HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
Number and Quantity	**Topic**	**Topic**	**Topic**	
The Real Number System				
Extend the properties of exponents to rational exponents				
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* 51/3 *to be the cube root of* 5 *because we want* (51/3)3 = 5(1/3)3 *to hold*, *so* (51/3)3 *must equal* 5.	**L2.1** Calculation Using Real and Complex Numbers		**L2.1** Calculation Using Real and Complex Numbers	
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**L2.1** Calculation Using Real and Complex Numbers		**L2.1** Calculation Using Real and Complex Numbers	
Use properties of rational and irrational numbers				
3. Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	**L1.1** Number Systems and Number Sense			
Quantities				
Reason quantitatively and use units to solve problems				
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.			**L2.3** Measurement Units, Calculations, and Scales	
2. Define appropriate quantities for the purpose of descriptive modeling.			**L2.3** Measurement Units, Calculations, and Scales	
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.			**L2.3** Measurement Units, Calculations, and Scales	
The Complex System				
Perform arithmetic operations with complex numbers				
1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	**L2.1** Calculation Using Real and Complex Numbers			
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.			**L2.1** Calculation Using Real and Complex Numbers	
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.			**L2.1** Calculation Using Real and Complex Numbers	CCSS STEM – in MI Algebra II

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
Represent complex numbers and their operations on the complex plane				
4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				*Michigan PreCalculus*
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus* 2 *and argument* 120°.				*Michigan PreCalculus*
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				*Michigan PreCalculus*
Use complex numbers in polynomial identities and equations				
7. Solve quadratic equations with real coefficients that have complex solutions.	**A1.2** Solutions of Equations and Inequalities			
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	**A3.3** Quadratic Functions			CCSS STEM – in MI Algebra I
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**A3.5** Polynomial Functions			CCSS STEM – in MI Algebra I
(+) Vector Quantities and Matrices				
Represent and model with vector quantities				
1. Understand that vector quantities have both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, *v*).		**L1.2** Representations and Relationships		CCSS STEM – in MI Geometry
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.		**L1.2** Representations and Relationships		CCSS STEM – in MI Geometry
3. Solve problems involving velocity and other quantities that can be represented by vectors.★		**L1.2** Representations and Relationships		CCSS STEM – in MI Geometry
Perform operations on vectors				
4. Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand that vector subtraction **v** – **w** is defined as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as		**L1.2** Representations and Relationships		CCSS STEM – in MI Geometry

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				
5. Multiply a vector **v** by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|\|c\mathbf{v}\|\| = \|c\|v$. c. Understand that when $\|c\|v \neq 0$, the direction of $c\mathbf{v}$ is either along **v** (for $c > 0$) or against **v** (for $c < 0$).				*Michigan PreCalculus*
Perform operations on matrices and use matrices in applications★				
6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.				*Michigan PreCalculus*
7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.				*Michigan PreCalculus*
8. Add, subtract, and multiply matrices of appropriate dimensions.				*Michigan PreCalculus*
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				*Michigan PreCalculus*
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				*Michigan PreCalculus*
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Understand a matrix as a transformation of vectors.				*Michigan PreCalculus*
12. Understand a 2 □2 matrix as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.				*Michigan PreCalculus*
Algebra	**Topic**	**Topic**	**Topic**	
Seeing Structure in Expressions				
Interpret the structure of expressions				
1. Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients.	**A1.1** Construction, Interpretation, and Manipulation of Expressions		**L1.2** Representation and Relationships **A1.1** Construction, Interpretation, and Manipulation of Expressions	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*			**A2.2** Operations and Transformations	
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	**A2.2** Operations and Transformations		**A2.2** Operations and Transformations	
Write expressions in equivalent forms to solve problems				
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	**A1.1** Construction, Interpretation, and Manipulation of Expressions **A1.2** Solutions of Equations and Inequalities		**A1.1** Construction, Interpretation, and Manipulation of Expressions	
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★			**L2.2** Sequences and Iteration	Also aligns with MI Recomended L2.2.4*
Arithmetic with Polynomials and Rational Expressions				
Perform arithmetic operations on polynomials				
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**A1.1** Construction, Interpretation, and Manipulation of Expressions		**A1.1** Construction, Interpretation, and Manipulation of Expressions	
Understand the relationship between zeros and factors of polynomials				
2. Understand the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.			**A1.1** Construction, Interpretation, and Manipulation of Expressions	
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**A3.5** Polynomial Functions			
Use polynomial identities to solve problems				
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* (*x*2 + *y*2)2 = (*x*2 – *y*2)2 + (2*xy*)2 *can be used to generate Pythagorean triples.*	**A1.1** Construction, Interpretation, and Manipulation of Expressions **A3.5** Polynomial Functions			
5. (+) Understand that the Binomial Theorem gives the expansion of			**L1.3** Counting and Probabilistic	CCSS STEM – in MI Algebra II

