

**CHAPTER 3: WHOLE NUMBER ADDITION AND SUBTRACTION**

**Whole Number Addition and Subtraction Overview:** There are four whole number topics (WN8-WN11) covering 22 grade level content expectations.

Whole number addition and subtraction topics (number of GLCEs):

- WN8 Meaning of addition and subtraction. (6)
- WN9 Addition and subtraction fact families and relationships. (5)
- WN10 Addition and subtraction (11)
- WN11 Multiple digit addition and subtraction (0)

**A. WHOLE NUMBER ADDITION AND SUBTRACTION GLCES, MEAP DATA, AND ACTIVITIES BY TOPIC**

Representations

Characteristics	Materials
• "Bustable" and Proportional	Straws and Beans
• Tradable and Proportional	Base-ten blocks
• Number Line	Demonstration Number Line (-120 to 120) Student Number Line (-40 to 40)
• Tradable and Non-Proportional	Money: dollars (1, 10, 100, 1000, 10,000, 100,000)

**WN8. MEANING OF ADDITION AND SUBTRACTION.**

**N.MR.00.08** Describe and make drawings to represent situations/stories involving putting together and taking apart for totals up to 10; use finger and object counting.

**N.MR.00.09** Record mathematical thinking by writing simple addition and subtraction sentences, e.g.,  $7 + 2 = 9$  and  $10 - 8 = 2$ .

**N.MR.01.09.** Compare two or more sets in terms of the difference in number of elements.

**N.MR.01.10** Model addition and subtraction for numbers through 30 for a given contextual situation using objects or pictures; explain in words; record using numbers and symbols; solve.\*

**N.MR.02.07** Find the distance between numbers on the number line, e.g., how far is 79 from 26? [Core-NC]

**N.MR.02.09** Given a contextual situation that involves addition and subtraction using numbers through 99, model using objects or pictures, explain in words, record using numbers and symbols; solve.\* [Core]

2005 MEAP Release Item Data				All Students						Students with Disabilities					
#	GLCE	MEAP	a	<x%	A	B	C	D	O/M	<x%	A	B	C	D	O/M
10	N.MR.02.07	Core	C		11	14	75		1		14	17	68		1
13	N.MR.02.07	Core	A		66	22	12		1		55	24	20		1
15	N.MR.02.07	Core	B		13	65	21		1		17	56	26		1
42	N.MR.02.09	Core	C		11	12	76		1		14	14	71		1
47	N.MR.02.09	Core	A		70	7	22		1		57	12	31		1
58	N.MR.02.09	Core	C		16	14	69		1		19	19	61		1

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2006 MEAP Release Item Data				All Students						Students with Disabilities					
#	GLCE	MEAP	a	<x%	A	B	C	D	O/M	<x%	A	B	C	D	O/M
7	N.MR.02.07	Core	B		19	53	28		1		22	45	33		1
8	N.MR.02.07	Core	B		8	76	15		1		13	63	24		1
9	N.MR.02.07	Core	A		83	10	7		1		69	16	14		1
25	N.MR.02.09	Core	B		19	59	22		0		23	45	32		0
26	N.MR.02.09	Core	C		12	11	77		0		13	13	73		0
27	N.MR.02.09	Core	C		4	4	92		1		7	7	86		0

### 2006 MEAP Grade 3

**7** How far is 56 from 33 on the number line below?



- ⊖ **A** 22
- ⊖ **B** 23
- ⊖ **C** 24

### 2006 MEAP Grade 3

**25** The second grade class read 76 books. The first grade class read 20 *fewer* books. How many books did the first grade class read?

- ⊖ **A** 20
- ⊖ **B** 56
- ⊖ **C** 96

### Meaning of Whole Number Addition:

- 1 Two meanings and/or representations of addition will be used:

Joining of two sets of objects:

The number two is necessary since addition (and subtraction as an inverse operation) is a binary operation and is done two sets at a time. Even with several addends, addition is done two at a time. For this type of addition, straws, money and base ten blocks will be used manipulatives which might be used with students for instruction.

Linear extension:

The number line will be used as the model for instruction with this type of addition. It can lead to an alternative algorithm which relies on counting by powers of ten. With this model addition need not be limited to non negative numbers at the beginning.

