension and Shape

NAEP	GLCE
Explore properties of paths between two points.	**G.GS.02.04** Distinguish between curves and straight lines and between curved surfaces and flat surfaces. **G.GS.03.01** Identify points, line segments, lines and distance. **G.GS.03.02** Identify perpendicular lines and parallel lines in familiar shapes and in the classroom. **G.GS.04.01** Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90) corner.
Identify or describe (informally) real world objects using simple plane figures such as triangles, rectangles, squares and circles and simple solid figures such as cubes, spheres, and cylinders.	**G.GS.00.01** Relate familiar three-dimensional objects inside and outside the classroom to their geometric name, e.g., ball/sphere, box/cube, soup can/cylinder, ice cream cone/cone, refrigerator/prism. **G.SR.02.05** Classify familiar plane and solid objects, e.g., square, rectangle, rhombus, cube, pyramid, prism, cone, cylinder, and sphere, by common attributes such as shape, size, color, roundness or number of corners and explain which attributes are being used for classification. **G.GS.03.03** Identify parallel faces of rectangular prisms, in familiar shapes and in the classroom.
Identify or draw angles and other geometric figures in the plane.	**G.GS.03.04** Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices. **G.GS.04.02** Identify basic geometric shapes, including isosceles, equilateral and right triangles, and use their properties to solve problems.
Describe attributes of 2- and 3- dimensional shapes.	**G.GS.00.02** Identify, sort and classify objects by attribute and identify objects that do not belong in a particular group. **G.GS.02.01** Identify, describe, and compare familiar two-dimensional and three-dimensional shapes, such as triangles, rectangles, squares, circles, semi-circles, spheres, rectangular prisms. **G.GS.03.04** Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices. **G.GS.03.06** Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).

Transformation of Shapes and Preservation of Properties

NAEP	GLCE
Identify whether a figure is symmetrical or draw lines of symmetry.	**G.TR.04.04** Recognize plane figures that have line symmetry.
Identify the images resulting from flips (reflections), slides (translations) or turns (rotations).	**G.TR.02.06** Recognize that shapes that have been slid, turned or flipped are the same shape, e.g., a square rotated 45° is still a square. **G.TR.04.05** Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.
Recognize which attributes change or don't change when plane figures are subdivided and rearranged.	**G.GS.02.02** Explore and predict the results of putting together and taking apart two-dimensional and three-dimensional shapes. **G.GS.03.04** Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices.
Match or draw congruent figures in a given collection.	

Relationships between geometric figures

NAEP	GLCE
Analyze or describe patterns of geometric figures by increasing number of sides, changing size or orientation (e.g. polygons with more and more sides).	**G.GS.00.03** Create, describe, and extend simple geometric patterns. **G.SR.01.03** Create and describe patterns, such as repeating patterns, and growing patterns using number, shape, and size.
Assemble simple plane shapes to construct a given shape.	**G.GS.01.01** Create common two-dimensional and three-dimensional shapes and describe their physical and geometric attributes, such as color and shape. **G.GS.02.02** Explore and predict the results of putting together and taking apart two-dimensional and three-dimensional shapes. **G.GS.03.06** Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).
Recognize 2-dimensional faces of 3-dimensional shapes.	**G.SR.03.07** Represent front, top, and side views of solids built with cubes. **G.SR.04.03** Identify and count the faces, edges, and vertices of basic three-dimensional geometric solids including cubes, rectangular prisms, and pyramids; describe the shape of their faces.
Describe and compare properties of simple and compound figures composed of triangles, squares, and rectangles.	**G.GS.02.02** Explore and predict the results of putting together and taking apart two-dimensional and three-dimensional shapes.

Position and Direction

NAEP	GLCE
Describe relative positions of points and lines using the geometric ideas of parallelism or perpendicularity.	**G.GS.03.02** Identify perpendicular lines and parallel lines in familiar shapes and in the classroom.
Construct geometric figures with vertices at points on a coordinate grid.	**G.LO.02.07** Find and name locations using simple coordinate systems such as maps and first quadrant grids.

Mathematical Reasoning

NAEP	GLCE
Distinguish which objects in a collection satisfy a given geometric definition and explain choices.	**G.GS.00.02** Identify, sort and classify objects by attribute and identify objects that do not belong in a particular group.

DATA ANALYSIS AND PROBABILITY: Grade 4

Data Representation

NAEP	GLCE
Read and interpret a single set of data.	**D.RE.01.02** Read and interpret pictographs. **D.RE.02.02** Read and interpret pictographs with scales, using scale factors of 2 and 3. **D.RE.03.01** Read and interpret bar graphs, in both horizontal and vertical forms. **D.RE.03.02** Read scales on the axes and identify the maximum, minimum, and range of values in a bar graph.
For a given set of data, complete a graph.	**D.RE.01.03** Make pictographs of given data, using both horizontal and vertical forms of graphs; scale should be in units of one and include symbolic representations, e.g., ÿ represents one child. **D.RE.02.01** Make pictographs using a scale representation, using scales where symbols equal more than one. **D.RE.04.01** Construct tables and bar graphs from given data.
Solve problems by estimating and computing within a single set of data.	**D.RE.02.03** Solve problems using information in pictographs; include scales such as "each represents 2apples". **D.RE.03.03** Solve problems using information in bar graphs, including comparison of bar graphs.

Characteristics of Data Sets

NAEP	GLCE
Given a set of data or a graph, describe the distribution	**D.RE.03.02** Read scales on the axes and identify the maximum, minimum, and range of values in a bar graph. **D.RE.04.02** Order a given set of data, find the median, and specify the range of values.
Compare two sets of related data.	**D.RE.03.03** Solve problems using information in bar graphs, including comparison of bar graphs. **D.RE.04.03** Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets.