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
$(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.			Reasoning	
Rewrite rational expressions				
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system			**A1.1** Construction, Interpretation, and Manipulation of Expressions	
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.			**A1.1** Construction, Interpretation, and Manipulation of Expressions	CCSS STEM – in MI Algebra II
Creating Equations				
Create equations that describe numbers or relationships				
1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	**A2.4** Models of Real-world Situations Using Families of Functions **A3.1** Lines and Linear Functions **A3.2** Exponential and Logarithmic Functions		**A2.4** Models of Real-world Situations Using Families of Functions **A3.6** Rational Functions	
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	**A1.2** Solutions of Equations and Inequalities		**A1.2** Solutions of Equations and Inequalities	
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	**A1.2** Solutions of Equations and Inequalities **A2.4** Models of Real-world Situations Using Families of Functions		**A1.2** Solutions of Equations and Inequalities **A2.4** Models of Real-world Situations Using Families of Functions	
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*	**A1.2** Solutions of Equations and Inequalities		**A1.2** Solutions of Equations and Inequalities	
Reasoning with Equations and Inequalities				
Understand solving equations as a process of reasoning and explain the reasoning				
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**A1.2** Solutions of Equations and Inequalities		**A1.2** Solutions of Equations and Inequalities	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**A1.2** Solutions of Equations and Inequalities		**A1.2** Solutions of Equations and Inequalities **A3.6** Rational Functions	
Solve equations and inequalities in one variable				
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Graph the solution set of an inequality on a number line.	**A1.2** Solutions of Equations and Inequalities			
4. Solve quadratic equations in one variable. a. Use the method of completing the square that transforms any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. This leads to the quadratic formula. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**A1.2** Solutions of Equations and Inequalities **A3.3** Quadratic Functions			
Solve systems of equations				
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**A1.2** Solutions of Equations and Inequalities			
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**A1.2** Solutions of Equations and Inequalities			
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.*	**A1.2** Solutions of Equations and Inequalities			
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.				*Michigan PreCalculus*
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 ⍰3 or greater).				*Michigan PreCalculus*
Represent and solve equations and inequalities graphically				
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a straight line).				
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**A1.2** Solutions of Equations and Inequalities **A2.1** Definitions, Representations, and Attributes of Functions			
Functions	**Topic**	**Topic**	**Topic**	
Interpreting Functions				
Understand the concept of a function and use function notation				
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*			**A2.1** Definitions, Representations, and Attributes of Functions	
Interpret functions that arise in applications in terms of the context				
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	
Analyze functions using different representations				
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	**A2.1** Definitions, Representations, and Attributes of Functions **A3.1** Lines and Linear Functions		**L2.3** Measurement Units, Calculations, and Scales **A2.1** Definitions, Representations,	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**A3.2** Exponential and Logarithmic Functions **A3.3** Quadratic Functions **A3.5** Polynomial Functions		and Attributes of Functions **A3.2** Exponential and Logarithmic Functions **A3.6** Rational Functions **A3.7** Trigonometric Functions	
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	**A2.1** Definitions, Representations, and Attributes of Functions **A2.2** Operations and Transformations **A3.2** Exponential and Logarithmic Functions **A3.3** Quadratic Functions		**A2.1** Definitions, Representations, and Attributes of Functions **A2.2** Operations and Transformations	
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**A2.1** Definitions, Representations, and Attributes of Functions		**A2.1** Definitions, Representations, and Attributes of Functions	
Building Functions				
Build a function that models a relationship between two quantities				
1. Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* c. (+) Compose functions. *For example, if f(t) is the height of a falling body after t seconds, f(t – 12) is the height of the same body dropped 12 seconds later.*	**A2.1** Definitions, Representations, and Attributes of Functions **A2.2** Operations and Transformations		**A2.1** Definitions, Representations, and Attributes of Functions **A2.2** Operations and Transformations	Part (c) and MI *A2.2.4-6 **(HSCE Recommended Expectation)**
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★			**L2.2** Sequences and Iteration **A2.1** Definitions, Representations, and Attributes of Functions	
Build new functions from existing functions				
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, k	**A2.2** Operations and		**A2.2** Operations and	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
f(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	Transformations		Transformations	
4. Find inverse functions. a. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1.* b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	**A2.2** Operations and Transformations		**A2.2** Operations and Transformations	MI A2.2.4*, A2.2.5*, A.2.2.6*
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.			**A3.2** Exponential and Logarithmic Functions	STEM and in MI Algebra II
Linear, Quadratic and Exponential Models				
Construct and compare linear, quadratic, and exponential models and solve problems				
1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Understand that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	**A2.3** Representations of Functions **A2.4** Models of Real-world Situations Using Families of Functions **A3.1** Lines and Linear Functions **A3.2** Exponential and Logarithmic Functions			
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	**A2.3** Representations of Functions **A3.1** Lines and Linear Functions **A3.2** Exponential and Logarithmic Functions		**L2.2** Sequences and Iteration **A2.3** Representations of Functions	
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	**A2.3** Representations of Functions **A3.1** Lines and Linear Functions **A3.2** Exponential and Logarithmic Functions **A3.3** Quadratic Functions **A3.5** Polynomial Functions			
4. For exponential models, express as a logarithm the solution to *a bct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.			**A2.3** Representations of Functions **A3.2** Exponential and Logarithmic Functions	
Interpret expressions for functions in terms of the situation they model				