Either type of representation allows several alternate algorithms.

2 Difficulties:

Regrouping:

Students in the early years of education are trained to focus on a page from top to bottom, left to right. This is necessary for reading. And unfortunately the first instruction for addition does not require regrouping. Thus, during their first experiences with formal addition, some students continue, regardless of what the teachers says, to add two digit numbers from left to right. Since there is no regrouping required in these problems, the left to right addition is reinforced. This is a “logical” process for the student since numbers are read left to right (i.e. two hundred thirty six). Many students continue to do column addition, resulting in the error illustrated on the right. It should be noted that “five hundred thirteen” is incorrect, however, “fifty thirteen” is an intermediate step in a mental algorithm used by a number of people, especially when the number are given orally and not written. Of course, there are left to right algorithms but are not the standard algorithm.

Error $\begin{array}{r} 28 \\ + 35 \\ \hline 513 \end{array}$ “Fifty thirteen”
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Addition as counting:

Marla was a little girl who had been taught that addition was counting up and subtraction was counting back with no other representation. For Marla eight plus four was “eight, nine, ten, eleven.” When confronted with a set of eight objects and another set of four objects, she would count and get eight and count the other set and get four, combine them and count twelve objects, but, still insist that eight plus four was eleven. When pressed her reaction was that combining sets and counting was “not addition, that addition was counting on.”

Activity 1: Straws and bundled straws. No regrouping.

Open the Place Value Chart on the table with the decimal portion folded under.

Make 25 on the top portion of the chart. Record the 25.

Make 34 below the 25. Record it.

Combine the ones, pulling them to the bottom of the sheet.

How many ones? Record it.

Combine the tens, pulling them to the bottom of the sheet.

How many tens? Record.

How many altogether? Record the sum.

Needed: Place Value Chart Record Sheet Straws
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Activity 2: Straws and bundled straws.

Regrouping.

Make 28 on the top of the Place Value Chart. Record the 28.

Make 37 below the 28 on the chart.

Record it.

Combine the ones, pulling them to the bottom of the sheet.

Are there enough to exchange for a ten

Alternate Recording Formats		
$\begin{array}{r} 28 \\ + 37 \\ \hline 15 \\ \hline 50 \\ \hline 65 \end{array}$	$\begin{array}{r} 28 \\ + 37 \\ \hline 5 \end{array} \rightarrow$	$\begin{array}{r} 28 \\ + 37 \\ \hline 65 \end{array}$

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bundle? (yes)

Make the exchange and record? (Note that there are several recording options.)

Combine the tens, pulling them to the bottom of the sheet.

How many tens? Record.

How altogether?

Whatever the recording format students should write 15 instead of “writing the 5 and carrying the 1.”

When that is done some students still write 15, in effect “writing the 1 and carrying the 5.”

### Activity 3: Money, Regrouping.

Open the Place Value Chart on the table with the decimal portion folded back.

On the top of the chart make 57 using money.

Below the 57 make 45 using money.

Write the addition problem on Place Value Record Sheet.

Combine the ones, pulling them to the bottom of the sheet.

Are there enough ones to exchange for a ten? (yes)

Make the exchange and record.

Combine the tens, pulling them to the bottom of the sheet.

Are there enough tens to exchange for a hundred? (yes)

Make the exchange and record.

How many hundreds? (one)

How many tens? (zero)

How many ones? (two)

How much altogether? (one hundred two)

Needed: Place Value Chart Place Value Record Sheet Money
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### Activity 4: Base Ten Blocks, Regrouping

Place the Addition Mat on the table.

On the top of the mat make 36 using Base Ten Blocks.

How many tens? (three) Record them.

How many ones? (six) Record them.

How much is that? (Thirty-six)

On the middle portion of the mat make 48 with base ten blocks.

How many tens? (four) Record them.

How many ones? (eight) Record them.

How much is that? (forty-eight)

Combine the ones, pulling them to the bottom of the sheet.

How many ones? (fourteen) Train them and compare the train with a ten rod.

Are there enough ones to exchange for a ten? (yes) Place the ten rod where you want to record the exchange in the algorithm.

