LIMESTONE.

22

LIMESTONE.

The study of the limestones of the northern part of the Lower Peninsula has been continued by Mr. Grabau, who is preparing a monograph upon them. Over 124 forms of the rich variety of fossils have been figured for reproduction. These studies have great scientific interest, as Michigan lies between and connects the eastern and western, or as Schuchert calls them Cumberland and Dakota Devonian seas.[1] They have also, however, a practical bearing. The limestone itself is of increasing value. It is shipped to Manistique and Gladstone to make acetate of lime; it is largely used in sugar manufacture. It is the raw material now of the Elk Rapids Portland Cement Co. which has recently consolidated with the Bay Shore Lime Co., as well as the Alpena Portland Cement Co. The rock Portland cements, are, I am told by practical engineers, no stronger, but smoother, and able to go farther than the marl cements.

Dr. Grabau writes:—"We still need about as many more illustrations as we have already made, and I trust you can give us an appropriation of $150.00 or $200.00 for this year. I will fully justify the expenditure in the report, when I think that I can easily prove the knowledge of the fossils is absolutely essential for the finding of limestone deposits of economic value and I also think that I can show that by means of fossils we can trace out anticlines in the limestone region, which may lead to the discovery of gas."

The development of the limestone business is illustrated by the Bay Shore Lime Co., recently consolidated with the Elk Rapids Portland Cement Co., concerning which the following facts are given. We have also, through the courtesy of the Elk Rapids Portland Cement Co., copies of analyses of their raw materials, showing their excellent character. The limestone is the Petoskey limestone, and the analyses are essentially similar in character to analyses given in 1901, 1902 reports. The analysis of the coal is given later.

BAY SHORE LIME CO.

"The Bay Shore Lime Co. is a corporation of this state, capitalized for $250,000,—$200,000 common and $50,000 preferred stock. It has been in business for the past 16 years at its present location and has built up a business which for several years past has paid good dividends on the capitalization. Its output of lime for the past ten years has increased from 16,500 to 118,517 barrels per annum, indicating a steady and profitable growth during the panic of 1892 to 1897, when other lime companies did very little. This is accounted for by the high grade of lime made from the rock in their quarries, which seems to be superior in quality and commands a better price than other limes in a great many markets.

---

[1] Am. Geologist, Sept., 1903, pp. 137 to 156.

"Mr. Sly, the president of the company, makes a sworn statement of their profits for the year 1901, after paying all salaries, repairs, etc., of $28,280.87. With the addition of its present stone-crushing plant, recently built, and two new kilns which are now nearly completed, the earnings of this company will be easily increased to $40,000 per annum, and can be made to exceed that amount by natural growth.

"The property of this company is all located at or near Bay Shore on the Pere Marquette railroad and Little Traverse Bay, seven miles west of Petoskey. They have four (4) large lime-kilns in operation and two building which will make 140 barrels of lime per day each. Their property consists of nearly nine (9) hundred acres of land, more than half of which is underlaid with limestone with a workable depth of from fifteen to sixty (15-60) feet. This is divided into several grades, one for building lime, the others for use in blast furnaces, wood alcohol and beet sugar plants, tanneries, paper mills, for making calcium carbide and Portland cements."

LIMESTONE.

Ig'n loss	49.91
Silica	3.37
Alumina	2.57
Iron oxide	
Lime	51.43
Magnesia	.50
Sulphur anhydride	.60
	100.38

SHALE.

Ig'n loss	17.50
Silica	56.00
Alumina	13.50
Iron oxide	6.50
Lime	2.50
Magnesia	2.00
Sulphur anhydride	.80
	98.80

W. G. BANKS.

"The limestone we use here is located about 2½ miles south of Petoskey, Mich., on the Pere Marquette R. R.

"The shale is located on Pine lake about 8 miles up from the city of Charlevoix, and comes to us by boat."

The development of the Detroit and Mackinac R. R. from Alpena toward Cheboygan, has opened a number of quarries which are in the same general horizon as Petoskey and Alpena.

Mr. A. N. Clark of the Alma Sugar works has been kind enough to give us results of a number of tests, from which it appears it is possible to get

calcium carbonate by the car load with but 2% or 3% of deleterious impurities. He writes, giving analyses below:—

Alma, Mich., Nov. 18, 1903.

. . . .

We have been using some Onaway limestone here at the factory. Some is good and some is poor. Runs from 90 to 98% $CaCO_3$. Some of it is of a rusty color and contains 6 to 8% sand and iron.

Yours truly,

A. N. CLARK.

PETOSKEY LIMESTONE.

A. N. CLARK, ANALYST.

	March, 1902.		April 28, 1902.[2]	
			Stratum 1.	Stratum 2.
Organic	.62[1]			
Insolubles	.44		1.03	1.21
$Al_2 Fe_2 O_3$	.43-	mainly Fe_2	.80	.70
$Ca CO_3$	96.58		83.05	90.62
$Mg CO_3$	2.-		15.11	7.41
$S O_3$		bare trace		
	100.05		99.99	99.94

[1] Hydrocarbon limestone smelling strong of petroleum when struck, has shale like appearance. Fine texture. Small sample sent.

[2] In kegs, two strata.

LIMESTONE NEAR LAKE HURON—NORTH OF ALPENA (NORTHEASTERN MICHIGAN).

	D-I.	E.	A.	D-2.	B.	C.
Insoluble	2.80	1.25	8.60	1.61	1.75	.40
$(O_3 Fe_2 Al_2) O$	.40	.40	1.40			
$Ca CO_3$	96.13	96.47	88.86	94.77	97.70	96.36
$Mg CO_3$	.68	1.60	.38	1.28	.60	3.—
Undetermined and $H_2 O$		.82	.76	2.34		.24
	100.01	100.—	100.—	100.—	100.05	100.—

COMPOSITE SAMPLE OF ALPENA LIMESTONE FROM ALMA SUGAR CO. (AUGUST, 1902.)

Insoluble & $(Fe_2 Al_2)O_3$	1.30	Insoluble and difference	1.—
$Ca CO_3$	97.60[1]	$Ca CO_3$	[1] 96.—
$Mg CO_3$	1.09	$Mg CO_3$	3.—
	99.99		100.—

[1] Checked three times volumetrically with permanganate gave 97.66 $Ca C O_3$ or 97.55. In car No. 146 gave 96.6 $Ca CO_3$.

SAMPLE OF LIMESTONE FROM ONAWAY.

Organic matter and moisture	1.30
$(Fe_2\ Al_2)O_3$	.40
$Ca\ CO_3$	97.—
$Mg\ CO_3$	1.70
	100.40

The following analyses also by Mr. H. R. Browne, are of interest, especially the very complete one of the Bellevue quarry in the carboniferous limestone, because this property has recently been purchased by the Burts of Saginaw, for a Portland Cement plant, and it is unusually complete.

BELLEVUE.

Soluble in H Cl:		
$Si\ O_2$	0.1208	
$Al_2\ O_3$	0.1008	
$Fe_2\ O_3$	0.2093	
Fe O	0.2916=	.4698%
$P_2\ O_5$	0.2692	
Mn	0.0449	
Arsenic	trace	
Ca O	53.300	
Mg O	.483	
$C\ O_2$	42.15	
$S\ O_3$	.098	
	97.0676	97.0676
Gangue insoluble in H Cl:		
$Si\ O_2$ soluble in $Na_2\ CO_3$	.354	
Insoluble	1.63	
S as pyrites	.175	
$Al_2\ O_3$	.369	
Ca O	.021	
Alkalies	.062	
	2.654	2.654
Organic matter	.1334	.1334
		99.8550

Three different beds of the Christiancy quarries in Monroe County show both a high grade and a magnesian limestone.

CHRISTIANCY QUARRIES.

	1	2	3
Insoluble (silica) in H Cl	0.49	0.61	3.31
Iron and alumina oxide	0.13	0.15	0.35
Calcium carbonate	97.59	97.64	77.78
Magnesium carbonate	1.66	1.55	18.48
	99.87	99.85	99.92
Difference (organic, etc.)	.13	.15	.08

Analyses by H. R. Browne.

CORRELATION ACROSS DETROIT RIVER.

INTRODUCTION.

Prof. Sherzer for us, and the Rev. Thomas Nattress of Amherstburg, on the Canadian side, are gathering very interesting information about the limestone along the Detroit river, upon which the various well records which I am able to turn in from my work from time to time, will throw much light.

I quote a letter of much interest from Mr. Nattress. The change of dip noted by Mr. Nattress, may be of importance in collecting oil. The Anderdon quarries near Amherstburg and Sibley quarries near Trenton, both dip southwest and seem to lie on the northeast side of a synclinal tongue of the Dundee or Corniferous limestone. A fault may bow the southwest side. To the north of them there should accordingly be a subordinate anticlinal or pitching fold, passing through Wyandotte.

St. Andrew's Manse,
Amherstburg, Ontario, Feb., 1903.

Dr. Alfred C. Lane,
State Geologist for Michigan,
Lansing.

Dear Sir:—The accompanying comparative statement of strata in (1) Monroe county, (a) "Monroe," and (b) "Dundee," and (2) Anderdon Tp., Essex county, Ontario, may not be without interest to you. I have a hope it may be of some service in getting at our problem in determining the relationship of said strata. I may say that the elevation of surface of white sand rock, here, is 552.00 above mean tide at New York. Mr. Dixon and I will ascertain the elevations of the successive outcrops of the Anderdon quarries. * * *

I believe I have found the point where the variation of dip begins. All I shall say just now is that I am obliged to admit an error in my former assertion that the sandstone (which is indeed "south of") is "superincumbent upon the Corniferous described in my paper." It is *not superincumbent.* This I now know. When, however, you become acquainted with the locality of the several outcrops, and note the very short distance in which the change of dip takes place, you will not be much surprised at my mistake.

There still remains to be explained the presence of the gray dolomite[1] superincumbent (in the Anderdon quarry) upon the Dundee (Corniferous) limestone. This gray rock is in no way to be mistaken for Hamilton.

Very truly yours,
THOMAS NATTRESS.

---

[1] Tests made by Mr. Nattress and myself in 1904, (letter of July 13), prove this *not* to be dolomite, though not so pure calcium carbonate as layers below. I expect to have the matter treated more fully in the Wayne county report.

A COMPARISON OF THE FORMATIONS IN MONROE COUNTY, MICHIGAN, AND ANDERDON TOWNSHIP, ESSEX COUNTY, ONTARIO.

	I. The Monroe beds:[1]	[1]Page.	II. The Dundee:	Page.	III. Anderdon:
Above the Sylvania.	(1) Dip. N. W. to N. (30 or 40 feet per mile.)	45	(1) "General dip to the N. W."	37	(1) Dip. "The approximate dip is ⅞ of an inch to the foot, S. by W."
	—W. 25° S. ("Water collects at S. W. corner.")	79	—"Christiancy quarry * * * 24 ft. per mile * * * W. N. W."	37	
	W. by S.	82			
	—"A fold * * * a high local dip"	87	—"Pulver quarry * * * 119 to the mile"	37	
Below the Sylvania.	—"Stripping increases to S. W."	92			
	—"Rock dips in four directions, from 2° to 5°"	95			
	—N. 61° W.	95			
Above the Sylvania.	2. Color: —"Gray to drab"	46	2. Color: "Light gray to brown * * * running into blue"	39	2. Color: The only rock that may be described as "drab" here is *not a dolomite*, but the pure limestone, lying between the *gray* rock above and the *brown* dolomite below; unless it be the rock next below the brown.
	—"Where weathered * * * of almost a chalky whiteness"	46			
	—"A light colored, finely laminated"	79	Cf. Orton, Ohio. "Is light colored, gray, drab and brown."	39	
	—"A drab and brown dol."	80			
	—"A firm grained drab dol., smooth"	91			
	—"Buffish gray"	82			
	—"Dark when damp, but drying to a light gray"	86			
	—"Buff"	87			
Below the Sylvania.	—"A light or dark drab dolomite"	92			
	—"A gray or drab dolomite"	92			
	—"A drab dol." * * * "Deep blue"	94			
	—"A gray to creamy white dol."	95			
	—"A light drab dol."	95			
	—"* * * covered with a black carbonaceous deposit"	95, 96			
	—"The rock weathers to a soft mealy substance of a creamy color"	97			
	Cf. Above and below Sylvania sandstone. "Bituminous matter is present locally, but does not impregnate the rock as in the case of the Dundee"	46	"Most of the beds have a strong oily odor, and semi-fluid bituminous matter frequently collects in the cavities of fossils"	39	None of our rock is impregnated with bituminous matter. Semi-fluid bituminous matter does not occur in cavities of fossils. There is a slight odor of oil in brown dol. from bed of river. There are thin layers of black bituminous substance in the same strata, one of them half an inch thick.
	Cf.	95, 96			
Sylvania.	3. Peculiarities: Weathering to a "chalky whiteness"	46	3. Peculiarities: "The upper beds are thin bedded and shattered"	39	3. Peculiarities: Some of our chert weathers chalky white,—some of our high grade limestone is thin bedded. Nothing brecciated; unless wavy surface layers of pure limestone. No ripple marks identified; nor mud cracks. No large concretionary bodies. No caverns. No ellipsoidal cavities, except where Favosites hemispherica has decayed away. No stylolites. No concretions observed
	—"Thin bedded"	47			
	—"Brecciated beds"	47			
	—"Ripple marks and mud cracks"	47			
	—"Immense concretionary bodies"	47			
Above the	—"In places cavernous"	79			
	—"Ellipsoidal cavities"	80			
	—"Peculiar stylolites"	89			
	—"Ellipsoidal concretions"	90			
	—"Thin wavy carbonaceous films traverse the rocks"	94			Similar films observable near the juncture of brown dol. and true limestone (perhaps in both) in Detroit river bed. There is no clay deposit between layers anywhere.
	—"Between the strata are layers of a soft putty-like clay which hardens on exposure"	94	"Seams of clay very frequently separate the layers"	39	
Below the Sylvania.	—"Breccia"	95			—No breccia observed. Sink holes of small size; but in the true limestone (where it is a surface area) and *not in the dolomitic rock*. There are rough surfaces in the true limestone. (Also at certain elevations in the gray rock with black carbonaceous deposit.)
	—"Sink holes * * * with diam. of 6 to 10 ft., * * * produced by solution of the dolomite"	114			
	—"The individual strata have very rough surfaces, covered with black carbonaceous deposit"	95, 96			—But this is the peculiar formation known as *crystallites*.
	—"Thin bedded fissured dol."	96	"A two foot bed of chert at the Sibley quarry"	41	—There is an apparently thin bed of chert in the brown dolomite (which may be "a fine crystalline massive quartz").
	4. Essential characteristic: The typical dolomite layers show the normal proportions of lime carbonate and magnesian carbonate; from 50 to 55 per cent of the former and 42 to 48 per cent of the latter ingredient	pp. 46, 82, 83, 86, 87, 92, 95	4. Essential characteristic: —"Essentially a pure limestone"	38	4. Essential characteristic: —Gray rock CaCo$_3$, Mg CO$_3$. 60 903 36 463.
			—"Carbonate of magnesia * * * may run up to 25 per cent"	39	—Brown rock 57 28 , 41 15.
			—"A very small amount of iron"	39	- Gray rock Fe$_2$ O$_3$, Al$_2$ O$_3$ = 0.230. —Brown dol. " " = 0.32. Between these is pure limestone.
	5. Crystals: —"Small cavities carry celestite, calcite, and a little native sulphur"	80	5. Crystals.		5. Crystals: *In the gray rock*, the characteristic crystal is *dog tooth spar*, small to large in size, and fluor crystals, in cavities; fluorite and *calcite* in light seams. In the brown dol. the characteristic crystals are *Scalenohedra of calcite* (carbonate of lime). Also *Strontianite* and *celestite*. Then there is the stratum of "minute closely cohering grains of quartz," = Sylvania.
	6. Strata: —"Thin bedded"	47	6. Strata: —"The upper beds are thin bedded and shattered"	39	6. Below: Your Dundee limestone corresponds closely with our pure limestone in bedding.
	—"Strata average eight inches"	92	"Deeper layers * * * reaching a thickness of several feet" Cf. Orton, Ohio, "The individual beds may reach a thickness of five feet"	39	—The gray rock beds measure (in descending order) 2,[1] 8, 4, 8 and 10 feet.
	—"Thin bedded fissured dol."	96	—"In the upper division * * * the beds seldom reach one foot in thickness"	39	—The brown dolomite is not thin bedded, though no measurements are to hand.
	7. Fossils: —"A stratum of gasteropod molds and casts"	79	7. Fossils: —See p. 227, Dr. Rominger's list	227	7. Fossils:
	—"Generally quite poor in fossils"	222			—See pp. 124, 125 of my paper in 1902 Bureau of Mines Report (Ontario), for fossils in the gray rock.
	—"Crinoids, corals and bryozoan remains are found occasionally" * * * "lamellibranchs rarely"	222	—Fenestella, Polypora, Orthoceras, Gomphoceras, Nautilus, etc.	227	—See pp. 126, 127, b. for fossils of brown dol.
	—Vide	pp. 223-225			
	—"Conularia, Tentaculites, Stropheodonta, Orthis * * * Conocardium, Platyceras, and numerous gasteropods, lamellibranchs, trilobites, crinoids, cyathophylloid, and favositoid corals"	226			

[1]Monroe County Report.