Experiments and Samples

NAEP	GLCE
None at 4th grade	

Probability

NAEP	GLCE
Use informal probabilistic thinking to describe events (i.e. likely and unlikely, certain and impossible) .	

NAEP	GLCE
Determine a simple probability from a context that includes a picture.	
List all possible outcomes of a given situation or event.	
Represent the probability of a given outcome using a picture or other graphic.	

ALGEBRA: Grade 4

Patterns, Relations, and Functions

NAEP	GLCE
Recognize, describe or extend numerical patterns.	**N.MR.00.10** Create, describe, and extend simple number patterns. **N.ME.01.01** Count to 110 by 1's, 2's, 5's, and 10's, starting from any number in the sequence; count to 500 by 100's and 10's; use ordinals to identify position in a sequence, e.g., 1st, 2nd, 3rd . **N.ME.02.01** Count to 1000 by 1's, 10's, and 100's starting from any number in the sequence. **N.ME.03.04** Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication.
Given a pattern or sequence, construct and explain a rule that can generate the terms of the pattern or sequence.	**N.ME.03.04** Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection between repeated addition and multiplication. **N.ME.03.05** Know that even numbers end in 0, 2, 4, 6, or 8; name a whole number quantity that can be shared in two equal groups or grouped into pairs with no remainders; recognize even numbers as multiples of 2. Know that odd numbers end in 1, 3, 5, 7, or 9, and work with patterns involving even and odd numbers.
Given a description, extend or find a missing term in a pattern or sequence.	
Create a different representation of a pattern or sequence given a verbal description.	
Recognize or describe a relationship in which quantities change proportionally.	

Algebraic Representations

NAEP	GLCE
Translate between the different forms of representations (symbolic, numerical, verbal or pictorial) of whole number relationships.	**N.MR.01.10** Model addition and subtraction for numbers less than 20 for a given contextual situation using objects or pictures; explain in words; record using numbers and symbols; solve. **N.MR.02.09** Given a contextual situation that involves addition and subtraction for numbers up to two digits: model using objects or pictures, explain in words, record using numbers and symbols; solve. **N.MR.02.14** Represent multiplication using area and array models. **N.MR.02.16** Given a simple situation involving groups of equal size or of sharing equally, represent with objects, words, and symbols; solve.
Graph or interpret points with whole number or letter coordinates on grids or in the first quadrant of the coordinate plane.	**G.LO.02.07** Find and name locations using simple coordinate systems such as maps and first quadrant grids.
Verify a conclusion using algebraic properties.	

Variables, Expressions, and Operations

NAEP	GLCE
Use letters and symbols to represent an unknown quantity in a simple mathematical expression.	**N.MR.01.13** Apply knowledge of fact families to solve simple open sentences for addition and subtraction, such as: + 2 = 7 and 10 – = 6. **N.MR.02.08** Find missing values in open sentences, e.g., 42 + = 57; use relationship between addition and subtraction. **N.MR.03.12** Find solutions to open sentences, such as 7 x = 42 or 12 ÷ = 4, using the inverse relationship between multiplication and division. **N.FL.04.12** Find unknowns in equations such as
Express simple mathematical relationships using number sentences.	**N.MR.00.09** Record mathematical thinking by writing simple addition and subtraction sentences, e.g., 7 + 2 = 9, 10 – 8 = 2. **N.MR.01.11** Understand the inverse relationship between addition and subtraction, e.g., subtraction "undoes" addition: if 3 + 5 = 8, we know that 8 – 3 = 5 and 8 – 5 = 3; recognize that some problems involving combining, "taking away", or comparing can be solved by either operation. **N.MR.02.13** Understand multiplication as the result of counting the total number of objects in a set of equal groups, e.g., 3 x 5 gives the number of objects in 3 groups of 5 objects, or 3 x 5 = 5 + 5 + 5 = 15. **N.MR.03.09** Use multiplication and division fact families to understand the inverse relationship of these two operations, e.g., because 3 x 8 = 24, we know that 24 ÷ 8 = 3 or 24 ÷ 3 = 8; express a multiplication statement as an equivalent division statement. **N.MR.03.10** Recognize situations that can be solved using multiplication and division including finding "How many groups?" and "How many in a group?" and write mathematical statements for those situations. **N.MR.04.07** Solve problems about factors and multiples, e.g., since 100 = 4 x 25, and 200 = 2 x 100, then 200 = 2 x 4 x 25 = 8 x 25.

Equations and Inequalities

NAEP	GLCE
Find the value of the unknown in a whole number sentence.	**N.MR.01.13** Apply knowledge of fact families to solve simple open sentences for addition and subtraction, such as: + 2 = 7 and 10 – = 6. **N.MR.02.08** Find missing values in open sentences, e.g., 42 + = 57; use relationship between addition and subtraction. **N.MR.03.12** Find solutions to open sentences, such as 7 x = 42 or 12 ÷ = 4, using the inverse relationship between multiplication and division. **N.FL.04.12** Find unknowns in equations such as

Appendix A.
Table 2A. Grade 8 Cross Matrix Table
Between NAEP objectives and GLCE expectations.