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.	**A2.3** Representations of Functions **A2.4** Models of Real-world Situations Using Families of Functions **A3.1** Lines and Linear Functions **A3.2** Exponential and Logarithmic Functions			
Trigonometric Functions				
Extend the domain of trigonometric functions using the unit circle				
1. Understand that the radian measure of an angle is the length of the arc on the unit circle subtended by the angle.			**A3.7** Trigonometric Functions	
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			**A3.7** Trigonometric Functions	
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π /3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π-*x*, π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number.			**A3.7** Trigonometric Functions	
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.			**A3.7** Trigonometric Functions	
Model periodic phenomena with trigonometric functions				
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ *NEW and not STEM*			**A2.4** Models of Real-world Situations Using Families of Functions **A3.7** Trigonometric Functions	
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.			**A1.2** Solutions of Equations and Inequalities **A3.7** Trigonometric Functions	
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★			**A1.2** Solutions of Equations and Inequalities **A2.4** Models of Real-world Situations Using Families of Functions **A3.7** Trigonometric Functions	
Prove and apply trigonometric identities				
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. *NEW and not STEM*				MI HSCE A1.1.7*
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				*Michigan PreCalculus*

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
NEW				
★Modeling	**Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.**			
Geometry	**Topic**	**Topic**	**Topic**	
Congruence				
Experiment with transformations in the plane				
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		**G1.1** Lines and Angles; Basic Euclidean and Coordinate Geometry **G1.6** Circles and Their Properties		
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).		**G3.2** Shape-preserving Transformations: Dilations and Isometries		
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.		**G3.1** Distance-preserving Transformations: Isometries		
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		**G3.1** Distance-preserving Transformations: Isometries		
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		**G3.1** Distance-preserving Transformations: Isometries **G3.2** Shape-preserving Transformations: Dilations and Isometries		
Understand congruence in terms of rigid motions				
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		**G3.1** Distance-preserving Transformations: Isometries		
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		**G3.1** Distance-preserving Transformations: Isometries **G2.3** Congruence and Similarity		
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		**G3.1** Distance-preserving Transformations: Isometries **G2.3** Congruence and Similarity		
Prove geometric theorems				
9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines,*		**L3.3** Proof **G1.1** Lines and Angles; Basic		