---

[1]Examination of a fresh break of this ledge after observing the same stratum on Pelee Island indicates thin bedding, not at first evident.

Notes on the Fossils in these Different Beds.

The "stratum of gasteropod moulds and casts" (p. 79) in bed A of the Woolmith quarries, above the Sylvania sandstone, *though differing entirely in color,* would remind one of a stratum of our brown dolomite, exposed in Detroit river bed, as to its gasteropod contents. The "occasionally" (p. 222) and "rarely" with reference to crinoids, bryozoans and lamellibranchs in the Monroe beds, apply here also; but do not apply with reference to our corals, especially in certain strata of the brown dolomite. *Tentaculites,* which you include among the fossils of the Monroe beds, does not occur here at all so far as observed, though it is abundant in the subsequent Hamilton formation, as at Bartlett's Mills and Arkona, Lambton county, Ontario.

Though there is a certain amount of identity between the forms found in the Monroe beds and the strata here, it would not appear to me to be greater than might be looked for from persistence of the fossil forms of one formation in the next later formation.

On the other hand, the Dundee fossils (p. 227) so far as enumerated, correspond very closely with our much longer list from the Anderdon quarries and the Detroit river bed exposures, a list that is made larger still when supplemented, as from "Geology of Canada, 1863," page 378.

But your Dundee is "essentially a pure limestone" (p. 38); whereas our high grade limestone appears to contain very few fossils, save a few cup corals and a few far-spreading Stromatoporoid, the low grade limestone (gray rock) above, and the dolomites below, being the fossil bearing strata.

(Signed) THOMAS NATTRESS.

In the Upper Peninsula, various well records, for which see the paper on Water Supply, have added to our knowledge of the geological column, with its heavy belts of limestone along the Lake Michigan shore. This has been treated in connection with the water supply.

Limestone Mountain, an interesting and isolated group of outcrops of the section up to and including the Niagara, has recently been made very much more accessible, by the line of the Mineral Range R. R. (D., S. S. & A.), which skirts the foot of it. It is so far from any other limestone, and so near points like Houghton, to which limestone is largely supplied, that I think it worth while to call attention to it. The Survey has done work upon it, which has never been published. This work was done very largely by W. L. Honnold, and a report on the fossils, written by myself over ten years ago, but the railroad and sundry new facts should be incorporated.

It was visited this year by Mr. A. G. Ruthven, who found it quite accessible, bare from recent fires, and with a limestone quarry proposed. Being so far from any other limestone, this might well be profitable, and I trust may be further studied in the near future. But I think it justice to Mr. Honnold that his work be given first, in addition to the fact that it will materially aid the next workers. I myself visited the district in June, 1894, and studied it with L. L. Hubbard and A. E. Seaman, having previously studied the fossils. We also visited other exposures of the limestone in two small hills on Sec. 7, T. 51 N., R. 34 W., which apparently had not been observed by previous geologists. It is not as high. All these hills rise from plains of flat clay apparently not unlike that at Mass City and elsewhere referred to. The plains appear to correspond in level to the towns of Houghton and Hancock, being from 90 to 205 feet above the lake, the level of Upham's Beaver bay beaches. From this general level of about 210 feet, the hills rise often in steep wave cut bluffs, the higher hill on Sec. 7 is about 250 feet above Lake Superior, 852 A. T., the lower limestone thin bedded, the upper with a bird's eye effect, with *Murchisonias, Crytodontas,* Rhynchonelloids, bryozoa, and Alveolites.

Limestone Mountain itself is fully 400 feet above Lake Superior, 1,002 A. T., 250 feet above the plains and 150 feet above the highest sandstone outcrop. The lines of wave cutting are at 325 feet, 240 to 270 feet, and at 210 feet I estimate (barometer). In addition to the specimens collected by Honnold and described below, we collected specimens 155067 to 166016, which show Maclurea, Ambonychia, Orthoceras, Murchisonia, Pleurotomaria, etc. 16571 and 16572 represent the Niagara, the remainder being apparently, Trenton. The following is a quotation from letter of W. L. Honnold, dated July 14th, 1894:—

"The principal question to my mind is: Whether the contact exposed is between the limestone, i. e., Calciferous and the L. S. sandstone=Potsdam, or between the Lower Magnesian (=Calciferous) and Madison sandstone. That is whether the Madison sandstone and Mendota limestone exist here. The sandstone I exposed under the limestone compares well with the Madison (better than with the Potsdam) and it may be that if excavations were made further west, the Mendota limestone would be found, and still farther west the Potsdam."

It is clear we have here in a short space, a section from the Niagara limestone down. The Lorraine and Utica shales have not been discovered, however, and the Trenton has not been well differentiated from the Calciferous. It affords most excellent opportunities for careful paleontologic study, which would be of great interest in connecting Minnesota with the east.

In connection with the limestone business, one of the discoveries of most interest to me is that shown by the samples of the Manistique well,[1] of the belts of real limestone, not dolomite, from 80 to 107 feet down, which should be the first rock encountered in a belt passing two or three miles north of town.

So far analyses of Niagara limestone have generally shown dolomite, and the presence of belts of chemically acceptable limestone, would be valuable to the calcium carbide and calcium acetate industries.

CEMENT MATERIAL.

The exhaustion of our timber resources is a present fact to this extent, that for many uses it is already too dear. There is, therefore, a great activity in the development of materials to replace it.

The lime and cement business is, therefore, in a flourishing condition, and we have given some notes about the Bay Shore Lime Co., consolidated with the Elk Rapids Portland Cement Co. We have recently published a report, Volume VIII, on the cement business, and so we have only to add occasional notes as they may develop.

The Michigan Alkali Co., having sold their Bellevue quarry, an analysis of which is elsewhere given, have acquired land around Alpena (see 1901

---

[1] See pp. 140 to 141. It may be in this belt that the Osborn caves lie.

report), containing limestone. It is probable that salt may also be obtained, thus making an ideal place for soda manufacture. (Alpena News, 5/11/03.)

In Branch county[1] it is suggested that the marl of Marble, Middle, Bartholomew, Wing, Long, and Coldwater lakes may soon be utilized for cement manufacture by draining the same. The Quincy plant is now on Middle lake.

Marl beds are reported on Maple river near Brutus;[2] South Monterey near Allegan,[3] and borings have disclosed it at Saugatuck,[4] and Union Pier. It is probable that most of the lakes along the Lake Michigan shore, may have some marl, but it is doubtful if it is of high quality.

It must not be forgotten, however, that almost all of our surface clays are highly calcareous and worthy the name of marls, according to dictionary definition, though not according to cement factory usage. Bog lime is a much better term for what they call marl. In the clay analyses given later, there can generally be allowed about twice as much limestone flour as there is of calcium magnesium oxides. We see accordingly, that there is often about a fourth calcareous matter in such clays. It is not surprising, therefore, that specimens come in for my judgment, taken for marl. So they are, but not in the Portland cement man's sense. There are marl beds and calcareous clays near Port Huron.

In regard to the question as to the amount of pyrites permissible in a coal to be used in cement manufacture, we have the following:—

Elk Rapids, Mich., December 15, 1903.

Mr. Alfred C. Lane,
State Geologist,
Lansing, Mich.

Dear Sir:—Replying to yours of the 9th inst. I wish to thank you very kindly for Volume No. 8, which you are sending us, and if there is any charge for it please send me bill and we will remit.

We are pleased to give you the information desired in regard to analysis of coal, and the effect of sulphur and pyrites of iron in the same, upon the clinker. We have found in our practice here that on the average not more than 3/10 of 1% of the total sulphur found in the coal is retained by the clinker. This will vary a good deal according to whether the coal is finely or coarsely ground and whether a large or small amount of air is used in burning. By the use of a large amount of air, we found the pyrites of iron in the sulphur was blown out. We have also found that the use of a shale containing quite a high per cent of pyrites of iron in the form of knobs, after it is incorporated in the slurry and ground, causes us no particular trouble as far as the sulphur is concerned. This resulted because of the fact that the oxidizing in the rotary is such that all the sulphur is driven out or oxidized to a condition which is not harmful.

We found no more sulphur in the clinker made from shale than will be given by the materials themselves, burned with the flame containing no sulphur. Of course, this is accomplished by careful burning and careful grinding of the coal.

---

[1] Detroit News, 8/23/1903.
[2] Petoskey Democrat. 7/4/1903.
[3] Grand Rapids Herald, 6/27/1903.
[4] Holland Sentinel, 5/9/1903.

Trusting this may be satisfactory, we remain,

Yours very truly,
ELK RAPIDS PORTLAND CEMENT CO.,
F. B. Moore, Sec'y.

Coal Analyses.

SHIPMAN CO.'S COAL (PITTSBURGH NO. 8) OHIO.

Volatile combustible	30.375%
Fixed carbon	46.175
Sulphur	5.09
Ash	18.36
	100%

CAMBRIDGE COAL, OHIO.

Volatile combustible	33.00 %
Fixed carbon	58.00
Sulphur	1.61
	92.61 %

W. C. CLARK CO.'S COAL, PITTSBURGH NO. 8, FROM OHIO.

Volatile combustible	36.455%
Fixed carbon	49.385
Sulphur	3.29
Ash	10.42
	100%

SUNDAY CREEK (WEST VA.) COAL.

Volatile combustible	35.19 %
Fixed carbon	49.55
Sulphur	1.30
Ash	13.96
	100%

WOLVERINE COAL, BAY COUNTY.

May 27, 1904.

Analysis of Wolverine No. 2 coal, by Dickman and Mackenzie, for Wickes Bros. Dried at 212 degrees F. Sample from car load lots.

H_2O	6.76
Volatile comb.	42.67
Fixed comb.	42.01
Ash	8.65
	100.10
Sulphur	3.50

B. T. U. 12,295.

This is the Upper Verne, according to Mr. Cooper.

Analysis of a coal from Bay City, Michigan.

Moisture	5.92
Volatile combustible	37.74
Fixed combustible	36.54
Ash	19.80
Sulphur	8.60

Sept. 14, 1903. E. D. CAMPBELL.

This may be the Lower Verne.

BUILDING AND ROAD MATERIALS.

The lumber supply of Michigan is not only approaching exhaustion, it is exhausted so far as the cheaper uses of wood are concerned. Instead of cedar blocks, plank, and corduroy roads, asphalt block, bithulithic or tar macadam, and macadam roads are coming in; instead of cedar fence posts, cement posts, and for building material instead of wood, brick or clay, sand or cement, concrete blocks, and concrete and steel constructions.

The asphalt block is made of Calumet and Hecla or Tamarack stamp sand, i. e., crushed quartz porphyry or felsite, cemented by asphalt. The following experiment on a limestone, a trap, and a conglomerate sand asphalt block was made, to show the relative durability.[1]

"The blocks were placed side by side near the periphery of a ten-foot marble rubbing bed, running thirty revolutions per minute. Each block was weighted, so that block and weights combined weighed forty-five pounds. They were all kept on the running bed forty-five minutes, sand and water being freely and properly supplied all the time. The limestone block was worn away one inch on its 4x12 inch face, the trap-rock three-eighths inch, and the conglomerate sand-block one-eighth inch. It is proposed to use the conglomerate sand in the factory mentioned above" (Detroit or Toledo).

The sand is shipped from the waste piles at Torch Lake to the factory at Toledo, and while water transportation is cheap it would seem as though a factory of these blocks could be started to good advantage at Lake Linden. This has been suggested.[2] The raw material could be improved by using the felsites of Mt. Houghton or the Bare Hills region[3] without doubt, which would not be decomposed as the wall rock of the copper lodes. But I do not think the advantage would pay for the cost of crushing.

The pavement of asphalt blocks is used in Pontiac and Ann Arbor and is well spoken of. It is also used in Kalamazoo. Occasional complaints are that the rock is crumbly under changes of temperature. Whether this is due to defects in the manufacture of particular lots I cannot say.

---

[1] Michigan Engineer, 1899, p. 110.

[2] Portage Lake Mining Gazette, October 22, 1903.

[3] See Vol. VI, Part 2, of Report of Geol. Survey of Michigan.

The "bithulithic" pavement[1] is meeting with much favor. It is essentially a macadam with a patent filler and binder of coal tar origin and bituminous nature. A full account will be found in the 1904 report of the Michigan Engineering Society. It has been used in Saginaw, Port Huron, Mt. Clemens, Bay City, Manistee, Kalamazoo and Pontiac. It should not be laid in cold weather. From the point of view of the economic geologist, the essential points are:

(1.) Like all macadams it must be well underdrained. On till or heavy clay a drainage two feet below the surface of road should be provided, and a bed of gravel or cinder, broken brick or other porous material laid.

The foundation proper is of 1 to 3 inches of broken stone, for which the various limestones of the State will do. The trap of the Upper Peninsula is better, and crushed field stone are usually all right. Layers of finer material should be laid on top, to fill the coarser material and a coat of bituminous cement applied. Then the surface coat of from 1¾ inch to 1-10 inch material mixed with cement is applied. Possibly stamp sand of the coarser grades could be used to good advantage here, and any way, a harder and less dusty rock like granite chips, crushed field stone or crushed trap is preferable to limestone for this top coat. A thin layer of stone chips is rolled into the surface, and this might well be waste jasper, crushed stamp sand, granite chips, or similar material, with which the State is well supplied. So far as raw materials are concerned Michigan should have good roads.

Perhaps an ordinary macadam might be converted by repeated sprinkling with crude oil into a somewhat bituminous macadam.[2]

As regards plain macadam and gravel roads I would call attention to the pamphlet "Modern Road Building,"[3] which, though an advertising circular for gratuitous circulation, contains many good hints, no matter what machinery may be used.

Road Materials in the Upper Peninsula.

The Upper Peninsula has a wealth of good road making materials. Around Ishpeming and Negaunee the waste rock of the iron mines, the "jaspilite," a red jaspery chert, is used. The chert makes a hard road and the iron gives it good cementing power. The main drawback is the bright red, paint-like dust, of which it is almost impossible to rid oneself. In damp weather these roads are fine, and a treatment with crude petroleum might be an improvement.

In some places the diorite, which is often the foot of the ore bodies, and makes most of the prominent knobs, is used. This is also a good material and the color is not so objectionable.

Northeast of Teal lake a crusher has been installed, crushing the gannister, or Mesnard quartzite. A road near the cemetery is finished with this. The same rock has been used near Marquette. It is very hard, but not tough, and does not cement so readily as some rock. There are also quantities of very fair gravel.

In the copper country the crushed trap of the copper mines is used and makes an excellent material. In a few places near the Allouez, broken conglomerate, which is practically broken felsite, is used. Both are among the best of materials.

---

[1] See advertising literature issued by Warren Bros. of Boston and New York.

[2] Year Book, Department of Agriculture, 1902.

[3] Port Huron Engine and Thresher Co., Port Huron, Mich.

Quartzite is on the whole probably the most widespread of materials, but usually does not cement well.

Hard limestone is found at Manistique and numerous other points and has good cementing powers, but for a combination of toughness with cementing powers, it will be hard to find a rock to excel the copper country traps and the iron country "diorites" or amphibolites.

Clays and Shales.

As regards paving brick, I have repeatedly called attention to the waste of the coal mines as furnishing suitable raw material.[1] The coal measure shales, like those naturally exposed at Flushing, often make good paving brick. The neighborhoods of Grand Ledge and Williamston, and Jackson are also promising. The exposures along the east side of Huron and Sanilac counties and of the Antrim shales in the northern part of the Lower Peninsula, as at Boyne Falls will also be likely to prove satisfactory. The Antrim shales are the same as the New Albany shales of Indiana. In the Upper Peninsula some of the slates near L'Anse would probably be all right, if the cost of grinding were not too much.

In as much as we have recently issued a report on clays we have only a few notes to add.

At Mass City, on Sec. 9, T. 50 N., R. 38 E., close to the little creek that flows through the center of the section and about 800 feet from the C. M. and St. P. R. R. station, is the newly opened brick yard of the M. Van Orden Cō. The brick are mainly light red. The bed of clay is some 190 feet deep as shown by borings,—all clay, but only the top 12 feet or so are now used. There are kettles of slightly sandier (6 per cent. sand) clay on top.