NUMBER AND OPERATIONS – Grade 8

Number sense

NAEP	GLCE
Use place value to model and describe integers and decimals.	**N.ME.04.15** Read and interpret decimals up to two decimal places; relate to money and place value decomposition. **N.ME.04.16** Know that terminating decimals represent fractions whose denominators are 10, 10 x 10, 10 x 10 x 10, etc. e.g., powers of 10. **N.ME.05.08** Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths.
Model or describe rational numbers or numerical relationships using number lines and diagrams.	**N.ME.04.17** Locate tenths and hundredths on a number line. **N.ME.04.20** Understand fractions as parts of a set of objects. **N.MR.04.21** Explain why equivalent fractions are equal, using models such as fraction strips or the number line, for fractions with denominators of 12 or less, or equal to 100. **N.MR.04.22** Locate and compare fractions on the number line, including improper fractions and mixed numbers with denominators of 12 or less. **N.MR.04.24** Know that fractions of the form $\frac{m}{n}$, where m is greater than n, are greater than 1 and are called improper fractions; locate improper fractions on the number line; express as mixed numbers. **N.ME.05.10** Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent. **N.ME.06.05** Order rational numbers and place them on the number line. **N.ME.06.17** Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposite sides and at equal distance from 0 on a number line. **N.ME.08.01** Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube.

NAEP	GLCE
	N.ME.08.03 Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g. $0.\bar{1} = \frac{1}{9}$; $0.\bar{3} = \frac{1}{3}$.
Write or rename rational numbers.	**N.MR.04.25** Write improper fractions as mixed numbers, and understand that a mixed number represents the number of "wholes" and the part of a whole remaining, e.g., $\frac{5}{4} = 1 + \frac{1}{4} = 1\frac{1}{4}$. **N.ME.04.18** Read, write, interpret, and compare decimals up to two decimal places. **N.MR.04.19** Write tenths and hundredths in decimal and fraction forms, and know the decimal equivalents for halves and fourths. **N.MR.05.07** Find the prime factorization of numbers between 1 and 50, express in exponential notation, e.g., $24 = 2^3 \times 3^1$, and understand that every whole number can be expressed as a product of primes. **N.ME.05.09** Understand percentages as parts out of 100, use % notation, and express a part of a whole as a percentage. **N.ME.05.11** Given two fractions, express them as equivalent fractions with a common denominator, but not necessarily a <u>least</u> common denominator, e.g., $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{4} = \frac{6}{8}$; use denominators less than 12, or factors of 100. **N.MR.05.22** Express fractions and decimals as percentages, and vice versa. **N.ME.06.06** Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations. **N.ME.06.07** Understand that a fraction or a negative fraction is a quotient of two integers, e.g., $-\frac{8}{3}$ is -8 divided by 3.
Recognize, translate between, or apply multiple representations of rational numbers in meaningful contexts.	**N.FL.06.13** Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers. **N.ME.08.01** Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its

NAEP	GLCE
	connection to the volume of a cube.
Express or interpret numbers using scientific notation from real-life contexts.	**N.ME.06.16** Understand and use integer exponents, excluding powers of negative numbers; express numbers in scientific notation.
Find or model absolute value or apply to problem situations.	**N.ME.06.20** Know that the absolute value of a number is the value of the number, ignoring the sign; or is the distance of the number from 0.
Order or compare rational numbers using various models and representations.	**N.MR.04.21** Explain why equivalent fractions are equal, using models such as fraction strips or the number line, for fractions with denominators of 12 or less, or equal to 100. **N.MR.04.22** Locate and compare fractions on the number line, including improper fractions and mixed numbers with denominators of 12 or less. **N.MR.04.23** Understand the relationships among halves, fourths and eighths and among thirds, sixths and twelfths. **N.ME.08.02** Understand meanings for zero and negative integer exponents. **N.ME.08.01** Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube.
Order or compare rational numbers including very large and small integers, decimals and fractions close to zero.	**N.ME.04.01** Read and write numbers to 1,000,000; relate them to the quantities they represent; compare and order. **N.ME.04.03** Understand the magnitude of numbers up to 1,000,000; recognize the place value's of numbers, and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds. **N.ME.05.10** Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent. **N.ME.05.11** Given two fractions, express them as equivalent fractions with a common denominator, but not necessarily a <u>least</u> common denominator, e.g., $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{4} = \frac{6}{8}$; use denominators less than 12, or factors of 100. **N.ME.06.05** Order rational numbers and place them on the number line.

Estimation

NAEP	GLCE
Establish or apply benchmarks for rational numbers and common irrational numbers in context.	**N.ME.08.04** Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g. $\sqrt{2}$, $\sqrt{3}$, π on the number line. **N.FL.08.06** Find square roots of perfect squares and approximate the square roots of non-perfect squares by locating between consecutive integers, e.g. $\sqrt{130}$ is between 11 and 12.
Make estimates appropriate to a given situation by: identifying when estimation is appropriate; determining the level of accuracy needed; selecting the appropriate method of estimation, or; analyzing the effect of an estimation method on the accuracy of results.	**N.FL.04.34** Estimate the answers to calculations involving addition, subtraction, or multiplication. **N.FL.04.36** Make appropriate estimations and calculations fluently with whole numbers using mental math strategies. **N.FL.07.09** Estimate results of computations with rational numbers.
Verify solutions or determine the reasonableness of results.	**N.FL.04.35** Know when approximation is appropriate and use it to check the reasonableness of answers; be familiar with common place-value errors in calculations.
Estimate square or cube roots of numbers less than 1,000 between two whole numbers.	**N.MR.07.06** Understand the concept of square root and cube root, and estimate using calculators. **N.FL.08.05** Estimate and solve problems with square roots and cube roots, using calculators. **N.FL.08.06** Find square roots of perfect squares and approximate the square roots of non-perfect squares by locating between consecutive integers, e.g. $\sqrt{130}$ is between 11 and 12.