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints		Euclidean and Coordinate Geometry		
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*		**L3.3** Proof **G1.1** Lines and Angles; Basic Euclidean and Coordinate Geometry **G1.2** Triangles and Their Properties **G2.3** Congruence and Similarity		
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*		**L3.3** Proof **G1.4** Quadrilaterals and Their Properties		
Make geometric constructions				
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*		**G1.1** Lines and Angles; Basic Euclidean and Coordinate Geometry		
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.				**No Alignment**
Similarity, Right Triangles, and Trigonometry				
Understand similarity in terms of similarity transformations				
1. Verify experimentally the properties of dilations: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		**G3.1** Distance-preserving Transformations: Isometries **G3.2** Shape-preserving Transformations: Dilations and Isometries		
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		**G2.3** Congruence and Similarity **G3.2** Shape-preserving Transformations: Dilations and Isometries		
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.		**G2.3** Congruence and Similarity **G3.2** Shape-preserving Transformations: Dilations and Isometries		
Prove theorems involving similarity				
4. Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*		**G1.2** Triangles and Their Properties **G2.3** Congruence and Similarity		

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.		**G2.3** Congruence and Similarity		
Define trigonometric ratios and solve problems involving right triangles				
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		**G1.3** Triangles and Trigonometry		
7. Explain and use the relationship between the sine and cosine of complementary angles.		**G1.3** Triangles and Trigonometry		
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★		**G1.3** Triangles and Trigonometry		
(+) **Apply trigonometry to general triangles**				
9. Derive the formula *A* = . *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		**G1.3** Triangles and Trigonometry		CCSS STEM – in MI Geometry
10. Prove the Laws of Sines and Cosines and use them to solve problems		**G1.3** Triangles and Trigonometry		CCSS STEM – partially in MI Geometry
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		**G1.3** Triangles and Trigonometry		CCSS STEM – in MI Geometry
Circles				
Understand and apply theorems about circles				
1. Prove that all circles are similar.		**G1.6** Circles and Their Properties		
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*		**G1.6** Circles and Their Properties		
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		**G1.1** Lines and Angles; Basic Euclidean and Coordinate Geometry **G1.6** Circles and Their Properties		
4. (+) Construct a tangent line from a point outside a given circle to the circle.		**G1.1** Lines and Angles; Basic Euclidean and Coordinate Geometry **G1.6** Circles and Their Properties		
Find arc lengths and areas of sectors of circles				
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		**G1.6** Circles and Their Properties	**A3.7** Trigonometric Functions	
Expressing Geometric Properties with Equations				
Translate between the geometric description and the equation for a conic section				

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.			**G1.7** Conic Sections and Their Properties	
2. Derive the equation of a parabola given a focus and directrix.			**G1.7** Conic Sections and Their Properties	*G1.7.4
3. (+) Derive the equations of ellipses and hyperbolas given two foci for the ellipse, and two directrices of a hyperbola.			**G1.7** Conic Sections and Their Properties	*G1.7.4 CCSS STEM – partially in MI Algebra II
Use coordinates to prove simple geometric theorems algebraically				
4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*		**L3.3** Proof **G1.4** Quadrilaterals and Their Properties **G1.7** Conic Sections and Their Properties **G2.3** Congruence and Similarity		
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	**A3.1** Lines and Linear Functions	**G1.1** Lines and Angles; Basic Euclidean and Coordinate Geometry		
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.				**No Alignment**
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★				
Geometric Measurement and Dimension				
Explain volume formulas and use them to solve problems				
1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		**G1.5** Other Polygons and Their Properties **G2.1** Relationships Between Area and Volume Formulas **G2.2** Relationships Between Two-dimensional and Three-dimensional Representations		
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		**G2.1** Relationships Between Area and Volume Formulas **G1.8** Three- Dimensional Figures		
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★		**G2.1** Relationships Between Area and Volume Formulas **G1.8** Three- Dimensional Figures		
Visualize relationships between two-dimensional and three-dimensional objects				
4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.		**G2.2** Relationships Between Two-dimensional and Three-dimensional		
Modeling with Geometry				

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
Apply geometric concepts in modeling situations				
1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★		**G1.8** Three- Dimensional Figures		
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★		**L2.3** Measurement Units, Calculations, and Scales		
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★				**No Alignment**
Statistics and Probability	**Topic**	**Topic**	**Topic**	
Interpreting Categorical and Measurement Data				
Summarize, represent, and interpret data on a single count or measurement variable				
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	**L1.2** Representation and Relationships		**S1.1** Producing and Interpreting Plots **(Removed from MI Algebra II 09/09)**	
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.			**S1.1** Producing and Interpreting Plots **S1.2** Measures of Center and Variation **(Removed from MI Algebra II 09/09)**	
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).			**S1.1** Producing and Interpreting Plots **(Removed from MI Algebra II 09/09)**	
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.			**S1.2** Measures of Center and Variation **S1.3** The Normal Distribution **(Removed from MI Algebra II 09/09)**	
Summarize, represent, and interpret data on two categorical and quantitative variables				
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	**L1.2** Representation and Relationships **S2.1** Scatterplots and Correlation			
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential*	**S2.1** Scatterplots and Correlation **S2.2** Linear Regression			