Analysis of clay from Mass City, by Elmer E. Ware, gave:—

Ann Arbor, Mich., Oct. 20, 1903.

SiO_2	52.12
Al_2O_3	14.95
Fe_2O_3	4.41
Ca O	10.21
Mg O	4.56
SO_3	.20
Loss on ignition	12.32
Difference	1.13
Total	100.00

Mr. Gustave Voelcker, superintendent, has analyzed all the clays around and finds them very low in free silica (quartz flour), the highest being 6 per cent. and less the further down they go, yet he says they are fairly resistent. The finely divided lime which the analyses show, he says, is not deleterious.

We have also analyses of the same clay made at the College of Mines, Houghton, for the management, given in the following letter:—

---

[1] See Vol. VIII, Part 1, of Report Geological Survey of Michigan. Also the Gateway, Jan., 1904. There is a brick plant at the Valley Coal Co.'s mine south of West Bay City.

Houghton, Mich., Dec. 4, 1903.

Alfred C. Lane,
State Geologist,
Lansing, Michigan.

My Dear Mr. Lane:—In reply to your favor of the 17th of November, in which you send me a copy of assay of our Mass City clay, would say that the same tallies very closely with Dr. Koenig's assay of our bottom clay. I give below copy of his assays of upper and middle and bottom belts. No. 1 being 7 to 8 feet thick; No. 2, 6 to 8 feet thick, and bottom unknown beyond 30 feet.

	1	2	3
Thick, feet	7 to 8 Top	6 to 8 Middle	30 & more Bottom
SiO_2	68.20	53.86	49.00
Al_2O_3	15.72	17.96	14.56
Fe_2O_3	4.32	5.72	4.64
CaO	2.00	5.22	10.54
MgO	1.00	2.42	3.98
Mn_3O_4	trace	trace	trace
K_2O, Na_2O	2.72	3.26	4.02
H_2O, CO_2	5.83	11.51	13.26
	100.00	100.00	100.00

Our experience shows that the combination of No. 1 and No. 2 makes a very hard brick, of uniform pink color. Very satisfactory in every way.

Respectfully yours,
(Signed) M. VAN ORDEN.

The peculiar thing about this clay as compared with others is that there is more lime and magnesium than carbon dioxide to combine with them, so that they must exist as silicates, and we may associate this with the fact that the clays lie close to the Copper Range, in the rocks of which calcium and magnesium silicates (such as laumonite, epidote, prehnite, chlorite, serpentine, labradorite olivine) are abundant. Many of these silicates are quite fusible, and it is probable that such clays would be quite fusible, especially No. 3, which would be almost a slip clay. Another plant has been talked of at Mass City as there is a large amount of clay there and also one has been proposed at Kenton.

A clay or shale encountered in a well at 40 feet near Daggett proved to be a hard white clay quite free from carbonates and highly resistent.

We have also an analysis of a surface clay from this part of the State, as follows:—

Dec. 8, 1900.

For W. S. Oberdorffer. Menominee Co., by F. S. Kedzie.

	No. 1.	No. 2.
Insol	60.96	61.42
$(Fe\ Al)_2O_3$	9.38	3.62
$CaCO_3$	11.81	22.77
Water, organic matter and difference	18.65	12.19
	100.00	

This "clay" is really a rock flour. There is very little alumina or kaolin about it. It must be very finely ground up silica and limestone, and would not, I should imagine, be a bad material to add to sand brick or cement brick.

There is a new brick yard at Munising, the Anna River Brick Yard. But at Munising a calciferous dolomite is also used for buildings.

The following is an analysis of a shale clay of the Michigan series, taken from near Alabaster and Sherman:—

Michigan Agricultural College.
Experiment Station.
Agricultural College, Mich., June 7, 1900.

R. A. McKay,
502 Phœnix Building,
Bay City, Michigan.

Dear Sir.—Herewith please find analysis of clay specimen:—

Silica	58.95%
Aluminum oxide	14.45
Iron oxide	7.60
Calcium oxide (lime)	2.94
Magnesium oxide	.86
Sulphuric anhydride (SO_3)	1.73
Alkalies calculated as potassium oxide (K_2O)	2.54
Water of combination	7.50
Organic matter and loss	3.43
	100.00

It is a good clay. It is entirely free from calcium carbonate and is to be recommended for its content of silica and freedom from grit.

Very respectfully,
(Signed) FRANK S. KEDZIE.

The following is a Maple Rapids drift clay. See record of well in Vol. VIII, Part II, p. 197.

SiO_2	49.90	51.00
$(Al_2Fe_2)O_3$	19.85	17.18
$CaCO_3$	18.32	17.31
$MgCO_3$	10.53	9.45
Total	95.60	95.54
Difference, alkalies and water	4.40	4.46
	100.00	100.00

F. S. KEDZIE.

The following is another drift clay from the Saginaw valley:—

Case School of Applied Science,
Chemical Laboratory.
Cleveland, February 5, 1900.

Mr. S. G. Higgins,
Saginaw, Michigan.

Dear Sir.—The sample brought me some time since by Mr. Dow contains:—

Silica (SiO_2)	41.2%
Alumina (Al_2O_3)	13.4%
Iron oxide (FeO_3)	5.0%
Lime (CaO)	10.4%
Magnesia (MgO)	0.5%
Sulphuric anhydride (SO_3)	0.5%

The material is more clay than marl, and would require the addition of lime or marl in order to make a Portland cement mixture. For this purpose it would be well adapted, as the sulphur and magnesia are low, and it contains little free sand.

Very truly,
A. W. SMITH.

Finally, we have a typical series of surface calcareous clays from near Detroit, good mainly for building brick, which have been analyzed in the laboratory of Prof. E. D. Campbell for Prof. W. H. Sherzer, who is studying them for his Wayne county report, and adds the following notes:—

Jacob Daniels & Bro. East of Pere Marquette and one-quarter mile south of Michigan avenue, Springwells.

No. 1. Middle of upper layer—2½ feet from surface and 2 feet from top of bed.

No. 2. Second or red layer—2½ feet from top of bed and 7 feet from surface.

No. 3. Third or blue layer—5 feet from top and 14½ feet from surface.

George H. Clippert & Bro. East of Pere Marquette and about one-half mile south of Michigan avenue.

No. 4. Upper layer—2½ feet from the surface or 2 feet from top; 4½ foot bed.

No. 5. Red or second layer—2 feet from top of bed or 7 feet from surface.

No. 6. Blue layer—5 feet from top of bed or 14 feet from surface.

L. D. Haggerty & Son. At Pere Marquette, just north of Michigan avenue, Springwells.

No. 7. Upper yellow brown layer—2 feet from surface and 1 foot from top of bed.

No. 8. Red or second layer—2 feet from top of bed or 5½ feet from surface.

No. 9. Blue layer 5 feet from top or 12½ feet from surface.

Lonyo Brick Co. About one mile east of Pere Marquette railroad and just north of Michigan avenue.

No. 10. Sample 5 feet from surface.

No. 11. Sample 2½ feet from surface.

Proctor Bros. One and one-half miles east of the P. M. R. R. and just north of Michigan avenue.

No. 12. 5 feet from surface.

No. 13. 2½ feet from surface.

M. Downey. Two miles east of P. M. R. R. and just south of Michigan avenue.

No. 14. 5 feet from surface.

No. 15. 2½ feet from surface.

At Haggerty's, Daniels' and Clipperts' there are three sharply defined beds—the upper yellowish-brown, 2 to 5 feet thick, considerably leached and not very plainly stratified. A second or "red layer"—yellow brown, but rusted and reddened—3 to 4 feet thick and very distinctly stratified. The third is a blue layer, fine grained and not leached; beautifully stratified. It overlies till and at Clippert's may run to a depth of 40 feet judging from borings. The clay is a lake deposit, back of the Detroit moraine, and the three beds may have been formed in the Maumee, Whittlesey and Warren stages—or the various stages of Lake Warren alone.

Eastward from the railroad the red layer gives out or passes into a yellow-brown one not to be separated from upper. Deposit of clay thins out towards Detroit and disappears on moraine. Main body to the east lies to the south of Michigan avenue.

WAYNE COUNTY CLAY ANALYSES.

BY ELMER E. WARE. UNDER THE DIRECTION OF PROF. E. D. CAMPBELL.

	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
Silica	57.18	57.55	56.16	55.86	57.58	74.77	57.75	55.53
Alumina	16.04	13.42	10.95	15.24	13.63	8.32	14.75	10.89
Ferric oxide	5.26	3.92	3.32	4.50	3.62	3.02	3.70	4.30
Calcium oxide	6.46	7.40	10.24	7.10	6.94	3.04	5.92	9.23
Magnesium oxide	4.60	5.96	4.86	5.05	4.68	2.52	4.68	5.36
Loss on ignition	10.07	11.43	13.95	12.23	11.18	5.07	9.90	12.51
Sulphuric anhydride	.01	.64	.04	.06	.65	.05	.06	.80
Sodium oxide	....	....	....	....	....	1.85	....	....
Potassium oxide	....	....	....	....	....	1.54	....	....
Difference	+.38	—.32	+.48	—.04	+1.72	—.18	+3.24	+1.38
Totals	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

December 31, 1903.

	No. 1.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	No. 15.
Silica	71.32	66.19	67.06	65.75	58.42	69.50	59.46
Alumina	12.73	12.78	16.78	15.04	13.44	14.84	14.88
Ferric oxide	4.88	5.18	4.30	5.26	4.70	4.92	4.40
Calcium oxide	1.70	3.37	1.70	3.10	5.86	1.92	5.18
Magnesium oxide	2.19	3.58	2.30	3.05	4.46	2.09	5.01
Sulphuric anhydride	.13	.06	.06	.06	.04	.16	.11
Loss on ignition*	4.67	7.42	4.98	6.85	10.08	4.42	9.38
Potassium oxide	....	....	.87	.74	....	....	....
Sodium oxide	....	....	2.69	2.45	....	....	....
Difference	2.38	1.42	—.74	—2.30	3.00	2.15	1.58
Totals	100.00	....	....	103.40	....	100.00	100.00

* Includes combined and hydroscopic water, organic matter, carbon dioxide, alkaline chlorides, etc.

ELMER E. WARE.

These analyses show beautifully the leaching out of the calcium oxide and sulphur.

CEMENT CONCRETE. CEMENT BRICK.

Another industry which has rapidly developed in this State is the Portland cement industry. As we have just issued a report on this subject[1] we need only add a few notes of current interest referring also to the heading limestone.

A sample of marl has been sent by James Searing of Lyons, from the Maple river bottom to Prof. Kedzie, which runs 96 per cent. Ca CO_3.[2]

The Lakeland deposits described by us, under the head Standard Portland Cement Co., are now being developed by the National Portland Cement Co. The average depth is said to be 18 feet, and there is a good deal of peat associated.[3]

On the other hand, the Standard Portland Cement Co. is said to be testing beds near Edgewater, 12 miles north of Frankfort. The contract is said to be for $350,000, by the Robert F. Wentz Engineering Co. The plant, like the Newaygo plant, expects to use water power which the Platte will furnish.[4]

Mr. Wentz writes:—

Toronto, Ont., Feb. 29, 1904.

Mr. Alfred C. Lane,
State Geologist,
Lansing, Michigan.

Dear Sir.—I received a copy of the State reports, etc., at Detroit, and in reply beg to thank your prompt attention to this matter. I have practically read over the entire matter on my way home, and in my spare

---

[1] Volume VIII, part 3.

[2] Michigan Investor, 12/16/1903. See also our report, Vol. VIII, part 3, p. 311.

[3] Detroit News, 2/11/1903.

[4] They have since turned to a limestone proposition near Charlevoix.

moments at the hotel. I am pleased to say that your investigation has been very thorough and the report, especially on marl, in which I am particularly interested, is very complete, containing a large amount of very interesting and important information. I shall be pleased to send you reports and results of analysis of Michigan raw material which I have made in the past several years. This I will do as soon as I possibly can.

I have recently made very valuable improvements in the manufacture of Portland cement from marl on the wet process, almost reducing the cost of burning to that of rock or dry process. The cost of burning however, is governed by the purity of the marl to a great extent. Marl containing a considerable proportion of organic matter is much more expensive to burn,[1] requiring a much larger amount of fuel as you will know, but with a pure marl, free from organic matter, I am able to produce with my process, from 180 to 225 barrels per kiln with a fuel consumption of about 125 pounds per barrel.

Yours very truly,
(Signed) ROBT. F. WENTZ ENGINEERING CO.,
Robt. F. Wentz, President.

A number of plants referred to in Volume VIII as in erection are, like the Fenton plant of the Egyptian Portland Cement Co., now in operation. In addition we have the El Cajon Portland Cement Company (with 425 acres near Alpena), the Onaway Cement Co. (which may also be a limestone proposition), the Aetna Portland Cement Co., near Fenton, the Logan Portland Cement Co. (the same areas as the Twentieth Century Portland Cement Co., referred to in Volume VIII), and the Palatine Portland Cement Co. (to operate on the marl beds of Genesee, Livingston, and Shiawassee counties), are among the newer propositions.

Not only is the Portland cement business growing in a substantial fashion, but industries dependent upon it, such as the making of concrete block, cement brick, and hitching and fence posts.[2]

One of the earliest places to use cement brick is Durand, where there is a church built of them. Other places are:—

The Utica Cement Brick and Block Co. (plant using gravel of Geo. B. Davis pit); Jas. Pryor & Son, Houghton (plant to use the stamp sand waste from copper mining, the old Franklin mine); F. P. Bradley, Hudson, Mich.; Ypsilanti Cement Block Co. (using Jollifer gravel pit near Plymouth, Wayne Co., i. e., an old lake gravel); Mason Tile and Cement Co., of Detroit (make cement brick and have tried some for paving), Dowagiac.

The Portland cement is always the more expensive material and should be used in quantities sufficient to fill the voids. Now even grained material has more voids, even though individual grains are very small, than gravel or sand of uneven grain, where the small grains can pack in between the large ones. Therefore in concrete it is necessary to mix gravel with sand enough to fill the voids, and then finally cement enough to fill the finer remaining voids. The theoretical maximum porosity is .4764 of the whole volume empty or about half and half. In an even grained sand, minimum porosity is 26 per cent. of the whole volume. An average clean

---

[1] These are commonly bluer or clay colored rather than white. L.

[2] Lansing Cement Post Co., American Cement Post Co., of Marshall, Hillsdale Fence Post Co., Geo. F. Demorest, Lapeer, and Pulfrey & Pond, St. Johns, hollow cement blocks.

beach sand will run about 40 per cent. voids, so that a cubic foot, which if it were solid quartz, would weigh about 2650 ounces (1000 ounces of water to the cubic foot), will weigh but 1590 ounces, or 100 pounds. The average specific gravity of Michigan sands and gravels according to my test is not far from 2.65. Thus if we take the number of ounces of water a cubic foot of a given sample of sand or gravel will absorb, and divide by 1000 we shall get the proportion of the volume which is pore space. Or, if we take the weight of a cubic foot in pounds and multiply by 0.6, or, more exactly, find the weight in ounces and divide by ten times the specific weight (say 2.65) we shall get the per cent. which is not pore space.

The per cent. of cement by volume required, therefore, will vary with the porosity of the sand. Two or three parts of sand to one part of cement is about all that it is wise to add. Any further dilution of the cement should be by coarser material. A common proportion may be 1 cubic yard broken stone or gravel, with one-half volume voids, requiring one-half yard of sand, which will have one-third its bulk voids, requiring so much or one-sixth cubic yard of cement, i. e., the proportion of gravel, sand or cement for one cubic yard of concrete will be 1 cubic yard gravel, one-half cubic yard sand, one-sixth cubic yard cement, and about the same proportions (a little less cement) by weight.

I do not think that pure sands will be so economical of cement as less assorted sands, beach and river sands and shingle, and as I have said, the kame and overwash gravels and eskers of the glacial epoch.

I understand that beside at Durand, concrete block plants are planned near Rochester, St. Charles, Lansing, Ypsilanti, etc.

The economic geologist is more especially concerned with the other raw material, the sand or the gravel. This should of course, be convenient to transportation and free from clay, mud or slimy material. But that does not mean that it must be very even gravel. Quite the contrary. "The best sand is that of very uneven sizes and sharp." The more uneven the sizes the smaller are the voids and the heavier the sand.

But here in Michigan the glacial gravels are often unassorted, so that little or no sand needs to be added[1] and in any case of material for any extensive proposition, one should have the amount of pore space tested.

SAND BRICK.

Another important industry in development is that of making sand brick. Among the most important papers are those of S. V. Peppel, formerly of the Ohio Geological Survey, now of "Rock Products," Louisville, Ky. I am also indebted to Prof. O. P. Hood for suggestions, and to Mr. W. O. Kutsche.