Make the exchange and record the fourteen.

Combine the tens, pulling them to the bottom of the sheet.

Are there enough tens to exchange for a hundred? (no)

How many tens? (eight) Record the eighty.

How many ones? (four) Record the four.

Needed: Addition Mat Base Ten Blocks
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Meaning of Whole Number Subtraction

Subtraction is two very different types; take away and comparison. The difference is best identified by contextual problems.

Take Away: John had 58 stamps. He gave 15 stamps to Sarah. How many stamps does John have left.

Comparison: John has 58 stamps and Sarah has 15 stamps. How many more stamps does John have than Sarah has?

In the first case, only 58 stamps are involved, but in the second case, there are 71 stamps involved. The physical operations used to solve these problems are also quite different.

Activity 1: Subtraction; Straws, No Regrouping

Write a take away question for this problem.

Open the Place Value Chart on the table.

Place 38 straws on the chart.

How many bundles of ten? (3)

How many unbundled straws? (8)

Write 38 on Record Sheet.

Take away 5 straws. (Move them to the bottom of the chart.)

Take away 20 (2 bundles) of straws and move them to the bottom of the chart.

Write – 25 below the 38.

How much is left? (13)

Write the difference.

$$\begin{array}{r} 38 \\ - 25 \\ \hline \end{array}$$

Needed:  
Place Value Chart  
Place Value Record Sheet  
Straws

Activity 2: Write a comparison question for this problem.

Place 38 straws on the top of the chart.

How many bundles of ten? (3)

How many unbundled straws? (8)

Write 38 on the Record Sheet.

Place 25 straws at the bottom of the chart.

How many bundles of ten straws? (2)

How many unbundled straws? (5)

Write –25 below the 38 on the Record Sheet.

Are there enough unbundled straws in the top number to match with the unbundled straws in the bottom number? (yes)

Match the unbundled straws and record the straws, which have not been matched.

Match the bundles of straws and record the number of bundles, which have not been matched.

How many straws have not been matched? (13)

Activity 3: Subtraction; Straws, Regrouping

Write a take away question for this problem.

Open the Place Value Chart on the table.

Place 43 straws on the chart.

How many bundles of ten? (4)

How many unbundled straws? (3)

Write 43 on the Record Sheet.

Is it possible to take 8 unbundled straws away without breaking a bundle of straws?  
(no)

Break a bundle of straws and place the in the ones portion of the chart

How many unbundled straws are there? (13)

Record this on the Record Sheet.

Take away 8 straws. (Move them to the bottom of the chart.)

Take away 20 (2 bundles) of straws and move them to the bottom of the chart.

Write - 28 below the 43.

How much is left? (15)

Write the difference.

$$\begin{array}{r} 43 \\ - 28 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ - 28 \\ \hline \end{array} \longrightarrow \begin{array}{r} \phantom{4}^3 \phantom{3}^1 \\ 43 \\ - 28 \\ \hline \end{array}$$

Activity 4: Write a comparison question for this problem.

Place 43 straws on the top of the chart.

How many bundles of ten? (4)

How many unbundled straws? (3)

Write 43 the Record Sheet.

Place 28 straws at the bottom of the chart.

How many bundles of ten straws? (2)

How many unbundled straws? (8)

Write -28 below the 43 on the Record Sheet.

Are there enough unbundled straws in the top number to match with the unbundled straws in the bottom number? (no)

Break a bundle of straws in the 43 and place them in the ones portion of the chart.

Record this on the sheet of paper.

Match the unbundled straws and record the straws, which have not been matched.

Match the bundles of straws and record the number of bundles, which have not been matched.

How many straws have not been matched? (13)

Activity 5: Subtraction; Money, Regrouping

Write a take away question for this problem.

Open the Place Value Chart on the table.

Place 54 on the chart using money.

How many of tens? (5)

How many ones? (4)

Write 54 on the Record Sheet.

Is it possible to take 8 ones away without breaking a ten? (no)

Exchange a ten for 10 ones and place then on the ones portion of the chart.