Number Operations

NAEP	GLCE
Perform computations with rational numbers.	**N.FL.04.10** Multiply fluently any whole number by a one-digit number, and a three-digit number by a two-digit number; for two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm. **N.FL.04.11** Divide numbers up to four-digits by one-digit numbers and by 10. **N.MR.04.27** Add and subtract fractions less than 1 with denominators 12 or less and including 100, in cases where the denominators are equal or when one denominator is a multiple of the other; e.g., $\frac{1}{12}+\frac{5}{12}=\frac{6}{12}; \frac{2}{25}+\frac{7}{50}=\frac{11}{50}$. **N.MR.04.29** Solve for the unknown in equations such as:

NAEP	GLCE
	$\frac{1}{8} + x = \frac{5}{8}$ *or* $\frac{3}{4} - y = \frac{1}{2}$. **N.MR.04.30** Multiply fractions by whole numbers, using repeated addition and area or array models. **N.MR.04.31** Use mathematical statements to represent problems that use addition and subtraction of decimals with up to two-digits; solve. **N.FL.04.32** Add and subtract decimals up to two decimal places. **N.FL.04.33** Multiply and divide decimals up to two decimal places by a one-digit whole number where the result is a terminating decimal, e.g., $0.42 \div 3 = 0.14$, but not $5 \div 3 = 1.\overline{6}$. **N.FL.05.04** Multiply a multi-digit number by a two-digit number; recognize and be able to explain common computational errors such as not accounting for place value. **N.FL.05.06** Divide fluently up to a four-digit number by a two-digit number. **N.FL.05.12** Find the product of two unit fractions with small denominators using area model. **N.FL.05.13** Divide a fraction by a whole number and a whole number by a fraction, using simple unit fractions. **N.FL.05.14** Add and subtract fractions with unlike denominators of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 100, using the common denominator that is the product of the denominators of the 2 fractions. **N.MR.05.15** Multiply a whole number by powers of 10: 0.01, 0.1, 1, 10, 100, and 1000; identify patterns. **N.FL.05.16** Divide numbers by 10's, 100's, 1000's, using mental strategies. **N.MR.05.17** Multiply one-digit and two-digit whole numbers by decimals up to two decimal places. **N.FL.06.04** Multiply and divide any two fractions, including mixed numbers, fluently. **N.FL.06.09** Add, subtract, multiply, and divide integers between -10 and 10; use number line and strip models for addition and subtraction. **N.FL.06.10** Add, subtract, multiply and divide positive rational numbers fluently. **N.FL.07.08** Add, subtract, multiply and divide negative rational numbers.
Describe the effect of multiplying and dividing by numbers including the effect of multiplying by zero or; a number less than zero, or; a number between zero	

NAEP	GLCE
and one; one or ; a number greater than one.	
Provide a mathematical argument to explain operations with two or more fractions.	**N.MR.06.01** Understand division of fractions as the inverse of multiplication.
Interpret rational number operations and the relationships between them.	**N.FL.04.12** Find unknowns in equations such as $a \div 10 = 25; 125 \div b = 25$ **N.MR.04.13** Use the relationship between multiplication and division to simplify computations and check results. **N.MR.05.21** Solve for the unknown in such equations as: $\frac{1}{4} + x = \frac{7}{12}$ **N.MR.06.03** Solve for the unknown in equations such as: $\frac{1}{4} \div \square = 1$, $\frac{3}{4} \div \square = \frac{1}{4}$ and $\frac{1}{2} = 1 \times \square$ **N.MR.06.08** Understand integer subtraction as the inverse of integer addition; add and subtract integers, using integers from 10 to -10.
Solve application problems involving rational numbers and operations using exact answers or estimates as appropriate.	**N.FL.04.14** Solve applied problems involving whole number multiplication and division. **N.FL.04.28** Solve fraction problems involving sums and differences for fractions where one denominator is a multiple of the other (denominators 2 through 12, and 100). **N.MR.05.05** Solve applied problems involving multiplication and division of whole numbers. **N.FL.05.18** Given an applied situation involving addition and subtraction of fractions, write mathematical statements describing the situation. **N.MR.05.19** Solve word problems that involve finding sums and differences of fractions with unlike denominators, using knowledge of equivalent fractions. **N.FL.05.20** Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money. **N.FL.06.14** For applied situations, estimate the answers to calculations involving operations with rational numbers. **N.FL.06.15** Solve applied problems that use the four operations with appropriate decimal numbers. **N.FL.07.07** Solve problems involving operations with integers.

Ratios and Proportional Reasoning

NAEP	GLCE
Use ratios to describe problem situations.	**N.MR.08.11** Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.
Use fractions to represent and express ratios and proportions.	**N.ME.05.23** Express ratios in several ways, given applied situations, e.g., 3 cups to 5 people, 3:5; $\frac{3}{5}$; recognize and find equivalent ratios. **N.ME.06.11** Find equivalent ratios by scaling up or scaling down.
Use proportional reasoning to model and solve problems.	**N.FL.07.02** Solve problems involving derived quantities. **N.FL.07.03** Calculate rates of change, including speed. **N.MR.07.04** Convert ratio quantities between different systems of units, such as feet per second to miles per hour. **N.FL.07.05** Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation a/b = c/d; know how to see patterns about proportional situations in tables. **N.FL.08.09** Solve problems involving compounded interest or multiple discounts. **N.MR.08.10** Calculate weighted averages such as course grades, consumer price indices, and sports ratings. **N.MR.08.11** Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.
Solve problems involving percentages.	**N.FL.06.12** Calculate part of a number given the percentage and the number. **N.FL.06.13** Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers. **N.MR.08.07** Understand percent increase and percent decrease in both sum and product form, e.g. 3% increase of a quantity x is x + .03x = 1.03x **N.MR.08.08** Solve problems involving percent increases and decreases.