A sand brick is a mass of sand or fine gravel (under 1-20 inch) cemented by a net work of silicates of lime with or without magnesia or other earths made by steaming under pressure, a mixture of sand or gravel with slacked lime.

The sand, as for concrete, should be of varying size, which may be provided by crushing some of it. "If we had a sand with a good assortment of sizes from coarse to fine, we ought to have sand finer than 150 mesh equal in amount to the weight of lime to be added." Clay is deleterious, but, if not over 10 per cent., may be counteracted largely by using

---

[1] See article by Geo. S. Pierson, Michigan Engineer, 1904.

more lime. It adds ease of working and the sand should not be quicksand, but mold well. To the sand 5 per cent. to 10 per cent. of slacked lime are added. A molding pressure of best about 15,000 pounds per square inch is used, and they are then exposed to steam at a pressure of 120 to 150 pounds, and temperature of 180° C. to 185° C., for say 8 to 10 hours. Mr. Peppel found that freezing actually improved the brick materially.

The absorption is said to be 8 to 10 per cent. of the weight of the dry brick. One Michigan specimen which I tested ran as high as 12 per cent. The crushing strength is better than that of the best sandstone if properly made, but a heavy pressure of 200 to 250 tons per brick is important to get the best results, and a uniform pressure is not always attained.

In regard to merits of different systems of manufacture, that is a matter for the mechanical engineer, and we would refer to the gentlemen whom I have mentioned above. Great advantages of all methods are, however, rapidity of production and economy of fuel.

Among the sand lime brick plants in operation or construction in the State are as follows:—

The Holland Brick Co., working under the patents of H. E. Brown of Coldwater, and making dolomitic sandstone brick, W. O. Kutsche, superintendent. They are working on sand at their plant corner of Fairbank avenue and Eleventh street, at Holland. The Jackson Pressed Brick Co. and the Battle Creek Dolomitic Brick Co., are also to use this process. This means that they use a slacked lime made of magnesian limestone. Most of the Monroe county lime (except the Christiancy quarries) is of this character. The quarries in the Niagara limestone along the west shore of Lake Michigan, as the Manistique, are pretty sure to be dolomitic lime. The Petoskey, Alpena, Bay Port, Sibley and Bellevue limestone is ordinarily not a dolomite, though layers may be found at Petoskey and Alpena which would yield it. There would be an advantage in combining this process with the making of cement, in that the dolomite lime is just the kind which is not wanted for cement. There is probably no especial advantages in quality in using magnesian lime. Peppel thinks the contrary.

The American Sandstone Brick Machinery Company has a plant at the south end of West Saginaw, near the glass works, well spoken of by Peppel, using a sand connected with the old Lake Algonquin beach line. The sand lies in a ridge running north of N. W. and cut by the M. C. R. R. It is at least 10 to 15 feet deep, with no apparent stratification. It is probably a wind or dune deposit between the Tittabawassee river inlet and the great lake of the time, on a projecting spit, and rain percolation since has destroyed what little stratification there was. It is *not* a quicksand, but molds well and has a little organic matter. This and a Muskegon plant are on the system of F. Komnick of Germany. Another plant is talked of for West Bay City. Peppel gives a description of the process.

Finally, we have the plants using Huennekes system. The parent company of the State is the Michigan Sand-Lime Brick Co., of Saginaw. They have a plant in operation using Saginaw river sand at the north end of the city near Saginaw, and as they have the rights for Michigan, are interested in plants in the copper country, Middleville, Detroit, and the Sault.

"C. W. Boyle and Geo. O. Cole, of 76th North Market street, Grand Rapids, Michigan, are taking preparatory steps for the organizing of a brick manufacturing plant. Mr. Boyle is the inventor of a new machine for manufacturing sand or chemical brick, on which he obtained a patent from the government, July 31, 1903. The new brick after leaving the machine is so soft that it cannot be handled, but after being subjected to a chemical treatment become harder and more durable, it is claimed, than any variety of brick now used for building purposes, and the brick can be manufactured and put on the market in 48 hours. One residence in this city has been built of this brick, which has proved satisfactory."

Mr. Peppel[1] remarks: "If the last five systems described, were all to work under parallel conditions, I do not believe that the cost of production would vary more than 35 cents per thousand." * * * "The cost of production in this country, independent of depreciation and interest on investment varies from $2.40 to $4.00 per thousand. The selling price ranges in different localities from $8.00 to $15.00 per thousand." * * * "I see no reason why plants should not be made independent of all the above mentioned patents or systems." * * * "There is no patent process of hardening with steam under pressure, since Dr. Michaelis of Germany allowed his patent to expire some years ago." * * * "We are not advocating sand lime brick for pavers." * * * "Those locations should be first selected where sand is abundant and fuel inexpensive.

The last remark seems to indicate Lake Superior as an appropriate region, and it is to be noted that freezing and thawing seems to strengthen them. But in general, Michigan is well supplied with sand, and while we have coal and peat, fuel is not and never will be as cheap as farther south, for there is more demand and less supply.

---

[1] S. V. Peppel, Louisville, Ky. Further contributions to the manufacturers of artificial sandstone and sand brick, p. 43.

TRANSMISSION OF HEAT INTO THE EARTH.

In several lines of investigation the question of conductivity or diffusivity of substances has come up as an important geological factor. Peat moss is a good non-conductor, and this lends it part of its practical value in growing nursery stock. Moreover it cannot be without influence upon peat bog vegetation that in the very heat of summer their roots are immersed in cold water while their leaves are exposed to the fierce heat of the summer sun. Again the heat diffusivity of the soil seems to indicate its porosity.[1]

From the temperatures of flowing wells we can draw some conclusions as to the depth from which their water is drawn, and in the same way we can infer from the temperature of springs the depth of their source. Mr. Frank Leverett has made practical use of this in studying the Ann Arbor water supply.

Finally in studying the temperatures at great depths and the rate of increase in mines we need to know the surface temperature from which the increase of temperature in going down begins, and in all these cases we find involved the fact that the temperature of the ground varies, being perceptibly affected for two or three feet by the daily range of temperature and for about twenty times greater range of depth, by the change from summer to winter. I found a large series of observations on soil temperatures by Prof. R. C. Kedzie which had never been completed, and I spent quite a while in finding a method of treating the subject which should be sufficiently accurate for the geological purpose and for the observations and yet not too technical.

We may treat the variation of temperature for a day and its spread down into the earth very nearly as though it were the only variation and suppose each day the top of the mercury of the thermometer rose and fell as a ship would rise and fall while a wave passed under it. If that is so and the period of variation is a day, the average of two observations half a day apart will give the average mean temperature of the day. If we subtract this mean temperature from observations of the temperature one-quarter of a day apart then the square root of the sum of the squares of the difference of two successive observations six hours apart will be the greatest difference for the day. We find this difference decreasing as we place our thermometer deeper in the earth, and the less the diffusivity of the soil for heat and the greater its porosity the less will the daily or annual differences of temperature be at a given depth. Moreover the quotient of the differences is proportional to the

---

[1] See pages 24 to 25 of this report.

exact time when this greatest difference from the mean occurs, and again we find this time is greater the deeper in the earth the thermometer is and the less the diffusivity of the ground.

The mathematical treatment below gives us the exact method of computation.

We may pick out from it certain facts of general interest. The range of surface soil temperature is greater than that of the air temperature. In a soil like that at Grayling or Lansing with the usual summer amount of moisture, etc., it takes about a day for the heat to penetrate two feet and the range at that depth is only about 5% of that at the surface. The annual variation would penetrate in a year about $\sqrt{365}=19$ times as deep or say 38 feet, and at 19 feet it would be coldest in summer. The range of variation of temperature at 38 feet would again be only about 5% of the range from the mean daily temperature the last of July to that the last of January. These figures vary of course according to the diffusivity.

Mathematical Treatment.

The variations of temperature at the surface of the soil may be taken as the result of a combination of a number of different variations of temperatures. These latter will differ in amplitude, that is to say, in range of temperature due to them. They will also differ in period, that is, the time which elapses between the occurrence of the greatest divergence, and the recurrence of the same, that is, between successive maxima. Two of the most prominent periods are the day and the year. They will also differ in the date at which the first maximum takes place, after any date that we take from which to begin to reckon our time, say midnight of January 1. It may be shown by Fourier's theorem that no matter how complicated the variation of temperature, it may be considered as the sum of a series of such harmonic variations,[1] each being expressed mathematically as—

$$(1)\ T = c\ \cos\left(2\,\pi\,\frac{t}{p} - l\right)$$

In this expression—

T is the divergence of temperature from the average, eliminating this particular variation, at a time t.

c is the amplitude, that is, the maximum divergence from the average.

p is the period for the particular term, the length of time between maxima.

π is 3.1416, but in estimating angles $2\ \pi = 360°$, so that $2\ \pi\ \frac{t}{p}$ is an angle which is to four right angles as the time t for which T is sought is to the time between maxima.

l is the value $2\ \pi\ \frac{t}{p}$ assumes at the time of the first maximum, and the time of it after the date of the beginning is $l.\ \frac{p}{2\pi}$

---

[1] Byerly, Fourier's Series, Chapter II.

Now, at a depth, which we will call x, the temperature, so far as it is due to the surface temperature, may be represented as the sum of a set of vibrations having the same periods, but the maxima will be less and later.[1] Each will be expressed mathematically as—

$$(2)\ T_x = c\ e^{\,-R}\cos\left(2\pi\frac{t}{p} - l - R\right)$$

In this expression—

T_x is the temperature of a point at the depth x at the time t as before.

c, π, p, t and l have the same meanings and values as in expression (1).

e is the exponential base 2.71828.

R depends on the retardation of the maxima which depends on the depth, the period, and the capacity of the ground to diffuse heat. This latter we will call K.

K is by Byerly called a^2 and by Kelvin $\frac{k}{c}$. It is proportional to the rate of change of temperature with the time at a given depth. It is also inversely proportional to the acceleration of the change of temperature for different depths at the same time; i. e., $D_t u = K D_x{}^2 u$. K is usually assumed to be invariable, though this is not strictly true, and in such case if the temperature was proportional to the depth, it could not change with the time.

It may be shown that—

$$(3)\ R = x\sqrt{\frac{D_x{}^2 u}{D_t\ u}\,\frac{\pi}{p}} = x\ \sqrt{\frac{\pi}{Kp}}$$

As to the units, x and $\sqrt{dx^2}$ are referred to the same unit, also dt and p, du and d^2u, and consequently a change in units does not affect R, nor does a change in temperature unit from degrees Fahrenheit to degrees Centigrade affect K. K is the conductivity and c the thermal capacity per unit volume, which is for water, 1; for most rock, about .5; and for air, .000307.

Obviously the diffusivity, or spread of heat, when the strata have to be heated up as the heat wave spreads, will be relatively less than when the conductivity or flow has become steady, if the body absorbs a good deal of heat in being heated up. One must be careful not to confuse the conductivity and the diffusivity, as is often done; and in part I myself, in my Annual for 1901, page 247, have mixed some conductivity and diffusivity data, as writers are sometimes obscure. The conductivity may be defined, for instance, as the number of calories, or heat units, which an indefinite slab, without varying the temperature, will transmit per square centimeter through a slab a centimeter thick, the difference of temperature of the two sides being 1°, while the diffusivity is the conductivity divided by the thermal capacity per unit volume. This is the specific heat per unit weight multiplied by the specific gravity. The former is calculable from the chemical composition, and is for most rock-forming minerals not far from .20. For water, it is

---

[1] See Byerly's Fourier's Series, Art. 69, much simplified here.

1.00. The specific gravity for rock-forming minerals is usually between 2.5 and 3.5, consequently the specific heat capacity per unit volume of the strata, except for porosity, is near .5 to .6. If the pores are filled with air, the heat capacity per unit volume is diminished; if filled with water, it is increased and the diffusivity diminished relative to the conductivity. Unless convection currents of water come in, therefore, a wet soil will propagate temperature changes less than a dry one. It may propagate heat from below more rapidly, but heat from above less rapidly.

To get the greatest values of the temperature T_x we shall have in eq. 2 $\cos\left(2\pi\frac{t}{p}-1-R\right)=1$, the greatest value a cosine can have, and then $1+R=2\pi\frac{t}{p}+2n\pi$, the *first* maximum (when n = o) being at the date $\frac{1p}{2\pi}$ at the surface, and at the date $\frac{(1+R)\,p}{2\pi}$ for the depth x. The difference $\frac{Rp}{2\pi}$ is the time it takes the heat to penetrate the distance x, which is from equation (3) —

$$(4)\quad \frac{x}{2}\sqrt{\frac{p}{K\pi}}=\frac{Rp}{2\pi}$$

Variations of long period move in more slowly than those of short period, but they are appreciable at a greater depth. For when we come to consider the amplitude or range of variation in temperature, it is the original amplitude multiplied by e^{-R}, i. e. $(0.36788)^R$, which is always a fraction decreasing from 1 as R increases from 0, and thus decreases with the depth also, but more slowly the better the diffusivity and the longer the period of variation.

The diffusivity (K) in materials such as those of which the earth's crust is composed, usually varies from .001 to .01,[1] if the unit of time is a second, and of distance a centimeter.

In the progress of fluctuations of temperature from the surface they soon tend to smooth out minor irregularities as they go down, and if not strictly harmonic, become more nearly so. Now for any one particular set of heat waves, say those due to day and night, as they go down—

(1) The times of maxima are retarded by an amount which is to the period (p) of the heat wave (in the instance cited 1 day) as R is to 2π, where—

$$2\pi=\frac{66}{7}\ \text{ and } R=x\sqrt{\frac{\pi}{Kp}}$$

(2) The amplitude of vibration is to the original amplitude as $1 : e^{-R}$; i. e., is $(.36788)^R$ of the original. We can for any one fluctuation take p = 1 if we choose a proper value of K.

---

[1] Annual Report 1901, page 247. B. O. Peirce. Rocks like traps have a diffusivity of about .006 to .007. That of the Calumet conglomerates is as high as .01, which is taken as a rough average of underground strata by Lord Kelvin.

TABLE OF DIFFUSIVITY.

K for different units of depth (x) and time (p).

	Second.	Hour.	Day.	Week (7 days).	Month (1-12 year).	Year.
Centimeter	1 (a)	3600	86400	610800	262800	31536000
Meter	.000001	.0036	.0864	.6108	.2628	31536
Inch	.154	556	13700	93600	40600	4870000
Foot	.001073	3.87	92.8	684	2830 (b)	33800 (c)

(a) Units of B. O. Peirce, Callendar, and McLeod.
(b) Units of Angstrom.
(c) Units of Kelvin.

Suppose, for instance, we ask to what distance from the surface does the variation amount to one per cent of that at the surface? Then

Amplitude $= c\,(0.36738)^R = .01c$

Canceling and taking logarithms.

$R\,(-.4343) = -2$ Therefore $R = 4.6 = x\sqrt{\pi/Kp}$

Suppose the variation is daily, if we let p = 1, and the diffusivity is (.01, in centimeters and seconds i. e.) .928 in foot day units then x is about 2.3 feet. The soil diffusivity will be usually from 2/3 to 1/5 of this, and since the daily variation of the surface soil temperature is not over 30° to 40°, the daily variation is usually less than a degree Fahrenheit, even at less than 2/3 foot below the surface.

It is plain, however, that when we come to consider thawing frozen ground, or vice versa, an enormous amount of heat will be absorbed in thawing the same out. The diffusivity may be easily ten times less. On the other hand, a warm rain may carry the temperature down very rapidly.

For instance, I made the following temperature observations on the peat bog of Mud Lake, Sec. 1, T. 1 S., R. 5 E., in Washtenaw County, July 16, 1903:—

	Fahrenheit.
Temperature of air in shade	86°
Temperature of air in sun, surface of bog	117°
Temperature at ½ inch depth	94°
Temperature at 2 inches	73°
Temperature at 4 inches	63° or 62°
Temperature at 12 inches	57° to 56.4°
Temperature at 18 inches	57°

At other places in that same bog the some day, we found at different times: In sedge marsh at 3 inches depth, 64°; at 4 inches depth, 50 feet within shade of Tamarack belt, 64° F.; in Cassandra belt, 3 inches below water level, 56°, and at 8 inches, 56½°; 2 feet deep in peat, 56.3°; away out on the floating bog, at 4 inches depth, 68°; at 18 inches, 66°. Now the maximum temperatures that day for adjacent stations of the weather service were 80° to 82°, and the minimum the previous night were from 46° to 55°.