How many ones are there? (14)

$$\begin{array}{r} \text{Problem:} \\ 54 \\ - 38 \\ \hline \end{array}$$

Needed:  
Place Value Chart  
Place Value Record Sheet  
Money

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Record this on the Record Sheet.

Take away 8 ones. (Move them to the bottom of the chart.)

Take away 30 (3 tens) and move them to the bottom of the chart.

Write  $-38$  below the 54.

How much is left? (16)

Write the difference.

$\begin{array}{r} 54 \\ -38 \\ \hline \end{array}$	→	$\begin{array}{r} 4\ 1 \\ \cancel{5}4 \\ -38 \\ \hline \end{array}$
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Activity 6: Write a comparison question for this problem.

Place 54 on the top of the chart using money.

How many tens? (5)

How many ones? (4)

Write 54 on Record Sheet.

Place 38 at the bottom of the chart using money.

How many tens? (3)

How many ones? (8)

Write  $-38$  below the 54 on the Record Sheet.

Are there enough ones in the top number to match with the ones in the bottom number? (no)

Exchange a ten into 10 ones and place them in the ones portion of the chart.

How many ones on the top portion of the chart? (14)

Write this on the Record Sheet.

Match the ones and record the ones, which have not been matched.

Match the tens and record the number of tens, which have not been matched.

How much has not been matched? (16)

Record the difference.

Activity 7: Subtraction; Base Ten Block, Regrouping

Write a take away question for this problem.

Place the Base Ten Subtraction Mat on the table.

Place 73 on the mat.

How many of tens? (7)

How many ones? (3)

Write 73 on a sheet of paper.

Is it possible to take away 8 ones without breaking a ten? (no)

Exchange a ten for 10 ones and place them on the ones portion of the mat.

How many ones are there? (13)

Record this on the sheet of paper.

Take away 8 ones. (Move them to the bottom of the mat.)

Take away 40 (4 tens) and move them to the bottom of the mat.

Write  $-48$  below the 73.

How much is left? (25)

Write the difference.

Problem:

73

-48

Needed:  
Base Ten Block Subtraction Mat  
Base Ten Blocks

$\begin{array}{r} 73 \\ -48 \\ \hline \end{array}$	→	$\begin{array}{r} 6\ 1 \\ \cancel{7}3 \\ -48 \\ \hline \end{array}$
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Activity 8: Show Power Point: Take Away Subtraction

Activity 9: Write a comparison question for this problem.

Place 73 on the top of the mat using Base 10 Blocks.

How many tens? (7)

How many ones? (3)  
 Write 73 on a sheet of paper.  
 Place 38 at the bottom of the mat using Base Ten Blocks.  
 How many tens? (3)  
 How many ones? (8)  
 Write  $-38$  below the 73 on the sheet of paper.  
 Are there enough ones in the top number to match with the ones in the bottom number? (no)  
 Exchange a ten into 10 ones and place them in the ones portion of the mat.  
 How many ones on the top portion of the mat? (13)  
 Record this on the sheet of paper.  
 Match the ones and record the ones, which have not been matched.  
 Match the tens and record the number of tens, which have not been matched.  
 How much has not been matched? (35)  
 Record the difference on the sheet of paper.

**Activity 10: Subtraction; Number Line**

Write a take away question for this problem.  
 Locate 32 on the number line.  
 Using the non-negative number line place the 17 above the 32.

**Problem:**

$$\begin{array}{r} 32 \\ -17 \\ \hline \end{array}$$

Needed:  
 Demonstration Number Lines  
 Student Number Lines

(Turning the non-negative number line over and placing the 0 above the 32 actually shows the take away more directly.)  
 Read the difference on the original number line. (15)

**Activity 11 Write a comparison problem for this problem.**

One such question could have been. How far from 17 to 32.  
 Locate 32 on the number line. Locate 17 on the number line.  
 Use the non-negative number line to measure “how far to 32 from 17.” This is done by placing the 0 over the 17 and reading the number above the 32. (15)

**Activity 12: Demonstration: Use the demonstration number line to show comparison subtraction using:**

- Two negative numbers. (subtract  $-17$  from  $-36$  and then subtract  $-36$  from  $-17$ )
- One positive number and one negative number. (subtract  $-45$  from 36 and then subtract 36 from  $-45$ )