Properties of number and operations

NAEP	GLCE
Describe odd and even integers and how they behave under different operations.	
Recognize, find or use factors, multiples or prime factorization.	**N.ME.04.04** Find all factors of a whole number up to 50, and list factor pairs. **N.ME.04.05** List the first ten multiples of a given one-digit whole number; determine if a whole number is a multiple of a given one-digit whole number, and if a one-digit number is a factor of a given whole number. **N.MR.04.07** Solve problems about factors and multiples, e.g., since 100 = 4 x 25, and 200 = 2 x 100, then 200 = 2 x 4 x 25 = 8 x 25. **N.MR.05.07** Find the prime factorization of numbers between 1 and 50, express in exponential notation, e.g., $24 = 2^3 \times 3^1$, and understand that every whole number can be expressed as a product of primes.
Recognize or use prime and composite numbers to solve problems.	**N.MR.04.06** Know that some numbers, including 2, 3, 5, 7, and 11 have exactly two factors (1 and the number itself) and are called prime numbers.
Use divisibility or remainders in problem settings.	**N.MR.05.01** Understand the meaning of division of whole numbers, with and without remainders; relate division to fractions and to repeated subtraction. **N.MR.05.02** Relate division of whole numbers with remainders to the form a = bq + r, e.g., 34 ÷ 5 = 6 r 4, so 5 · 6 + 4 = 34; note remainder (4) is less than divisor (5).
Apply basic properties of operations.	**N.ME.04.09** Multiply two-digit numbers by 2, 3, 4, and 5, using the distributive property, e.g.,21 x 3 = (1 + 20) x 3 = (1 x 3) + (20 x 3) = 3 + 60 = 63. **N.FL.04.10** Multiply fluently any whole number by a one-digit number, and a three-digit number by a two-digit number; for two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm. **A.PA.07.11** Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition.

Explain or justify a mathematical concept or relationship.	**N.ME.06.07** Understand that a fraction or a negative fraction is a quotient of two integers, e.g., $-\frac{8}{3}$ is -8 divided by 3. **N.ME.06.18** Understand that rational numbers are quotients of integers (nonzero denominators), e.g., a rational number is either a fraction or a negative fraction. **N.ME.06.19** Understand that 0 is an integer that is neither negative nor positive.

MEASUREMENT: Grade 8

Measuring physical attributes

NAEP	GLCE
Compare objects with respect to a length, area, volume, angle measurement, weight, or mass.	**M.TE.04.10** Identify right angles and compare angles to right angles. **G.TR.05.01** Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4 and full turns.
Estimate the size of an object with respect to a given measurement attribute.	
Select and use appropriate measurement instruments to determine or create a given length, area, volume, angle, weight, or mass.	**M.UN.04.01** Measure using common tools and select appropriate units of measure. **G.GS.05.02** Measure angles with a protractor and classify them as acute, right, obtuse or straight.

Solve mathematical or real-world problems involving perimeter or area of plane figures such as triangles, rectangles, circles, or composite figures.	**M.TE.04.06** Know and understand the formulas for perimeter and area of a square and a rectangle; calculate the perimeters and areas of these shapes and combinations of these shapes using the formulas. **M.TE.04.07** Find one dimension of a rectangle given the other dimension and its perimeter or area. **M.TE.04.08** Find the side of a square given its perimeter or area. **M.PS.04.09** Solve contextual problems about perimeter and area of squares and rectangles in compound shapes. **M.PS.05.05** Represent relationships between areas of rectangles, triangles and parallelograms using models. **M.TE.05.06** Understand and know how to use the area formula of a triangle and represent using models and manipulatives. **M.TE.05.07** Understand and know how to use the area formula for a parallelogram and represent using models and manipulatives. **G.SR.08.03** Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems. **G.SR.08.04** Find area and perimeter of complex figures by subdividing them into basic shapes (quadrilaterals, triangles, circles). **G.SR.08.05** Solve applied problems involving areas of triangles, quadrilaterals, and circles.

Solve problems involving volume or surface area of rectangular solids, cylinders, prisms, or composite shapes.	**M.TE.04.04** Measure surface area of cubes and rectangular prisms by covering and counting area of the faces. **M.PS.04.11** Solve contextual problems about surface area. **M.TE.05.08** Build solids with unit cubes and state their volumes. **M.TE.05.09** Use filling (unit cubes or liquid), and counting or measuring to find the volume of a cube and rectangular prism. **M.PS.05.10** Solve applied problems about the volumes of rectangular prisms using multiplication and division and using the appropriate units. **M.PS.06.02** Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets). **M.TE.06.03** Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides, using formulas. **G.SR.08.06** Know the volume formulas for generalized cylinders ((area of base) x height), generalized cones and pyramids ($\frac{1}{3}$(area of base) x height) and spheres ($\frac{4}{3}\pi$ (radius)^3), and apply them to solve problems. **G.SR.08.07** Understand the concept of surface area and find the surface area of prisms, cones, spheres, pyramids, and cylinders.
Solve problems involving indirect measurement such as finding the height of a building by comparing its shadow with the height and shadow of a known object.	
Solve problems involving rates such as speed or population density.	**N.FL.07.02** Solve problems involving derived quantities. **N.FL.07.03** Calculate rates of change, including speed. **N.MR.07.04** Convert ratio quantities between different systems of units, such as feet per second to miles per hour. **N.MR.08.11** Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.