July (1903)	14	15	16	17	Ave, July	Ave. Aug.
Ann Arbor	72 —52	74—45	79 —51	81—57	82.5—60.2	76 0—56.2
Ypsilanti	71 —50	72—44	76 —55	81—58	80.5—58.2	85.0—54.9
Average	71½—51	73—44½	77½—53	81—57½	81.5—59.2	75 5—55 5
Average for day	61	59½	65	69	70 4	65½

The range of soil temperature for the day could hardly have been over (117–65) 52 x 2 = 104°. The amplitude at 1 foot and 1.5 foot would then be at a diffusivity $K = .928$ for units of a foot and day 15.9° F. and 6.35° F. respectively and the maximum or the minimum would be reached 29 /100 of a day and 44 /100 of a day later respectively, than on the surface, and a greater range means a greater diffusivity. To get a temperature of almost 10° below the daily mean at a foot, as we have, we should have to assume a lower minimum soil temperature than is probable, and greater diffusivity than is really likely. Finally, the possibility of the temperatures at a foot and a foot and a half diverging from the daily mean 10° on account of the cold the night before, utterly breaks down, in view of the relative constancy of the temperatures both in depth and at various times of day.

We must then take the other alternative that the temperatures are practically unaffected at a foot's depth by the daily variation, the amplitude of which at that depth is reduced to not over 2°, and then the diffusivity (c.g.s.) will be not over .004 to .005. From considering the temperature at 1 foot as fixed by the annual variation in temperature we find that the greatest summer temperature at a foot depth would be nearly thirty-seven days later than at the surface[1]. This leads to an extremely low diffusivity, like that of pumice or dry sand, and shows what a good nonconductor peat may be taken to be. The result may, however, be affected by evaporation and consequent depression of temperature, the peat acting like the wet flannel wrapper of a Hindoo water bottle.

Suppose we have observations at intervals of one-fourth of the period of variation in temperature apart, i. e., in the case of the daily variation six hour intervals, we have, calling them

---

[1] R (—.4343)=log. 2-/104 or log. 2/24, according to the amplitude of the surface variation taken = —1.716 + or — 1.079 +.

$\therefore R = 3.96$ or $2.48 + \therefore$ if $x = 1$ and $p = 1$. From eq. (3).

$K = \frac{\pi}{R^2 +} = 0.2$, or 0.5 in day foot units = .002 or .0056 c.g.s.

Supposing that the temperature at 1 to 1½ foot depth 56.6° is fixed by the annual variation (it is probably 2° or 3° too low on account of previous cold days). which is, or should be. just about a maximum (70°.6 daily mean) at the surface on July 17; reckoning the time from that same date last summer ($l = 0$), taking this time p = unity = 1 year (the diffusivity will accordingly be referred to years and feet), we have—

$$T_x = (56°.6 - 46°.6) = 10° + = (c \doteq 24° = 70.6 - 46.6)\, e^{-R} (\cos 2\pi \frac{t}{p} - l - R)$$

$$= 24 \cos (2\pi - R)\, e^{-R} = 24 \cos R/e^{R}$$

$= \log 24 + \log \cos R - R$ ($\log e = .4343$) $= -.38$. This equation is nearly satisfied if $R = (\frac{37}{360})\, 2\pi = .646$ in foot year units. $\frac{37}{360} \times \frac{365}{} = 37$ days, which is the difference between the time of greatest heat at the depth of 101½ feet at which 56°.6 is taken and at the surface. The maximum mean daily temperature at a foot depth is over a month later than at the surface. And if $R = .646 = x\sqrt{\frac{\pi}{Kp}}$

$\therefore K = \frac{\pi}{R^2} = 7.6$ in year foot units = .0002 in c. g. s.

$T_1 + M, T_2 + M, T_3 + M, T_4 + M$, M being the average temperature.

$$T_1 = c\, e^{-R} \cos (2\pi \frac{t}{p} - l - R)$$

$$T_2 = c\, e^{-R} \left[\cos (2\pi \left(\frac{t}{p} + 1/4\, p\right) - l - R) = \sin (2\pi \frac{t}{p} - l - R)\right]$$

$$T_3 = c\, e^{-R} \left[\cos (2\pi \left(\frac{t}{p} + 2/4\, p\right) - l - R) = -\cos (2 \frac{t}{p} - l - R)\right]$$

$$T_4 = c\, e^{-R} \left[\cos (2\pi \left(\frac{t}{p} + 3/4\, p\right) - l - R) = \sin 2\pi \frac{t}{p} - l - R\right]$$

The sum of two observations a half period apart $T_1 + M$ and $T_3 + M$ or $T_2 + M$ and $T_4 + M$ will give us twice the mean average temperature. Then we can get T_1, T_2, etc., readily. Squaring the sums, we get—

$T^2_1 + T^2_2 = c^2 e^{-2R} (\cos^2 + \sin^2 = 1)$ = the square $(c\, e^{-R})^2$ of the amplitude at the depth x, from which we can readily find R, if we know c. Or, if we have a series of different observations at different depths, we can eliminate c between them. Also dividing—

$\frac{T_2}{T_1} = \tan (2\pi \frac{t}{p} - l - R)$, from which we can also compute R.

There are thus two ways of computing the diffusivity of heat. In the former case we estimate from the decrease in amplitude or amount of maximum variation and that is all we need to know. But if we have a series of timed observations, we can also eliminate the diffusivity from the time at which this maximum temperature occurs for various depths.

To illustrate these principles and get some idea of the diffusivity near the surface we may proceed as follows:

Between 1888 and 1892, measurements of the soil temperatures at different depths were made at Grayling and at Lansing.[1]

These were read three times a day, at 7 a m , 1 p. m., and 7 p. m., i. e , six hour intervals, but not in the winter. Therefore they do not help us as regards the blanketing effect of the snow in winter, and a study of the annual variation must be very imperfect. We can, however, get some comparative results as to daily variation and as to diffusivity, and apply the formula above; and it is well to notice also that the composition and porosity of the Grayling soil has been tested, as well as its diffusivity.[2]

The addition of morning and evening observations should give us twice the daily mean, free from the daily variation.

Also $$\frac{T_{7\text{ A. M.}}}{T_{1\text{ P. M.}}} = \frac{T_{7\text{ P. M.}}}{T_{1\text{ P. M.}}} = \tan \overline{l + R}.$$

From this we derive $l + r$, which is proportional to the time after 1 P. M. that the daily maximum arrives. Finding this for various depths, and remembering that $R = x\sqrt{\frac{\pi}{K}}$ and that we know the depths, and the

---

[1] Report State Board of Agriculture, 1888, pages 154-164; 1889, pages 85-101; 1890, pages 143-152; and 1891, pages 92-97.

[2] Bulletin State Board of Agriculture, Bull. 99, p, 11, also this report above. pp. 15, 16.

period (p) is taken as 1, we shall have expressions with only l and K, so that if we have these observations for two different depths we can find both l and K.

Lansing.

Taking for instance, the observation for September, 1891, at the Agricultural College, we have:—

Depth.	7 A. M.	1 P. M.	7 P. M.	7 A. M. + 7 P. M.	Mean.
3 in.	61.3	72.3	69.2	130.5	65.25
6 in.	63.6	67.2	68.1	131.7	65.85
9 in.	64.8	63.8	66.5	131.3	65.65
12 in.	65.1	65.1	65.7	130.8	65.4
24 in.	64.3	64.4	64.3	128.6	64.3
Average					65.29

If there were only daily harmonic variations to account for, the column of means should be identical. The variation in the means is due to sine terms of a period longer or shorter than a day. Throughout we assume that we can isolate pure harmonic vibrations corresponding to the single curve. This is not strictly true, but the farther from the surface the more the curves tend to become such. With the exception of the 24-inch thermometer, which is distinctly lagging in temperature, owing to the annual variation, the variation is less than $\frac{1}{4}$°.

We have, accordingly, as variations from the means for the various depths:—

Depth inches	T_7	T_1	T_7	$\frac{T\ 7\text{ P. M.}}{T\ 1\text{ P. M.}}$	1 + R in degrees 15°=1 hr.	1+R	$\frac{1+R}{x}$	K in inch and day units	K in cgs. units	$c\,e^{-R}$
3	—3.95	7.05	3.95	.561	29° 20′	.51	.17	105	.0083	8.1
6	—2.25	1.35	2.25	1.65	58° 40′	1.02	.17	105	.0083	2.6
9	— .85	—1.85	.85	— .458	155° 20′	2.72	.30	35	.00275	2.0
12	— .3	— .3	.3	—1.	135°	2.55	.20	78.5	.0062	1.42
					360° or	6.28	.26			
24	— .0	+1.1	+ .0	—0.	180°	+3.14	+.23	46	.0036	1.1

From the two shallowest depths we infer that the hottest part of the day at the surface was almost exactly 1 P. M. and l = 0. The diffusivity[1] will vary between .003 and .008. Apparently it takes about four hours for the daily heat wave to penetrate half a foot, but nearly twenty-four hours for it to penetrate two feet.

Grayling.

Let us now apply this same analysis to July, 1888, at Grayling:—

Depth.	7 A. M.	1 P. M.	7 P. M.	7 A. M. + 7 P. M.	Mean.
3 in.	63.92	75.49	75.52	139.44	69.72
6 in.	66.52	71.10	74.06	140.58	70.29
9 in.	68.28	68.97	71.79	140.07	70.02
12 in.	68.26	68.02	69.74	138.00	69.00
24 in.	66.80	66.77	66.60	133.40	66.70

---

[1] The lower diffusivity may be due to the greater dampness of the lower layers.

The variations in the means shows the importance in influence of periods with others terms than 1 day. In the first three depths the difference is not over $\frac{1}{4}$°. Obtaining T we have:—

x	T_x (7 A. M.	T_x (1 P. M.	T_x (7 P. M.	$T_7 \div T_1$	$(1+R)\frac{360}{2\pi}$	1 + R	Time that maximum is retarded.
3 in.	—5.80	+5.78	+5.80	+1.003	45°	.79	3h.2
6 in.	—3.77	+0.81	+3.77	+4.65	77°	1.35	5h.4
9 in.	—1.74	—1.05	+1.77	—1.69	90 + 31°	2.12	8h.1
12 in.	—0.74	—0.98	+0.74	—0.75	180° —37°	2.49	9h.55
24 in.	+0.10	+0.07	—0.10	—1.43	360 —55°	5.31	20h.04

We obtain, using the values at three inches and six inches to determine l, l =2 (.79) — 1.35 = .23, indicating that the warmest part of the day at the surface was nearly an hour after 1 P. M. The values for R will be: .57, 1.12, 1.89, 2.26, and 5.08 respectively; and of R ⁄ x = $\sqrt{\frac{\pi}{Kp}}$ .19, .19, .21, .19, and .21 respectively, a fairly uniform rate averaging .020 in inch-day units, which will lead to a diffusivity of 78.5 in inch-day units, or .006 in c.g.s.

For amplitudes we find, for the $\sqrt[2]{T_{x7}^2 + T_{1x}^2}$ successively, 8.17°, 3.36°, 2.30°, 1.23°, and 0.71° respectively. If we assume the maximum divergence from the average at the surface is 15°, we shall obtain for values of R: .61, 1.33, 1.88, 2.50, and 3.06, and for R ⁄ x .20, 0.22, 0.21, and 0.14, which agree quite fairly (except the last, where the amplitude at two feet is so slight as to be greatly affected by the error of observation, and may be affected by the yearly variations.)

Thus we may conclude that the mean diffusivity at the point where the observations were made at Grayling was on that July close to .006, the average surface temperature between 69° and 70°, while the daily range of the soil temperature was 14.6 above and below; i. e., from 56° F. to 84° F.

We have also tried one other month, April, 1889. The results were as follows: The mean range of temperature at three inches was greater than that of the air, so the surface range of soil temperature may have been much greater. The whole variation was not nearly so great as in July. By taking the differences from the mean value of the temperature at any two times six hours apart, and squaring them, we obtain the square of the greatest difference at that depth as before, and accordingly find at 3, 6, 9, 12, and 24 inches respectively, maxima of 8.1, 2.62, 2.03, 1.34, and 1.21 respectively. From these again the diffusivity may be computed. If we assume R = 51 for three inches, we find a maximum at the surface of 13.3 above the mean, the mean daily variation being about 27°, which harmonizes well with atmospheric data, and we find for successive values of the retarding factor, R, values of 1.62, 1.9, 2.3, 2.5, respectively. Then R ⁄ x will be .17, .27, .21, .20, .11, respectively, or an average of .18, or for the diffusivity in units per day, .98, or in c.g.s. units, .007. The average diffusivity computed from the retardations of the maximum was .006.

We see, then, that records of the temperature at different depths give us check methods of determining the diffusivity and that only three observations a day, at intervals of six hours, will help us out pretty well. And after we have the diffusivity determined, we can readily figure the amplitude of a heat or cold wave at a given depth, and how long it will take to get down there.

It is scientifically desirable that the observations taken with so much care by Prof. R. C. Kedzie be worked up completely to obtain the different diffusivities of the soil. Important light on soil physics would probably be obtained. The direct geological bearing is not great enough to warrant my undertaking the work. The results above give us an idea of the diffusivity.

The above treatment must not be taken as a thorough and accurate treatment. Byerly, above cited, gives the necessary mathematical foundation, and I also have the following papers which work out especial cases in detail.

Louis Saalschütz: Astronomischen Nachrichten, Altona, 1861, "Ueber die Wärmeveränderungen in den höheren Erdschichten unter dem Einplüsse des nicht periodischen Temperatur wechsels an der Oberfläche."

Angstrom: Nova Acta regiae Societatis Upsaliensis, pp. 147-224, series III, Vol. I, 1855—"Température de la terre." The same pp. 50-72, Vol. III, 1861, "Conductibilité des corps."

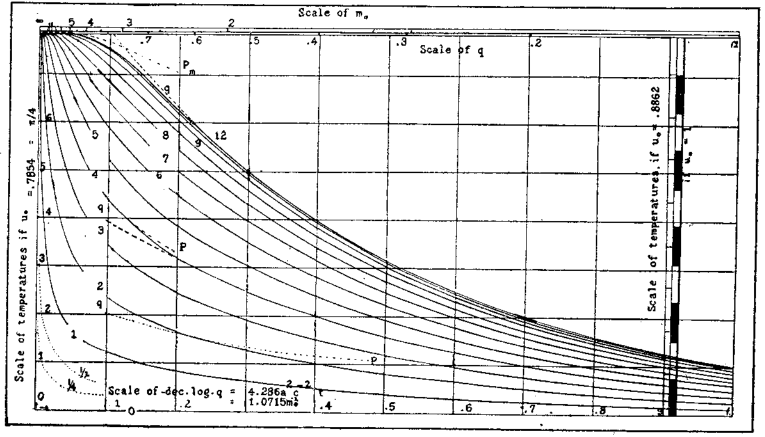
B. O. Pierce: Proceedings of the American Academy of Sciences, August, 1898, October, 1899, June, 1900, May, 1903, a series of papers.

Callendar aud McLeod: Transactions of the Royal Academy of Canada, 1895-1896, a series of papers on soil temperature observations.

GRAIN OF ROCK.

In connection with the Isle Royale report[1] already referred to I developed a theory that the coarseness of grain in igneous rocks was not only somewhat dependent on the rapidity of cooling, but sometimes depended on it in so definite a way as to be capable of mathematical expression. I found that if a mineral was formed in a time $T_1 — T_2$ during a drop in temperature $U_1 —U_2$, then the area of a cross section of an average grain was proportional to the time and inversely proportional to the drop, or dependent on $\frac{T_1 — T_2}{U_1 — U_2}$.

BULL. GEOL. SOC. AM. VOL. 14, 1902, PL. 54



CURVES OF COOLING—MARGIN TEMPERATURE CONSTANT

FIG. 1. This shows the cooling of an igneous sheet, the margins of which are kept at a constant temperature, taken as 0° and represented by the bottom line of the diagram. The temperatures are represented by ordinates, and three different scales of temperature are given, corresponding to different values of the initial temperature. The main scale at the left is the same as used in the Isle Royale report. To the right is the scale if, as is assumed by Queneau, u_0 =.8862, and also if the initial temperature of the igneous sheet (that is, its excess of temperature over the surrounding rock) is taken at 1. The abscissas to the right represent the lapse of time, the main scale at the bottom being proportional to dec. log. q and t, and the scales above being proportional to m_0 and q, which are defined in the text. Curves for twelve points from the center to the margin are drawn in full lines, and in dotted lines two curves still closer to the margin, one twenty-fourth and one forty-eighth of the distance to the center respectively. Also for curves 12, 4, and 2 we indicate by dashes the points where the curves of the approximate solutions which we have used in practice leave the curve which we have taken them to represent, to show how small the error is which we make in using them.