Subtract 45 from -36 (start at 45, stop at -36)

Negative one, negative two, ..., negative five, negative fifteen, ..., negative seventy-five, ..., negative eighty-one

Subtract -36 from 45 (start at -36, stop at 45)

One, two, ..., six ..., sixteen ..., twenty-six ..., seventy-six, ..., seventy-seven, ..., eighty-one

Activity 13: Show the Power Point: Addition Fact Strategies

Basic Addition Fact Strategies:

- Addition Fact Strategies (zero more, one more, two more, doubles, near doubles  $\pm 1$ , combinations of ten, how much more to ten and how much more?)

Fact Families:

- Example:  $6 + 5 = \square$ ,  $5 + 6 = \square$ ,  $\square + 5 = 11$ ,  $6 + \square = 11$

Composition and decomposition of numbers to add.

Activity 14: Demonstration: How many to 10 and how many more

Needed: Tens Frames
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**WN9. ADDITION AND SUBTRACTION FACT FAMILIES AND RELATIONSHIPS.**

**N.ME.01.04** Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100.

**N.ME.01.08** List number facts (partners inside of numbers) for 2 through 10, e.g.,  $8 = 7 + 1 = 6 + 2 = 5 + 3 = 4 + 4$ ;  $10 = 8 + 2 = 2 + 8$ .

**N.MR.01.11** Understand the inverse relationship between addition and subtraction, e.g., subtraction “undoes” addition: if  $3 + 5 = 8$ , we know that  $8 - 3 = 5$  and  $8 - 5 = 3$ ; recognize that some problems involving combining, “taking away”, or comparing can be solved by either operation.

**N.FL.01.12** Know all the addition facts up to  $10 + 10$ , and solve the related subtraction problems fluently.

**N.MR.01.13** Apply knowledge of fact families to solve simple open sentences for addition and subtraction, such as:  $\square + 2 = 7$  and  $10 - \square = 6$ .

**WN10.ADDITION AND SUBTRACTION (NO REGROUPING, REGROUPING WITH AND WITHOUT REGROUPING)**

**N.FL.01.15** Calculate mentally sums and differences involving: a two-digit number and a one-digit number without regrouping, a two-digit number and a multiple of 10.

**N.FL.01.14** Add three one-digit numbers.

**N.FL.01.16** Compute sums and differences through 30 using number facts and strategies, but no formal algorithm.\*

**N.MR.02.08** Find missing values in open sentences, e.g.,  $42 + \square = 57$ ; use relationship between addition and subtraction. [Ext-NC]

**N.FL.02.10** Add fluently two numbers through 99, using strategies including formal algorithms; subtract fluently two numbers through 99.\* [Core-NC]

**N.FL.02.11** Estimate and calculate the sum of two numbers with three digits.\* [Core-NC]

**N.FL.02.12** Calculate mentally sums and differences involving: three-digit numbers and ones; three-digit numbers and tens; three-digit numbers and hundreds. [NASL]

**N.FL.03.06** Add and subtract fluently two numbers, through 999 with regrouping and through 9,999 without regrouping.\* [Core-NC]

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**N.FL.03.07** Estimate the sum and difference of two numbers with three digits (sums up to 1000), and judge reasonableness of estimates. [Core-NC]

**N.FL.03.08** Use mental strategies to fluently add and subtract two-digit numbers. [NASL]

**N.FL.04.08** Add and subtract whole numbers fluently. [Ext-NC]

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63	N.MR.02.08	E Core	C		21	13	65		1		24	17	58		1
3	N.FL.02.10	Core	C		4	4	91		0		9	8	82		0
5	N.FL.02.10	Core	C		4	22	73		1		8	27	64		1
12	N.FL.02.10	Core	A		44	31	24		1		38	32	29		1
4	N.FL.02.11	Core	B		11	73	16		1		12	64	23		0
8	N.FL.02.11	Core	B		16	68	15		1		19	60	21		0
14	N.FL.02.11	Core	C		8	35	55		1		11	38	50		1
3	N.FL.03.06	Core	B		2	89	3	5	0		6	77	7	10	1
6	N.FL.03.06	Core	A		77	6	14	3	0		63	8	21	7	0
10	N.FL.03.06	Core	D		6	8	3	82	0		9	13	7	70	0
11	N.FL.03.07	Core	A		57	16	22	5	0		44	20	26	9	0
12	N.FL.03.07	Core	A		51	29	12	7	0		38	34	15	12	0
14	N.FL.03.07	Core	B		16	51	20	13	0		19	38	25	18	0
60	N.FL.04.08	E Core	D		2	3	2	93	0		3	6	5	85	0