Systems of Measurement

NAEP	GLCE
Select or use appropriate type of unit for the attribute being measured such as length, area, angle, or volume.	**M.UN.04.01** Measure using common tools and select appropriate units of measure. **G.GS.05.02** Measure angles with a protractor and classify them as acute, right, obtuse or straight.

Solve problems involving conversions within the same measurement system such as conversions involving square inches and square feet.	**M.TE.04.05** Carry out the following conversions from one unit of measure to a larger or smaller unit of measure: meters to centimeters, kilograms to grams, liters to milliliters, hours to minutes, minutes to seconds, years to months, weeks to days, feet to inches, ounces to pounds (using numbers that involve only simple calculations.) **M.UN.05.01** Recognize the equivalence of 1 liter, 1000 ml and 1000 cm^3 and include conversions among liters, milliliters, and cubic centimeters. **M.UN.05.02** Know the units of measure of volume: cubic centimeter, cubic meter, cubic inches, cubic feet, cubic yards, and use their abbreviations: cm^3, m^3, in^3, ft^3, yd^3. **M.UN.05.04** Convert measurements of length, weight, area, volume, and time within a given system, using easily manipulated numbers. **M.UN.06.01** Convert between basic units of measurement within a single measurement system, e.g., square inches to square feet.
Estimate the measure of an object in one system given the measure of that object in another system and the approximate conversion factor.	**M.UN.05.03** Compare the relative sizes of one cubic inch to one cubic foot, and one cubic centimeter to one cubic meter.
Determine appropriate size of unit of measurement in problem situation involving such attributes as length, area, or volume.	
Determine appropriate accuracy of measurement in problem situations and find the measure to that degree of accuracy.	**M.PS.04.02** Give answers to a reasonable degree of precision in the context of a given problem.
Construct or solve problems involving scale drawings.	**G.TR.07.03** Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor. **G.TR.07.04** Solve problems about similar figures and scale drawings. **G.TR.07.06** Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r^2.

GEOMETRY: Grade 8

Dimension and Shape

NAEP	GLCE

Draw or describe a path of shortest length between points to solve problems in context.	**G.LO.08.02** Find the distance between two points on the coordinate plane, using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.
Identify a geometric object given a written description of its properties.	
Identify, define, or describe geometric shapes in the plane and in three-dimensional space given a visual representation.	
Draw or sketch from a written description polygons, circles, or semicircles.	**G.SR.07.01** Use a ruler and other tools to draw squares, rectangles, triangles and parallelograms with specified dimensions. **G.SR.07.02** Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.
Represent or describe a three-dimensional situation in a two-dimensional drawing from different views.	**M.PS.06.02** Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets).
Demonstrate an understanding about the two- and three- dimensional shapes in our world through identifying, drawing, modeling, building, or taking apart.	**G.GS.04.02** Identify basic geometric shapes, including isosceles, equilateral and right triangles, and use their properties to solve problems. **G.TR.05.01** Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, 3/4 and full turns.

Transformation of Shapes and Preservation of Properties

NAEP	GLCE
Identify lines of symmetry in plane figures or recognize and classify types of symmetries of plane figures.	**G.TR.04.04** Recognize plane figures that have line symmetry.
Recognize or informally describe the effect of a transformation on two-dimensional geometric shapes.	**G.TR.04.05** Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.
Predict results of combining, subdividing, and changing shapes of plane figures and solids.	**G.SR.06.05** Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors, and justify informally.

Justify relationships of congruence and similarity, and apply these relationships using scaling and proportional reasoning.	**G.GS.06.02** Understand that for polygons, congruence means corresponding sides and angles have equal measures. **G.TR.06.03** Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems. **G.TR.06.04** Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection. **G.TR.07.03** Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor. **G.TR.07.05** Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments. **G.TR.07.06** Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r^2.
For similar figures, identify and use the relationships of conservation of angle and of proportionality of side length and perimeter.	**G.GS.06.01** Understand and apply basic properties of lines, angles, and triangles.

Relationships between geometric figures

NAEP	GLCE

Apply geometric properties and relationships in solving simple problems in two and three dimensions.	**G.GS.05.03** Identify and name angles on a straight line and vertical angles. **G.GS.05.04** Find unknown angles in problems involving angles on a straight line, angles surrounding a point and vertical angles. **G.GS.05.05** Know that angles on a straight line add up to 180° and angles surrounding a point add up to 360°; justify informally by "surrounding" a point with angles. **G.GS.05.06** Understand why the sum of the interior angles of a triangle is 180° and the sum of the interior angles of a quadrilateral is 360°, and use these properties to solve problems. **G.GS.05.07** Find unknown angles using the properties of: triangles, including right, isosceles, and equilateral triangles; parallelograms, including rectangles and rhombuses; and trapezoids. **G.GS.06.01** Understand and apply basic properties of lines, angles, and triangles.
Represent problem situations with simple geometric models to solve mathematical or real-world problems.	
Use the Pythagorean theorem to solve problems.	**G.GS.08.01** Understand at least one proof of the Pythagorean Theorem; use the Pythagorean Theorem and its converse to solve applied problems, including perimeter, area, and volume problems. **G.LO.08.02** Find the distance between two points on the coordinate plane, using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.
Describe or analyze simple properties of, or relationships between, triangles, quadrilaterals, and other polygonal plane figures.	**G.GS.06.01** Understand and apply basic properties of lines, angles, and triangles.
Describe or analyze properties and relationships of parallel or intersecting lines.	**G.GS.04.01** Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.