---

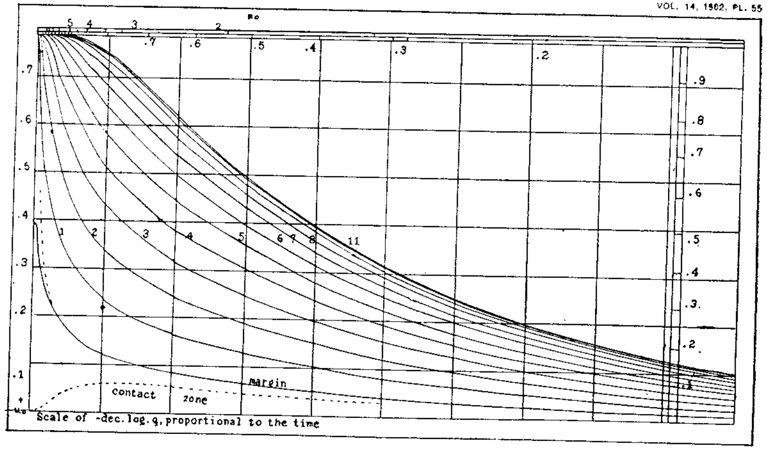
[1] Vol. VI, Part I. This subject has been worked on independently by Tammann and Doelter. I have not seen all the original papers, only the summaries given in Neues Jahrbuch, 1903, 1, and Central Blatt, 1903, p. 608. The *power of crystallization* is defined as the number of crystal granules forming in a unit volume in a unit of time. The velocity of crystallization is the linear velocity of growth in millimeters per minute. It seems to depend upon the number of degrees below the temperature of saturation, respectively fusion, at which the crystallization takes place and apparently attains a maximum some 15° below and more, according to Doelter's ideas, 20° to 25° below the fusion point for augite and plagioclase, nearer the temperature of solidification than of fusion.

Pressure does not make much difference except as it may retain H_2O, etc., but the crystallization is very sensitive to addition even of chemically neutral substances.

Glass formation may be due either to slow velocity of crystallization or weak power of crystallization.

During the first few degrees below saturation temperature, microlites and elongate forms tend to occur, later come the more equal sized grains. In augite the length of the grains formed in 100 minutes is not ten times that formed in ten minutes, but Doelter's cylinder, which cooled in 100 minutes, had a slow cooling margin and a more rapidly cooling center, a case we shall refer to later.

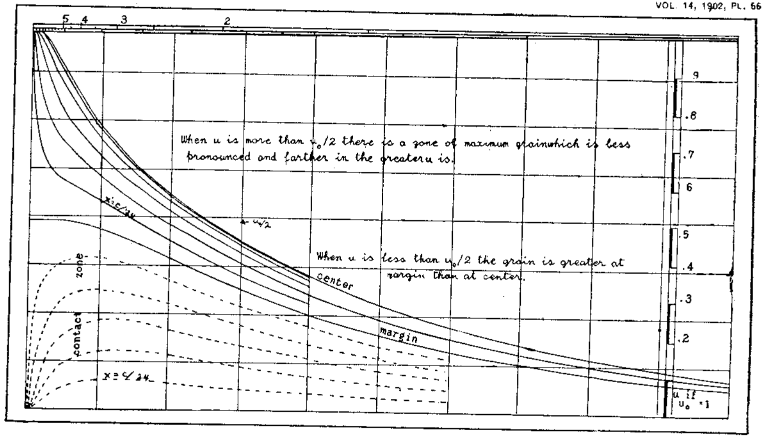
BULL. GEOL. SOC. AM. VOL. 14, 1902, PL. 55



CURVES OF COOLING–NARROW EFFECTIVE CONTACT ZONE.

FIG. 2. Illustrating cooling, as in Fig. 1, where there is a narrow effective contact zone, one-twelfth the width of the dike. The dotted line shows the rise and fall of temperature in the middle of the contact zone. There is also shown by dashes the point where curve 2 of Fig. 1 diverges from the curve of marginal temperature.

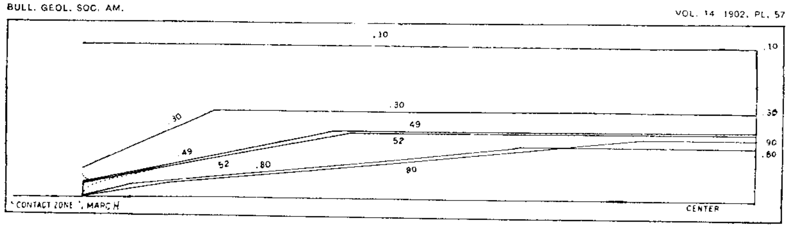
BULL. GEOL. SOC. AM. VOL. 14, 1902, PL. 56



CURVES OF COOLING–EFFECTIVE CONTACT ZONE BROAD.

FIG. 3. This figure is based on Fig 1, and is for similar conditions. The full lines show tangents to the curve of grain of an injected igneous sheet (supposing the same to be proportional to the square root of the slowness of cooling) as derived from the approximate equations referred to in the text. The position of the point relative to the margin is represented by horizontal coördinates, the average linear dimensions of grain corresponding by vertical coördinates, while the decimals attached to each curve show the ratio corresponding to it of the consolidation–to the initial temperature. The lines in dashes are the actual curves sketched in, where they diverge markedly from the tangents.

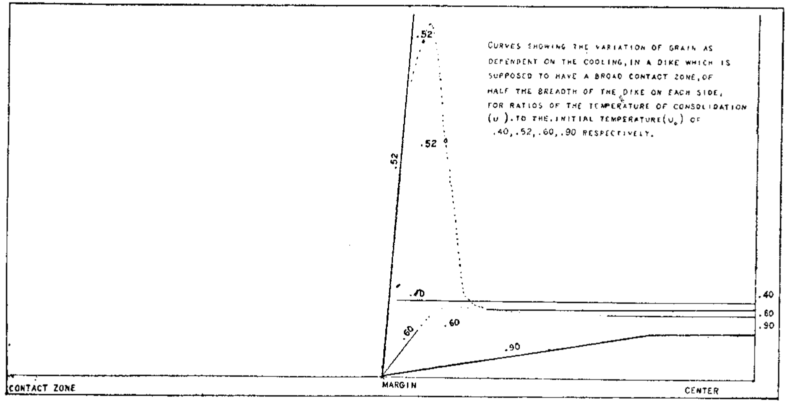
BULL. GEOL. SOC. AM. VOL. 14 1902, PL. 57



TANGENTS TO CURVES OF GRAIN–NARROW CONTACT ZONE

FIG. 4. Illustrating the variation of grain from margin, (to the left) to center (to the right) in flows, intrusive dikes, or sills which have a narrow contact zone, by the tangents to the curve of grain. The decimal figures adjacent to the curve indicate the ratio of the temperature of consolidation to that of initial injection or effusion, the temperature of the country rock being taken at 0°.

BULL. GEOL. SOC. AM. VOL. 14, 1902, PL. 58



TANGENTS TO CURVES OF GRAIN–BROAD CONTACT ZONE

FIG. 5. Similar to Fig. 4, but illustrating the variation in grain in case the contact zone is broad. It stands in the same relation to Fig. 3 that Fig. 4 does to Fig. 2.

Many of the remarks and the theoretical results given below will be only slightly modified in case the above formula is not strictly true if only it be granted that the grain is some continuous expansible function of the slowness of cooling increasing with the same and zero when the latter is zero. In the Isle Royale report I worked out the theory of grain especially on the supposition that the top and bottom of the sheet were kept at a constant temperature, by water or air currents or otherwise, and found that this supposition illustrated by Fig. 1 fitted the facts fairly well for the augite of the Keweenawan traps and is an important help in studying the sections. Certain principles there enumerated it may be well to repeat here. It will be more convenient, however, to compare the grains by their linear dimensions than by the areas of their cross sections, and, if we call the average linear dimension[1] the grain, then:

(1). The grain will vary as the square root of the slowness of cooling.

---

[1] Determined by taking the largest grain in a number of different areas, usually five.

(2). Points at the same distance from a surface of fixed temperature will, if other conditions are similar and the grain was formed before the center cooled appreciably, have the same grain regardless of the size of the sheet. This was found experimentally to be true in the Keweenawan flows.

(3). The grain in such a case would be proportional to the distance from the bottom or top of the flow. This was found to be nearly true for the augite of a common type of flow mainly composed of labradorite and augite, and probably nearly a eutectic mixture, of which the Greenstone is a type.

This is illustrated in a plate of drill cores prepared for my next annual report while this is passing through the press.

This ratio of the increase of grain of the augite to the distance from the margin was found to be .0002, two millimeters for ten meters or about one inch for 400 feet. More accurate theory, as will be shown in this paper, indicates a more rapid increase close to the margin of the flow so that the first millimeter is attained within ten feet.

To illustrate the method of computation we will take the observations as given on page 245 of the Isle Royale report on thin sections 15252 to 15258 and reduce the distance to millimeters, allowing for fractions not given there. There is, however, an uncertainty of probably one-half of a foot (152 millimeters) in most of the distances. We obtain the table below:

Section.	Distance in mm.	Difference.	Grain in mm.	Difference.	Rate of difference = S=A or C.	Ax.	B.
15258.	0		0				
		296 ± 152		1.2-5	.00422		
5257	296 ± 152		1.25			.06	1.19
		7,054 ± 152		1.65	.00022		
15256	7,350 ± 152		2.90			1.48	1.32
		8,550 ± 152		1.1	.00013		
15255	15,900 ± 152		4.			3.42	.58
		6,200 ± 152		1.	.00016		
15254	22,100 ± 152		5.			4.74	.26
		6,800 ± 152		2.	.00029		
15253	28,900 ± 152		7.			6.2	.80
		8,600 ± 152		2.—	.00023		
15252	36,500 ± 152		9–6			7.84	1.16
Average from 15257 to 15252					.00021		.885

In this table the observations in thin-section 15258 are uncertain, for it is decomposed, and those on 15252 and 15253 are not very satisfactory, for the grain is so coarse that it would require a number of sections to give us a fair idea. It is obvious, however, that from 15257 clear in to the center, represented by 15252, the gradient is fairly steady and somewhere about .0002, which is what we found for the group on page 129 of the Isle Royale report (.06 / 304).

Taking the gradient from 15257 to the center, we find it to be (9-1.25) / (36,500—296) or .000213. Assuming this gradient from the margin, we find the column headed "Ax" and the difference between these and observed values will give us the average value of B. These differences are given in the column headed "B". Thus we can, as we shall later see, find the contact zone y′, which equals B / A, or 2,400 millimeters.[1]

But we did not find this to be true for all flows, for there was another law in evidence:

(4). Other things being equal the greater the abundance of its constituent molecules, the coarser the grain.

PRACTICAL APPLICATIONS.

Some practical applications of the above theories may be suggested.

1. A thick sheet of trap very fine grained as to ground mass is pretty sure to be a felsite or "ashbed diabase" = (porphyritic melaphyre or andose) type, more alkaline than the normal ophites.
2. From the coarseness of the lustre-mottling of even a small exposure of ophite a considerable bed may be inferred. For instance mottles an inch across would mean a bed some 800 feet thick. This may help to fill gaps in cross sections, and save trenching or exploring for amygdaloids uselessly near.
3. The direction in which the mottles or grain gets finer is probably (faults, etc., excepted) the direction of the nearest amygdaloid. Dr. Hubbard has used this successfully in cross cutting from shafts, and other exploratory work which he directed.
4. As Dr. Hubbard has suggested, if a flow appears by a chemical or petrographic analysis to be a normal ophite, but when cut by a drill hole or otherwise the grain does not increase at the normal rate of 400 feet per inch diameter of grain but at some other rate it might be possible to infer the dip.[2] But on account of the difficulty of determining the average rate of increase of grain accurately, and the liability of the drill hole to veer somewhat, and a possibility that an immediately succeeding flow may have modified the rate of increase this method can only be used with extreme caution, as a suggestion of probable dip. I have, however, used it.
5. In sinking or drifting sudden variations in grain may serve to call attention to faults and slips, and a normal diminution in grain forewarn of an amygdaloid or conglomerate.

In surface flows and for minerals formed immediately after the intru-

---

[1] I believe this is really too high, and that part of the coarseness of grain at the margin, which makes B and hence y so large, is formed in flowing. I suspect y′ should be about two-thirds of the value used.

[2] The new rate would be (400 ÷ cos dip) feet per mile, in a vertical shaft or drill hole.

sion of a dike, before the temperature of the wall, which may be considered to start in at a temperature half way between that of the country rock and the intruded lava, has perceptibly varied, the facts agree nearly enough with the assumption of a constant marginal temperature, but in general we really ought to allow for a greater or less width of contact zone. In my Isle Royale report I showed how the contact zone could be allowed for, and yet I assumed that for the Keweenawan traps and corresponding intrusive diabases, the contact zone could be counted as insignificant. In this so far as the intrusives are concerned I was mistaken, and I also misled somewhat Mr. Queneau.[1]

He guided by Woodward, nevertheless, furnished a rapid method of computing the early and marginal temperatures, with the help of which I have been able to put the theory of intrusive sheets with a contact zone into much more calculable shape The labor of getting numerical results, and the apparent agreement of numerical results obtained in the Isle Royale report with the possibilities of the case, had previously led me to be too easily satisfied.

The following paper is strictly technical and puts the matter in more satisfactory shape.

GENERAL FORMULÆ FOR ROCK GRAIN.

We may briefly summarize the results of the mathematical investigations as follows:

MARGINAL TEMPERATURE FIXED.

In an injected sheet of uniform temperature and diffusivity whose walls are kept at a fixed temperature the temperature may be expressed in two ways, the former most applicable at the early stages of cooling, the latter in the latter stages of cooling.

Curves showing the manner of cooling, at first fast at the margin and slow at the center, then later slow at the margin and faster at the center are given in Fig. 1.

$$(1)\quad \frac{u}{u_o} = P_m - (P_{m_o+m} - P_{m_o-m}) + (P_{2m_o+m} - P_{2m_o-m}) - - -$$

$$(2)\quad \frac{\pi}{4}\,\frac{u}{u_o} = q \sin \pi\, x/c + \frac{q^9}{3} \sin 3\,\pi\, x/c + \frac{q^{25}}{5} \sin 5\,\pi\, x/c - - -$$

where:

u is the temperature after a certain time (t);
u_o is the initial temperature;
t is the time from the beginning of cooling;
x is the distance from the margin;
c is the thickness of the flow;
a^2 is the diffusivity, K it was called in the article on soil temperatures.

---

[1] See "size of grain of igneous rocks in relation to the distance from the cooling wall," by Augustin L. Queneau, School of Mines Quarterly, XXIII, 181-195, January, 1902. Contributions from the Geological Department of Columbia University; Amer. Jour. Science. November, 1902, Volume XIV, page 393.

$m = x/2a\sqrt{t}$
$m_o = c/2a\sqrt{t} = c\, m/x$
$\text{nat log } q = -\pi^2 a^2 c^{-2} t = (\pi/2m_o)^2$
$P_m = \frac{2}{\sqrt{\pi}} \int_o^m - m^2 dm$
$\pi = 22/7 = 3.1416$

Formula (1) is the Woodward-Queneau formula and converges most rapidly at early times and near the margin when m_o is greatest.

This is apparent from the table below p. 222, in which P_m varies but little from 1 as soon as m is much over 2.

Formula (2) I used in the Isle Royale report[1] and converges much more rapidly for large values of t the time or q. When formula (1) ceases to give a result approximate to the third decimal place, a couple of terms of formula (2) will usually give it. The linear grain if we call (T_1-T_2) dt and (U_1-U_2) du will be expressed by the formula (where k depends on the power of crystallization of the mineral under the given conditions and will be *assumed* independent of the temperature and time, except as provided):

$$(3)\quad g = k\sqrt{\frac{dt}{du}}$$

For ordinary igneous magmas where probably according to Doelter du is often not over 40° and u_o and u are to be reckoned in hundreds of degrees dt and du may be treated by the infinitesimal calculus or that of finite differences.

In any table of the values of u in the terms of t, such as are given in Volume VI and here, we should find the ratio dt: du as the quotient of the corresponding consecutive differences of argument and functions respectively, for the same value of x^2.

But a glance at equation (2) shows, what is not so clear in (1), that it is only the ratios u/u_o ,x/c and a/c which are involved in determining t, moreover our tables are not constructed directly with t as an argument, though u/u_o is the function. It will therefore be convenient to express (3) otherwise and introduce instead of t $m_o = \frac{c}{2a\sqrt{t}} = \frac{1}{2(a/c)\sqrt{t}}$ we have then:

$$(4)\quad g = \frac{k}{\sqrt{u_o}} \cdot \frac{c}{a} \sqrt{\frac{-dm_o}{2m_o^3}} \sqrt{\frac{1}{-du/u_o}}$$

Now in studying the variation of grain from center to margin in any one sheet $\frac{ck}{a\sqrt{u_o}}$ may be supposed not to vary. I have denoted it by K.