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for scatter plots that suggest a linear association.				
Interpret linear models				
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**S2.2** Linear Regression			
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	**S2.**1 Scatterplots and Correlation			
9. Distinguish between correlation and causation.	**S2.**1 Scatterplots and Correlation			
Making Inferences and Justifying Conclusions				
Understand and evaluate random processes underlying statistical experiments				
1. Understand that statistics allows inferences to be made about population parameters based on a random sample from that population.			**S3.1** Data Collection and Analysis **(Removed from MI Algebra II 09/09)**	
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*			**S3.1** Data Collection and Analysis **(Removed from MI Algebra II 09/09)** **S4.1** Probability **(Removed from MI Algebra II 09/09)** **S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
Make inferences and justify conclusions from sample surveys, experiments, and observational studies				
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			**S3.1** Data Collection and Analysis **(Removed from MI Algebra II 09/09)**	
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling			**L2.4** Understanding Error **S3.1** Data Collection and Analysis **(Removed from MI Algebra II 09/09)**	
5. Use data from a randomized experiment to compare two			**S3.1** Data Collection and Analysis	MI S4.1.3*

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
treatments; use simulations to decide if differences between parameters are significant.			**(Removed from MI Algebra II 09/09)**	
6. Evaluate reports based on data.			**S3.1** Data Collection and Analysis **(Removed from MI Algebra II 09/09)** **S4.1** Probability **(Removed from MI Algebra II 09/09)** **S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
Conditional Probability and Rules of Probability				
Understand independence and conditional probability and use them to interpret data				
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).			**L1.3** Counting and Probabilistic Reasoning **(Removed from MI Algebra II 09/09)** **S4.1** Probability **(Removed from MI Algebra II 09/09)** **S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.			**L1.3** Counting and Probabilistic Reasoning **(Removed from MI Algebra II 09/09)** **S4.1** Probability **(Removed from MI Algebra II 09/09)**	
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.			**L1.3** Counting and Probabilistic Reasoning **(Removed from MI Algebra II 09/09)** **S4.1** Probability **(Removed from MI Algebra II 09/09)**	
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*			**S4.1** Probability **(Removed from MI Algebra II 09/09)** **S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a*			**S4.1** Probability **(Removed from MI Algebra II 09/09)** **S4.2** Application and	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
smoker with the chance of being a smoker if you have lung cancer.			Representation **(Removed from MI Algebra II 09/09)**	
Use the rules of probability to compute probabilities of compound events in a uniform probability model				
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.			**S4.1** Probability **(Removed from MI Algebra II 09/09)** **S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.			**S4.1** Probability **(Removed from MI Algebra II 09/09)**	
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.			**S4.1** Probability **(Removed from MI Algebra II 09/09)**	
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.			**S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
Using Probability to Make Decisions				
Calculate expected values and use them to solve problems				
1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.			**S1.1** Producing and Interpreting Plots **S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	**L2.1** Calculation Using Real and Complex Numbers		*S3.1.4-6, *S3.2.1 **(HSCE Recommended Expectations)** *S4.1.3 **(HSCE Recommended Expectation)**	
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*			*S3.1.4-6, *S3.2.1 **(HSCE Recommended Expectations)** *S4.1.3 **(HSCE Recommended Expectation)**	
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*			*S3.1.4-6, *S3.2.1 **(HSCE Recommended Expectations)** *S4.1.3 **(HSCE Recommended Expectation)**	

HS CC Mathematics	MI HSCE – Algebra I	MI HSCE - Geometry	MI HSCE – Algebra II	Other / Comments
Use probability to evaluate outcomes of decisions				
5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*			*S3.1.4-6, *S3.2.1 **(HSCE Recommended Expectations)** *S4.1.3 **(HSCE Recommended Expectation)**	
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).			**S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).			**S4.2** Application and Representation **(Removed from MI Algebra II 09/09)**	