Position and Direction

NAEP	**GLCE**
Describe relative positions of points and lines using the geometric ideas of midpoint, points on common line through a common point, parallelism, or perpendicularity.	
Describe the intersection or two or more geometric figures in the plane.	

Visualize or describe the cross section of a solid.	
Represent geometric figures using rectangular coordinates on a plane.	

Mathematical Reasoning

NAEP	GLCE
Make and test a geometric conjecture about regular polygons.	**G.TR.07.05** Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments.

DATA ANALYSIS AND PROBABILITY: Grade 8

Data Representation

NAEP	GLCE
Read or interpret data, including interpolating or extrapolating from data.	
For a given set of data, complete a graph and then solve a problem using the data in the graph.	**D.RE.04.01** Construct tables and bar graphs from given data. **D.RE.05.02** Construct line graphs from tables of data; include axis labels and scale.
Solve problems by estimating and computing with data from a single set or across sets of data.	**D.AN.07.03** Calculate and interpret relative frequencies and cumulative frequencies for given data sets
Given a graph or a set of data, determine whether information is represented effectively and appropriately.	**D.RE.07.01** Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and-whisker plots, and select appropriate representation to address specific questions.
Compare and contrast the effectiveness of different representations of the same data.	

Characteristics of Data Sets

NAEP	GLCE

Calculate, use, or interpret mean, median, mode, or range.	**D.RE.04.02** Order a given set of data, find the median, and specify the range of values. **D.AN.05.03** Given a set of data, find and interpret the mean (using the concept of fair share) and mode. **D.AN.05.04** Solve multi-step problems involving means. **D.AN.08.01** Determine which measure of central tendency (mean, median, mode) best represents a data set, e.g. salaries, home prices for answering certain questions, and justify the choice made.
Describe how mean, median, mode, range or interquartile ranges relate to the shape of the distribution.	**D.AN.07.04** Find and interpret the median, quartiles, and interquartile range of a given set of data.
Identify outliers and determine their effect on mean, median, mode or range.	
Using appropriate statistical measures, compare two or more data sets describing the same characteristic for two different populations or subsets of the same population.	**D.RE.04.03** Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs; read bar graphs showing two data sets. **D.RE.05.01** Read and interpret line graphs, and solve problems based on line graphs, e.g., distance - time graphs, and problems with two or three line graphs on same axes, comparing different data.
Visually choose the line that best fits given a scatterplot and informally explain the meaning of the line. Use the line to make predictions.	**D.AN.07.02** Create and interpret scatter plots and use an estimated line of best fit to answer questions about the data.

Experiments and Samples

NAEP	GLCE
Given a sample, identify possible sources of bias in sampling.	**D.AN.08.02** Recognize practices of collecting and displaying data that may bias the presentation or analysis.
Distinguish between a random and nonrandom sample.	
Evaluate the design of an experiment.	

Probability

NAEP	GLCE
Analyze a situation that involves probability of an independent event.	**D.PR.08.06** Understand the difference between independent and dependent events, and recognize common misconceptions involving probability, e.g. Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the 4^{th} roll as she was on any previous roll.

Determine the theoretical probability of simple and compound events in familiar contexts.	**D.PR.06.02** Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.
Estimate the probability of simple and compound events through experimentation or simulation.	
Use theoretical probability to evaluate or predict experimental outcomes.	**D.PR.08.04** Apply the Basic Counting Principle to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams.
Determine the sample space for a given situation.	
Use sample space to determine the probability of the possible outcomes of an event.	**DA.PR.08.03** Compute relative frequencies from a table of experimental results for a repeated event and be able to answer questions about the results, using relationship of probability to relative frequency. **D.AN.08.07** Compute relative frequencies from a table of experimental results for a repeated event; understand the relationship of experimental probability to relative frequency; answer questions regarding the results.
Represent probability of a given outcome using fractions, decimals, and percents.	**D.PR.06.01** Express probabilities as fractions, decimals or percentages between 0 and 1; know that 0 probability means an event will not occur, and that probability 1 means an event will occur.

Deleted: probable

Deleted: fraction

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ALGEBRA: Grade 8

Patterns, Relations, and Functions

NAEP	GLCE
Recognize, describe or extend numerical and geometric patterns using tables, graphs, words, or symbols.	
Generalize a pattern appearing in a numerical sequence or table or graph using words or symbols.	
Analyze or create patterns, sequences, or linear functions given a rule.	
Identify functions as linear or nonlinear or contrast distinguishing properties of functions from tables, graphs, or equations.	
Interpret the meaning of slope or intercepts in linear functions.	

Algebraic Representations

NAEP	GLCE
Translate between the different representations of linear expressions using symbols, graphs, tables, diagrams, or written descriptions.	**A.RP.07.02** Represent directly proportional and linear relationships using verbal descriptions, tables, graphs and formulas, and translate among these representations.
Analyze or interpret linear relationships expressed in symbols, graphs, tables, diagrams, or written descriptions.	**A.PA.07.01** Recognize when information given in a table, graph or formula suggests a proportional or linear relationship. **A.PA.07.03** Given a directly proportional or linear situation, graph and interpret the slope and intercept(s) in terms of the original situation; evaluate $y = kx$ for specific x values.
Graph or interpret points that are represented by ordered pairs of numbers on a rectangular coordinate system.	**A.RP.07.10** Know that the graph of $y = k/x$ is not a line; know its shape, and know that it crosses neither the x nor the y-axis.
Solve problems involving coordinate pairs on the rectangular coordinate system.	**A.RP.06.02** Plot ordered pairs of integers and use ordered pairs of integers to identify points in all 4 quadrants of the coordinate plane.
Make, validate, and justify conclusions and generalizations about linear relationships.	**A.PA.07.05** Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.
Identify or represent functional relationships in meaningful contexts including proportional, linear, and common nonlinear in tables, graphs, words, or symbols.	**A.RP.06.08** Understand that graphs and tables can suggest relationships between quantities. **A.PA.06.09** Graph and write equations for linear functions of the form $y = mx$ and solve related problems, e.g., given n chairs, the "leg function" is $f(n) = 4n$: if you have 5 chairs, how many legs? ; if you have 12 legs, how many chairs? **A.RP.06.10** Represent simple relationships between quantities, using verbal descriptions, formulas or equations, tables, and graphs, e.g. perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches. **A.PA.07.09** Recognize inversely proportional relationships in contextual situations; know that quantities are inversely proportional if their product is constant, e.g., the length and width of a rectangle with fixed area, and that an inversely proportional relationship is of the form $y = k/x$ where k is some non-zero number. **A.RP.08.01** Identify and represent linear functions, quadratic functions, and other simple functions including