The expression $\sqrt{-du/u_o}$ if u/u_o and $\frac{x}{c}$ be given may be found from the tables on pages 221 and 222 by subtracting successive values of u/u_o and so may be the expression $\sqrt{\frac{-dm_o}{2m_o^3}}$ or an average value of the same which is the same for any column of the table I.

---

[1] See Byerly, Fourier's series, Article 60, ex. 1.

[2] I. e. we deal with the partial derivative of t relative to u, x being supposed constant.

We may write:

$$(5)\quad \frac{c}{a}\sqrt{\frac{-dm_o}{2m_o^3}} = f\left(\frac{x}{a}, \frac{u}{u_o}, \frac{a}{c}\right) = \frac{c}{a}M_o = \frac{x}{a}\sqrt{\frac{-dm}{2m^3}} = \frac{x}{a}M = \sqrt{dt}$$

so we have $g = KM_o / \sqrt{du/u_o}$.

It is also at times worth while to replace dt by

$$d\left(m = \frac{x}{2a\sqrt{t}} = \frac{x/c}{2a / c\sqrt{t}}\right)$$

dt (a partial derivative,—x constant) $= \frac{x}{a}\sqrt{\frac{-dm}{2m^3}}$ and

$$(7)\quad g = \frac{k}{\sqrt{u_o}}\,\frac{x}{a}\sqrt{\frac{-dm}{2m^3}}\sqrt{\frac{1}{du/u_o}} = K.\frac{x}{c}\sqrt{\frac{-dm}{2m^3}/d\,u/u_o}.$$

From equation (1) and table (2) it appears that when $m_o > 2.5$ $u/u_o = P_m$ so that $m = P^{-1}$ u/u_o and although theoretically and in general a function of x,m is so no more. Moreover if u/u_o is small m is nearly proportional to it and $m = \sqrt{\pi/4}$ $u/u_o = \frac{1}{1.125}\frac{u}{u_o} = .8863$ u/u_o.
If therefore we assume

$$(8)\quad h = \sqrt{\frac{-dm : (u/u_0)^3}{-du/u_0 : m^3}}$$

it will be for a considerable range of temperature close to .8863, for a still wider range be $\frac{u/u_0}{m}$, and for all early times be practically independent of t so that it will be convenient to introduce it in (7) and find

$$(9)\quad g = K.\frac{x}{c}\cdot\frac{1}{h\sqrt{2\,(u/u_0)^3}}$$

$$(10)\quad g = Cx \text{ where } C = \frac{K}{ch\sqrt{2\,(u/u_0)^3}}$$

and C in early times, i. e. for the grain close to the margin, is a function of u/u_0 and of K/c, i. e., k, a, and u_0, not involving c nor x.

As soon as the cooling has proceeded so far that there is a belt of uniform grain at the center (the cooling is expressed by the first term of Eq. 2) the grain at the center E may be written [1].

$$(11)\quad g = E = K/\pi\sqrt{u/u_o}$$

Suppose we have the rate of increase of grain at the margin determined by observation and also the grain at the center. Then we may determine when at the rate of increase at the margin continued the grain would be of a coarseness equal to that at the center.[2] We may call this (x′) the width of the belt of increasing grain. It is:

$$(12)\quad \frac{x'}{c} = \frac{E}{C} = \frac{h\sqrt{2}}{\pi}\,\frac{u}{u_O} = .4\ u/u_O$$

---

[1] From (3) by differentiation of the first term of (2).

[2] Point of intersection of the tangents to the curve of grain at center and at margin.

MARGINAL TEMPERATURE NOT FIXED.

When we come to consider the case that the sheet is supposed to have a heated contact zone, we find it again important to consider the conditions close to the margin and those at the center, for in each case we find relatively simple expressions for the grain, though the intermediate grain does not depend on them so simply.

First, we can see that a formula which we found in the Isle Royale report will enable us readily to construct the curves of decreasing temperature, which are shown in figures 2 and 3, in the former when the contact zone supposed heated is narrow and in the latter when it is as wide as the distance to the center of the dike.

This formula is as follows:—If v represents the temperature at a time t of a point at a distance of z from the center of a sheet the thickness of which is 2w with a contact zone of breadth y on either side, so that the whole zone affected or c is equal to 2w+2y. then—

$$(13)\quad v = \tfrac{1}{2}\,(u_{w+z} + u_{w-z})$$

where u_{w+z} is the temperature which would exist at the same time t after the beginning of cooling and conditions otherwise the same (except that its margins are kept at a constant temperature) in a sheet whose thickness was equal to c at a distance of w+z from the margin and u_{w-z} is the temperature at a point at a distance of (w–z) from the margin in the same sheet. Or if (w–z) becomes negative—that is, the point lies in the contact zone—we must assume

$$(14)\quad u_{w-z} = -u_{z-w} \text{ (in the contact)}$$

Now if we let x′ be the distance of the point in the sheet with a contact zone from the margin of the sheet—that is $x' = \frac{c}{2} - z - w$

$$(15)\quad v = \tfrac{1}{2}\,(u_{x'+2y} + u_{x'})$$

So if we take y to be $\frac{1}{24}$ of c, then for various values of x′ from 0 to 11 the center we shall have as corresponding values of v respectively one-half the values of curve 2 of 1, values half way between curves 1 and 3, 2 and 4, 4 and 6, 5 and 7, 6 and 8, 7 and 9, 8 and 10, and at the center the values of curve 11 for the same time. These curves are drawn in Fig. 2 which illustrates, the cooling, therefore, in case the contact zone affected is only narrow.[1]

Similarly if y be 6, the margin will have half the values of curve 12 Fig. 3 and the center will have the values of curve 6 for the same times. Thus we can obtain the curves of these figures either graphically (very rapidly) or by taking means of appropriate rows of the table of solutions of the case where the margin is kept at a fixed temperature, a table which is given in the Isle Royale report for an initial temperature of .7854, and here for the initial temperature of 1. (Table I.)

Thus expressing the grain of the sheet with a contact zone y on each side as a certain function g ( ) of x′, of u the temperature of crystallization of u_o temperature of injection, of y the width of the contact zone

---

One might ask, why should narrow and broad contact zones be assumed? The answer would be, to allow for the varying diffusivity of the country rock and its capacity to absorb heat by the convection or evaporation of water, or by chemical reactions.

and of k, to include all other factors, such as diffusivity, crystallization power, etc., we have:

AT THE CENTER.

Here.

(16) $v = u_w$ whence we have.

(17) Center grain

$g\ (k,\ x = c/2,\ v,\ u_o,\ y,\ c) = g\ (k,\ x = w,\ u_w,\ u_o,\ y = o,\ c.)$

The expression for the grain at the center will be then,

$$(18)\quad g\ (k,\ x = c/2 \text{ or } x' = w,\ u,\ u_o,\ y,\ c) = \frac{k}{\sqrt{u_o}} \sqrt{dt} / \sqrt{-du/u_o}$$

$$= K\ \frac{a}{c} f\left(\frac{w}{c},\ \frac{v}{u_o},\ \frac{a}{c}\right) \sqrt{-dv/u_o}.$$

In this expression and in the table we must confine ourselves to the row for which most nearly $x = w/c$, and find the point for which most nearly u/u_o has the value v/u_o, and then interpolate to find $\frac{a}{c} f\left(\frac{w}{c}, \frac{v}{u_o}, \frac{a}{c}\right)$. Or if the cooling has gone on so far that there is a belt of constant grain at the middle ($m_o > 2.5$ practically) by Eq. 11.

$$(19)\quad g\ (k,\ x = \frac{c}{2} \text{ or } x' = w,\ u,\ u_o,\ y,\ c) = \frac{K}{\pi \sqrt{u/u_o}} = E$$

AT THE MARGIN.

(Consolidation temperature nearer that of country rock.)

Here.

(20) $v = \frac{1}{2}\, u_{2y}$ whence we find that the grain at margin if $u < u_o/2$ is, since $\sqrt{dv} = \frac{\sqrt{du}}{\sqrt{2}}$.

(21) $g\ (k,\ x = y \text{ or } x' = 0,\ v,\ u_o,\ y,\ c) = \sqrt{2}\ g'\ (k,\ x = 2y,\ u,\ u_o,\ y = o,\ c)$,

or in general,

$$g\ (k,\ x' = o,\ u,\ u_o,\ y,\ c) = \sqrt{2}\ \frac{K}{\sqrt{u_o}} \sqrt{dt} / \sqrt{du/u_o}$$

$$(22)\quad = K\sqrt{2} f\left(\frac{2y}{c},\ \frac{u}{u_o},\ \frac{a}{c}\right) / \sqrt{du/u_o},$$

or if u/u_o is so low that this value falls in the second period, i. e., $m_o < 2.5$.

(23) $g\ (k,\ x' = o,\ u,\ u_o,\ y,\ c) = K\ \sqrt{2} / \pi\sqrt{u/u_o} = K / \pi\sqrt{v/u_o}$.

Comparing equations (22) and (23) with (18) and (19) we infer that if the temperature of consolidation is low enough the grain may be the same from center to margain. Otherwise it will be coarser at the margin if $u < u_o/2$.

(Consolidation temperature nearer that of injection.)

In as much as $v = 1/2\ u_{2y}$ at the margin, so long as u_{2y} has not dropped from u_o, v will be equal to $u_o/2$. During this time the cooling will be as though the temperature at the margin were fixed at 0. It

will be the same as that of a sheet of constant marginal temperature (which is taken as 0) of which the initial temperature will be $u_o/2$ and u will be $v - \frac{u_o}{2}$. In as much as we have expressed the grain in terms of the ratio u/u_o we can now write

(24) $g\ (k,\ x = x' + y,\ v,\ u_o,\ y,\ c) = g\ (k,\ x',\ v - \frac{u_o}{2},\ \frac{u_o}{2},\ 0,\ 2w)$,

where $2w = c - 2y$ and going back to equations 3 to 10 we find from (4) and (5)

$$(25)\quad g = \frac{k}{\sqrt{\frac{u_o}{2}}} \cdot f\left(\frac{x'}{2w} \cdot \frac{2v}{u_o} - 1,\ \frac{a}{2w}\right) / \frac{1}{\sqrt{-d\left(\frac{2v}{u_o} - 1\right)}}$$

$$(26)\quad = \frac{K}{c} \cdot \frac{x' \qquad \sqrt{2}}{-h' \sqrt{2\left(\frac{2u}{v_o} - 1\right)^3}} = C'x'$$

Where h' stands in the same relation to $\frac{x'}{2w} \cdot \frac{2v}{u_o} - 1$ as h to $\frac{x}{c}$ and $\frac{u}{u_o}$, i. e., h' is for points close to the margin crystallized in early times independent of $\frac{x'}{2w}$, and this will be true for the linear rate of increase of grain (tangent to the curve of grain) at the margin, and while it depends on $\frac{2v}{u_o} - 1$ it will vary only between .7 and 9. if $\frac{2v}{u_o} - 1$ is less than .966, i. e., $\frac{v}{u_o}$ is $< .984$. That is unless the crystallization is practically coincident with the consolidation h' is roughly constant.

$$(27)\quad h' = \sqrt{\frac{-d\,\frac{x'}{2a\sqrt{t}}}{d\left(\frac{2v}{u_o} - 1\right)} \left(\frac{\frac{2v}{u_o} - 1}{\frac{x'}{2a\sqrt{t}}}\right)^3}$$

$$= \frac{a}{x'} \sqrt{\frac{dt}{d\,\frac{2v}{u_o} - 1} \cdot \left(\frac{2v}{u_o} - 1\right)^3} = \frac{2w}{x'}\left(\frac{a}{2w} \cdot \sqrt{dt}\right) \sqrt{\frac{\left(\frac{2v}{u_o} - 1\right)^3}{d\,\frac{2v}{u_o} - 1}}$$

$$= \frac{2w}{x'} (M_o) \sqrt{\frac{\left(\frac{2v}{u_o} - 1\right)^3}{d\,\frac{2v}{u_o} - 1}}$$

in which t is a function of $\frac{x'}{2w}$ and $\frac{2v}{u_o} - 1$ and $\left(\frac{a}{2w} \sqrt{dt}\right) = M_c$ must be taken from the table I, using $\frac{x'}{2w}$ for the distance, $\frac{2v}{u_o} - 1$ for the temperature.

INTERMEDIATE ZONE.

We observe in figure 2 that the curve of cooling of the extreme margin thus indicated soon becomes coincident with a curve of cooling of the sheet of reference Fig. 1 with width c and fixed marginal temperature of 0. If that is so,

$$(28) \quad v = \tfrac{1}{2}\, u_{2y} = u_{y'}$$

when 2y represents the distance of the curve of cooling of the sheet of reference with which it becomes coincident. If the cooling has gone so far that it can be represented by the first term of $\frac{\pi}{4}\,\frac{u}{u_o} = q \sin \pi\,\frac{x}{c}$, we can see that (24) can be true and that

$$(29) \quad \frac{\pi}{4}\,\frac{v}{u_o} = \frac{1q}{2} \sin \frac{\pi\ 2y}{c} = q \sin \frac{\pi\ y'}{c}$$

gives us a simple equation to solve to find y', which can always be found.

$$(30) \quad \frac{\pi\ y'}{c} = \sin^{-1} \tfrac{1}{2} \sin \pi\ \frac{2y}{c}$$

But we also see that if y is so small, that is, the contact zone so small, that $\sin 2\pi y'/c$ is proportional to $2\pi\, y/c$ then y' will be the same proportional to y, and more generally, if it is so small that $\sin n\pi\, y/c$ is proportional to $n\pi\, y/c$ to the number of terms of (2) covered by n will y' be proportional to y and eq (28) be true. Not only that, but in equation (1) if $v/u_o = P\,\frac{y'}{2a\sqrt{t}}$ and $P\,\frac{2\,y'}{2\,a\sqrt{t}}$ is nearly proportional to $\frac{y'}{2\,a\sqrt{t}}$ which it is if $\frac{v}{u_o} < .33$, then to the same extent will:

$$(31) \quad \frac{v}{u_o} = \tfrac{1}{2}\,\frac{u_{2y}}{u_o} = \tfrac{1}{2}\,(\text{constant} \times)\,\frac{2\,y}{2\,a\sqrt{t}}$$

$$= (\text{constant} \times)\,\frac{y'}{2\,a\sqrt{t}}$$

and $v = u_{y'}$

That is to say if the contact zone y is small enough so that $2v = u_{2y}$ becomes .33 before the center has cooled perceptibly,[1] or the marginal temperature becomes say 1 / 6 of the initial, the cooling at that margin will be the same as at a corresponding point of the sheet of reference whose walls are kept cold. At the same time the cooling at the center is the same as that of another sheet in the reference sheet, at a distance $\frac{c}{2} - y$ from the wall. The cooling on two planes, center and margin being the same as on two planes of the reference sheet, the cooling between must be the same, except that due allowance must be made for the different distance apart of the two sets of planes. The distance of corresponding planes being thus proportional we obtain a formula:

$$(32) \quad g\,(k, x, u, u_o, y, c,) = g\,\left(k, \frac{c - 2\overline{y + y'}}{c - 2y}\, x' + y',\ u,\ u_o,\ 0,\right)$$

if 2 sin y π c = sin 2 y π c nearly.

At the center formula (32) reduces to formula (18) and writing (32) (29) $= g = Ax' + B$ and comparing it with (3), (7) and (9) we find

$$(33) \quad A = \frac{K}{c}\,\frac{c-2y-2y'}{c-2y} \cdot \frac{1}{h\sqrt{2(u/u_o)^3}}$$

$$(34) \quad B = K\,\frac{y'}{c} \cdot \frac{1}{h'\sqrt{2(u/u_o)^3}}$$

The equation (29) is the equation of another tangent to the curve ordinarily or a line parallel to one. It will only appear in the case of small contact zones at the point of minimum curvature. It may be distinguished from the other tangent to the curve of grain by the fact that it does not give a 0 grain at the margin. We notice, too, that

$$(35) \quad \frac{B}{A} = y' \cdot \frac{c-2y}{c-2y-2y'} = y' \cdot \frac{2w}{2w-2y'} = y' \cdot \frac{w}{w-y'}$$

This is always positive unless $y + y' > 1/2$, i. e., unless the contact zone is greater than the width of the dike > c/4 on each side, and is for small contact zones practically proportional to the contact zone.

If a series of observations of grain give a value of B negative or very small it is pretty likely that they refer to the equation $y = C'\,x'$ and that the value of B is due to the errors of observation.

From the formulæ above given not only can the grain be roughly constructed from the data K, u, u_o, y and c, but more important still, conversely if from observation we can construct the curve of grain for a given mineral with such accuracy that we can locate the tangents in question we can then draw inferences regarding the size of the dike, and the heat at injection.

From the curves of grain we may infer the order of crystallization of the different minerals.