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66	N.MR.02.08	E Core	A		75	16	8		1		64	21	14		1
13	N.FL.02.11	Core	C		18	27	54		1		22	33	44		1
14	N.FL.02.11	Core	B		19	72	8		1		23	62	14		1
15	N.FL.02.11	Core	A		77	14	8		1		68	16	16		1
7	N.FL.03.06	Core	C		3	4	69	23	0		6	7	51	35	0
8	N.FL.03.06	Core	B		2	83	7	8	0		4	68	12	17	0
9	N.FL.03.06	Core	C		5	6	86	3	0		8	10	75	7	0
10	N.FL.03.07	Core	A		43	28	18	11	0		30	32	22	16	0
11	N.FL.03.07	Core	A		53	31	9	8	0		40	34	13	13	0
12	N.FL.03.07	Core	C		12	15	65	8	0		13	20	55	12	0
64	N.FL.04.08	E Core	C		3	2	94	1	0		6	3	88	2	0

**2006 MEAP Grade 4**

**10** Farmer Green has 412 chickens, and Farmer Brown has 285 chickens. Which is *closest* to how many more chickens Farmer Green has than Farmer Brown?

- A** 100
- B** 200
- C** 300
- D** 700

1. Non-standard and standard algorithms. Show the relationship of the model to the algorithm.
  - a. Partial Sums
  - b. Regroup at the bottom

- c. Short (Standard) Algorithm
- 2. Computation
  - a. Facts
  - b. Simple computation

**WN11.MULTIPLE DIGIT ADDITION AND SUBTRACTION (Subtraction with regrouping across two zeroes.)**

- 1. a. Basic Fact Strategies: Subtraction  
(refer to Addition Fact Strategies power point)
  - b. Fact Families:
    - i. Example:  $11 - 5 = \square$ ,  $11 - 6 = \square$ ,  $\square - 5 = 6$ ,  $\square - 6 = 5$
  - c. Composition and Decomposition of numbers to subtract.
- 
- 2. Non-standard and standard algorithms. Show the link between the model and the algorithm.
    - a. Partial Differences
    - b. Counting Up
    - c. Short (Standard) Algorithm



**C. SUPPLEMENTAL MATERIALS** Available at [www.MichiganMathematics.org](http://www.MichiganMathematics.org)

Chapter 3: Whole Number Addition and Subtraction (April 2007) (PDF, 294 KB)

All 8.5x11 Prints for Place Value and Numeration (PDF, 4,364 KB)

-120 to 120 Vertical Number Line (10 pages)

-40 to 40 and -20 to 20 Opposing Number Line (1 page)

0 to 40 Number Line Drawing Mat (1 page)

No Number Line (1 page)

0 to 99 Chart (1 page)

1 to 100 Chart (1 page)

Blank Hundreds Grid (1 page)

Money Bills (6 pages)

Place Value Mat (3 pages)

Record Sheet (1 page)

Base Ten Blocks (5 pages)

Tens Frames (2 pages)

Addition/Subtraction Mat (PDF, 11 KB)

Subtraction Mat (PDF, 23 KB)

Power Point Presentations:

Addition Fact Strategies (Power Point, 364 KB)

Addition with Regrouping Base 10 Blocks (Power Point, 167 KB)

Take Away Subtraction with Regrouping Base 10 Blocks (Power Point, 68 KB)

Comparison Subtraction with Regrouping Base 10 Blocks (Power Point, 207 KB)

Diagnostic Inventories:

Whole Number Addition Diagnostic Inventory (PDF, 55 KB)

Whole Number Subtraction Diagnostic Inventory (PDF, 58 KB)