	inverse functions ($y = k/x$), cubics ($y = ax^3$), roots ($y = \sqrt{x}$), and exponentials ($y = a^x$, $a > 0$), using tables, graphs, and equations. **A.PA.08.02** For basic functions, e.g. simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others. **A.PA.08.03** Recognize basic functions in problem context, e.g. area of a circle, volume of a sphere, and represent them using tables, graphs, and formulas. **A.RP.08.04** Use the vertical line test to determine if a graph represents a function in one variable. **A.RP.08.05** Relate quadratic functions in factored form and vertex form to their graphs, and vice versa; in particular, note that solutions of a quadratic equation are the x-intercepts of the corresponding quadratic function. **A.RP.08.06** Graph factorable quadratic functions, finding where the graph intersects the x-axis and the coordinates of the vertex; use words "parabola" and "roots"; include functions in vertex form and those with leading coefficient –1.

Variables, Expressions, and Operations

NAEP	GLCE
Write algebraic expressions, equations, or inequalities to represent a situation.	**A.FO.06.03** Use letters, with units, to represent quantities in a variety of contexts, e.g., y lbs., k minutes, x cookies. **A.FO.06.04** Distinguish between an algebraic expression and an equation. **A.FO.06.05** Use standard conventions for writing algebraic expressions, e.g., $2x + 1$ means "two times x, plus 1" and $2(x + 1)$ means "two times the quantity $(x + 1)$". **A.FO.06.06** Represent information given in words using algebraic expressions and equations.

Perform basic operations, using appropriate tools on linear algebraic expressions.	**A.FO.06.11** Relate simple linear equations with integer coefficients to particular contexts, e.g., $3x = 8$ or $x + 5 = 10$, and solve. **A.FO.06.12** Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution. **A.FO.06.13** Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same solutions. **A.FO.06.14** Solve equations of the form $ax + b = c$, e.g., $3x + 8 = 15$, by hand for positive integer coefficients less than 20, using calculators otherwise, and interpret the results. **A.FO.06.07** Simplify expressions of the first degree by combining like terms, and evaluate using specific values. **A.FO.07.13** From applied situations, generate and solve linear equations of the form $ax + b = c$ and $ax + b = cx + d$, and interpret solutions. **A.FO.07.12** Add, subtract and multiply simple algebraic expressions of the first degree, e.g., $(92x + 8y) – 5x + y$, or $– 2x (5x – 4)$, and justify using properties of real numbers.

Equations and Inequalities

NAEP	**GLCE**
Solve linear equations or inequalities.	**A.FO.08.11** Solve simultaneous linear equations in two variables, by graphing, by substitution and by linear combination; estimate solutions using graphs; include examples with no solutions and infinitely many solutions. **A.FO.08.12** Solve linear inequalities in one and two variables, and graph the solution sets. **A.FO.08.13** Set up and solve applied problems involving simultaneous linear equations and linear inequalities.
Interpret "=" as an equivalence between two expressions and use this interpretation to solve problems.	**A.FO.08.10** Understand that to solve the equation $f(x) = g(x)$ means to find all values of x for which the equation is true; e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution).
Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically.	**A.PA.07.04** For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.

Interpret relationships between symbolic linear expressions and graphs of lines by identifying and computing slope and intercepts.	**A.PA.07.06** Calculate the slope from the graph of a linear function as the ratio of "rise/run" for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that linear functions have slope that is a constant rate of change. **A.PA.07.07** Represent linear functions in the form $y = x + b$, $y = mx$, and $y = mx + b$, and graph, interpreting slope and y-intercept. **A.FO.07.08** Know that the solution to a linear equation corresponds to the point at which its graph crosses the x-axis.
Use and evaluate common formulas.	**M.TE.04.06** Know and understand the formulas for perimeter and area of a square and a rectangle; calculate the perimeters and areas of these shapes and combinations of these shapes using the formulas. **M.TE.05.06** Understand and know how to use the area formula of a triangle and represent using models and manipulatives. **M.TE.05.07** Understand and know how to use the area formula for a parallelogram and represent using models and manipulatives. **M.TE.06.03** Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides, using formulas. **G.SR.08.03** Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems. **G.SR.08.06** Know the volume formulas for generalized cylinders ((area of base) x height), generalized cones and pyramids ($\frac{1}{3}$(area of base) x height) and spheres ($\frac{4}{3}\pi$ (radius)^3), and apply them to solve problems. **A.FO.08.07** Recognize and apply the common formulas: $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$ $(a + b)(a - b) = a^2 - b^2$; represent geometrically.