A glance at a few typical curves of grain will show us what inferences may be drawn without any special calculation. Fig. 4 shows a number of curves of grain of different values of u/u_o and a small contact zone. Fig. 5 gives curves of grain when the contact zone is large. As the contact zone becomes smaller and smaller the curves approach those appropriate to the case when the contact belt is kept at a constant temperature.

We may make the following comments:

(1) The shape of the curves depends wholly on the ratio of the initial temperature to that of the crystallization of the mineral whose grain we are studying.

(2) Unless these temperatures are very close together, however, the grain at the center does not vary with their ratio, but is proportional to

---

[1] And table I shows that 2y must be about $\frac{1}{24}\,\frac{x}{c}$, for then when the temperature at the center is still .9626, the temperature for $x = \frac{2c}{24}$ is .3048.

($kc/a\sqrt{u_0}$), being greater the greater the size of the sheet and the less the diffusivity and temperature of consolidation.

The last condition may be put more suggestively by saying that the grain at the center is greater the nearer the temperature of the country rock is to the temperature of consolidation.

(3) The grain at the margin should be 0 if the initial temperature is more than twice the temperature of consolidation.

(4) The grain will be greater at the margin than at the center if the initial temperature is more than twice the temperature of consolidation, but if it is much more the grain will be practically even throughout, as with some aplites (curve .40 of Fig. 5).

(5) If there is a broad contact zone and the initial temperature is not far below twice the temperature of consolidation, there will be a belt of coarsest grain not at, but parallel to the margin (curve .52 of Fig. 4).

(6) When the contact zone is quite small and the initial temperature below the twice the temperature of consolidation the increase of grain will at first be linear and very rapid, then the linear and less rapid. This leads to the following practically somewhat puzzling uncertainty.

Suppose we have found by observation a nearly linear increase of grain. We can then locate a tangent to the curve of grain, which will be either that of formula (9) or that of formula (11). But which? Theoretically we can tell by this sign that the equation of (26) should lead to a value of the grain at the margin of zero, while the equation (29) should not lead to a value of 0 at the margin, but one of B, proportional to the effective contact zone y′.

Practically, unavoidable errors, of observation will be shown by a divergence of the equation from giving a zero value of the grain at the margin, even if it is really referrible to the equation (26) $g = C'x'$. But if the values of B come out negative, indicating a negative contact zone, they are likely to be due to error, and once more as C′ is independent of the width of the dike or of the contact zone it is more likely to be fairly uniform for the same type of rock.

(36). Thus from (19) to (26) we find: $\frac{E}{C'c} = \frac{h'}{\pi}\sqrt{\frac{(2\frac{v}{u_0}-1)^3}{v/u_0}}$

As long as u/u_0 is low enough, i. e., < .80 eq (36) expresses the relation of $E/C''c$. We can construct a curve or compute a table to express $E/C'c$ in the terms of v/u_0 (Fig. 6) and from such a curve and the observation of the grain at the middle, the rate of increase of grain at the margin and the probable breadth of the zone affected $c = 2w + 2y$ we can at once infer the ratio of the crystallization temperature to the initial temperature. In this expression the actual width of the dike or the ratio of contact zone to dike does not appear. This is because by the time the temperature has dropped to .80 uo the contact zone and dike are cooling together as one mass.

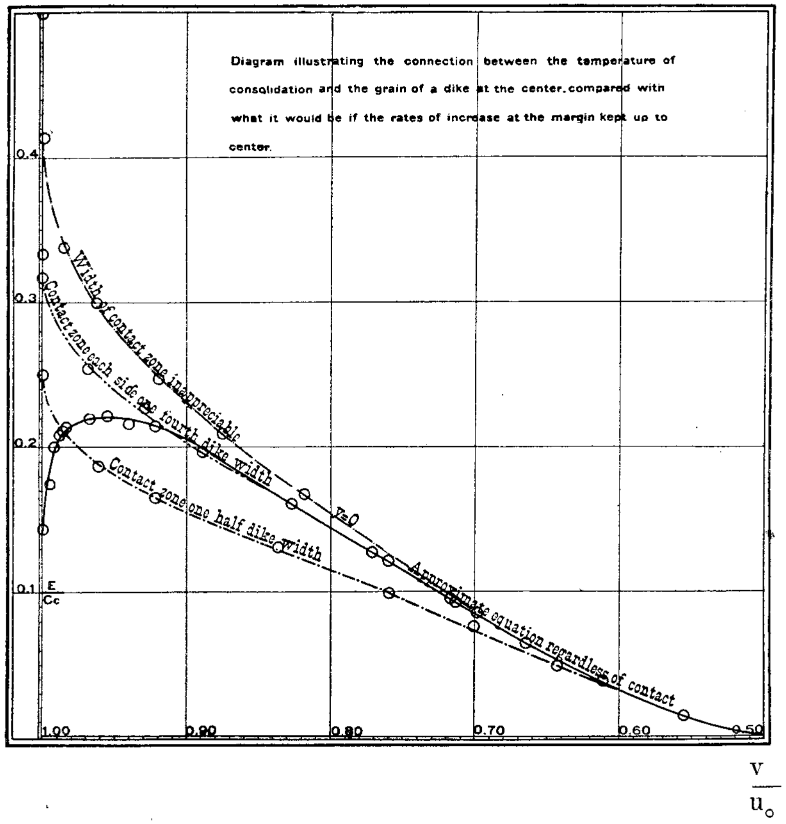


FIG. 6. Showing E/C′c or the ratio of the grain at the center to that which would be reached if the rate of increase at the margin C′ kept up for a distance c, equivalent to the whole zone affected.

But if the crystallization is earlier and we have to use eq. (18) instead of eq. (19) it will not be so. The actual grain will diverge from that of E and $\frac{E}{C'c}$ from that given by (36). But as the crystallization temperature approaches u/u_0 as we have seen the increase of grain becomes more and more linear and E approaches $\frac{C'w}{C'c} = \frac{w}{c}$. An ordinary range of contact zone may perhaps be from $y = o$ to $y = w$, and for that range of contact zone, the point which $\frac{E}{C'c}$ approaches will range from .5 to .25. The accompanying figure 6 gives $\frac{E}{C'c}$ for various values of u/u_0 and also the ratio of $E/C'c$ as derived assuming three values of y, to wit: $y = o$, $y = \frac{w}{2}$, and $y = w$ and computing as follows: Going back to (18) we have for the center $g = K\frac{a}{c}f\left(\frac{w}{c}\ \frac{v}{u_0}{}^{*}\frac{a}{c}\right)\sqrt{-dv/u_0} = KM_0 / \sqrt{-dv/u_0}$. We will assume y = 0 then $\frac{w}{c} = 1/2$ and we must use the row 12 $\left(\frac{x}{c} = \frac{12}{24}\right)$ of table I. Taking this row we notice that $\frac{v}{u_0}$ dropped from .8742 to .7597 when M_0 has the average value of .1234. Accordingly $\frac{g}{K} = \frac{.1234}{\sqrt{.1145}} = .368. = \frac{E}{K}$

The average value of the temperature we will take for this interval to be $\frac{(.8742 + 7597)}{2} = .8169$. Then $\left(\frac{2v}{u_0} - 1\right)$ will be .6338. Now for the temperature at the margin table II holds, and we find $P^{-1}\left(\frac{2v}{u_0} - 1\right) = m' = \frac{x'}{2a\sqrt{t}}$ to be .630. Taking the nearest differences we find that for an increase in m′ from .63 to .64 or dm′ = .0100 there is a change in Pm′ or $\left(2\frac{v}{u_0} - 1\right)$ of .0076 so that going back to (27) and (26)

$$\frac{cg}{Kx'} = \frac{cC'}{K} = \sqrt{2}\ \frac{1}{\sqrt{.0076}}\sqrt{\frac{.0100}{2(.6338)^3}\ \frac{(.630)^3}{(.6338)^3}} = \frac{1}{.445}$$

Thus the ratio of grain at center E / K to c C′ / K is

$$\frac{E}{c\,C'} = .368 \Big/ \frac{1}{.445} = .165^1$$

Similarly we may go on to compute the values for the drop of temperature given by values of the temperature in the same row and adjacent columns of table I. Each row will correspond to a different value of the contact zone. C′ c / K may be also found and the quotient, E / C′c Some of their values are shown in Table III below and the curves showing the dependence of E / C′c upon v / u_0 are shown in figure 6 for different contact zones.

In a very rough way the value of the actual grain relative to C′ c is proportional to the amount by which the temperature of crystallization is nearer the initial temperature than the country rock temperature. In fact if the contact zones have the width of the dike the actual grain E is to the grain which there would be if the marginal increase continued to the center $\left(\text{i. e., } \frac{C'c}{4}\right)$ very roughly as the amount by which the consolidation temperature is nearer to the initial temperature and exceeds the consolidation marginal median temperature, is to the median temperature.

---

[1] These calculations are made with a slide rule and are not very accurate.

TABLE I.

Of u / u_0 when u_0 = 1 for various values of x (distance from margin) and q; (— dec log q = 1.0715 m_0^2).

x′:c	1.00	.99	.98	.97	.96	.95	.94	.93	.92	.91	.90	.80	.70	.60	.50	.40	.30	.20	.10	q = 0.00	$24\frac{x}{c}$
.0416	1.0000	0.6444	0.4852	0.4040	0.3613	0.3172	0.2835	0.2688	0.2519,	0.2363	0.2241	0.1553	0.1229	0.1013	0.0834	0.0665	0.0499	0.0332	0.0117	0	1
.0833	1.0000	.9353	.8058	.7111	.6404	.5862	.5309	.4964	.4897	.4533	.4217	.3048	.2372	.2007	.1653	.1319	.0966	.0658	.0329	0	2
.1250	1.0000	.9968	.9493	.8102	.8306	.7810	.7357	.6973	.6635	.6341	.6077	.4433	.3569	.2975	.2443	.1950	.1428	.0974	.0487	0	3
.1666	1.0000	1.0000	.9909	.9660	.9331	.8977	.8632	.8306	.8002	.7719	.7459	.5667	.4521	.3861	.3190	.2547	.1910	.1273	.0637	0	4
.2083	1.0000	1.0000	.9999	.9919	.9779	.9590	.9372	.9193	.8910	.8682	.8460	.6882	.5584	.4690	.3883	.3100	.2325	.1550	.0775	0	5
.2500	1.0000	1.0000	1.0000	.9973	.9940	.9858	.9744	.9607	.9455	.9294	.9129	.7598	.6422	.5308	.4508	.3602	.2701	.1759	.0900	0	6
.2916	1.0000	1.0000	1.0000	.9997	.9986	.9959	.9909	.9838	.9751	.9651	.9541	.8289	.7135	.6077	.5054	.4040	.3030	.1974	.1010	0	7
.3533	1.0000	1.0000	1.0000	1.0000	.9996	.9989	.9971	.9940	.9897	.9841	.9766	.8813	.7718	.6616	.5513	.4410	.3308	.2204	.1102	0	8
.3750	1.0000	1.0000	1.0000	1.0000	1.0000	.9997	.9993	.9980	.9961	.9933	.9897	.9189	.8168	.7041	.5878	.4704	.3528	.2352	.1149	0	9
.4166	1.0000	1.0000	1.0000	1.0000	1 0000	.9999	.9996	.9994	.9985	.9974	.9956	.9437	.8488	.7349	.6141	.4919	.3689	.2459	.1229	0	10
.4583	1.0000	1.0000	1.0000	1.0000	1.000	1.0000	.9999	.9998	.9995	.9990	.9981	.9580	.8678	.7534	.6306	.5048	.3787	.2524	.1262	0	11
.5000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	9999	.9997	.9994	.9987	.9626	.8742	.7597	.6357	.5091	.3819	.2546	.1273	0	12
	Inf.	15.673	11.053	8.9994	7.774	6.9347	6.3149	5.8305	5.4905	5.1146	4.8389	3.325	2.64	2.20	1.834	1.645	1.435	1.24	1.035	0	m_0

.0319 .0323 .0325 .0328 .0333 .0334 .0335 .1056 .1144 .1234 .1370 .1684 .1701 .20 .264 .604 M_0

TABLE II.

Values of the probability integral, P_m: *Sometimes* $= u / u_o$, *if* $m = \frac{x}{2a\sqrt{t}} = \frac{x}{c} m_o$.

m	0	Difference.	1	Difference.	2	Difference.	3	Difference.	4	Difference.	5	Difference.	6	Difference.	7	Difference.	8	Difference.	9
0.0	0.0000	113	0.0113	113	0.0226	112	0.0333	113	0.0451	113	0.0564	112	0.0676	113	0.0789	112	0.0901	112	0.1013
0.1	.1125	111	.1236	112	.1348	111	.1459	110	.1569	111	.1680	110	.1790	110	1900	109	.2009	109	.2118
0.2	.2227	108	.2335	108	.2443	107	.2550	107	.2659	106	.2763	106	.2860	105	.2974	105	.3079	104	.3183
0.3	.3286	103	.3389	102	.3491	102	.3593	101	.3694	100	.3794	099	.3893	099	.3992	098	.4090	097	.4187
0.4	.4284	096	.4380	095	.4475	094	.4469	093	.4662	093	.4755	092	.4748	090	.4937	090	.5027	090	.5117
0.5	.5205	087	.5292	087	.5379	086	.5465	084	.5549	084	.5633	083	.5716	082	.5798	081	.5879	080	.5959
0.6	.6039	078	.6117	077	.6194	076	.6270	076	.6346	076	.6520	074	.6494	072	.6566	072	.6638	070	.6708
0.7	.6778	069	.6847	067	.6914	067	.6981	066	.7047	065	.7112	063	.7175	063	.7238	062	.7300	061	.7361
0.8	.7421	059	.7480	058	.7538	057	.7595	056	.7651	056	.7707	054	.7761	053	.7814	053	.7867	051	.7918
0.9	.7969	050	.8019	049	.8068	048	.8116	047	.8163	046	.8209	045	.8254	045	.8209	043	.8342	043	.8385
1.0	.8427	041	.8468	040	.9508	040	.8548	038	.8586	038	.8624	037	.8661	037	.8698	035	.8733	035	.8768
1.1	.8802	033	.8835	033	.8868	032	.8900	031	.8931	030	.8961	030	.8991	029	.9020	028	.9048	028	.9076
1.2	.9103	027	.9130	025	.9155	026	.9181	024	.9205	024	.9229	023	.9252	023	.9275	022	.9297	022	.9319
1.3	.9340	021	.9361	020	.9381	019	.9400	019	.9419	019	.9438	018	.9456	017	.9473	017	.9490	017	.9507
1.4	.9523	016	.9539	015	.9554	015	.9569	014	.9583	014	.9597	014	.9611	013	.9624	013	.9637	012	,9649
1.5	.9661	012	.9673	011	.9684	011	.9695	011	.9706	010	.9716	010	.9726	010	.9737	009	.9745	010	.9755
1.6	.9713	009	.9772	008	.9780	010	.9788	008	.9796	008	.9804	007	.9811	007	.9818	007	.9825	007	.9832
1.7	.9838	006	.9844	006	.9850	006	.9856	005	.9861	006	.9867	005	.9872	005	.9877	005	.9882	004	.9886
1.8	.9891	004	.9895	004	.9899	004	.9903	004	.9907	004	.9911	004	.9915	003	.9918	004	.9922	003	.9925
1.9	.9928	003	.9931	003	.9934	003	.9937	002	.9939	003	.9942	002	.9944	003	.9947	002	.9949	002	.9951
2.0	.9953	002	.9955	002	.9957	002	.9959	002	.9961	002	.9963	001	.9964	002	.9966	001	.9967	002	.9969
2.1	.9970	002	.9972	001	.9973	001	.9974	001	.9975	001	.9976	001	.9977	002	.9979	001	.9980	....	.9980
2.2	.9981	001	.9982	001	.9983	001	.9984	001	.9985	....	.9985	001	.9986	001	.9987	....	.9987	001	.9988
2.3	.9989	....	.9989	001	.9990	....	.9990	....	.9991	....	.9991	001	.9992	....	.9992	....	.9992	001	.9993
2.4	.9993	....	.9993	001	.9994	....	.9994	000	.9994	001	.9995	....	.9995	....	.9995	....	.9995	001	.9996
2.5	.9996	....	.9996	....	.9996	....	.9997	....	9997	....	.9997	....	.9997	....	.9997	....	.9997	011	.9998
2.6	.9998	....	.9998	....	.9998	....	.9998	....	.9998	....	.9998	....	.9998	....	.9998	....	.9998	001	.9999
2.7	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
2.8	....	....	....	....	....	....	....	....	....	....	....	....	.9999	001	....	....	....	....	....

TABLE III.

Ratio of grain at center to C′ c.

if y=0		y=1/2 w		y=w		General Approximate.	
v/u_o	E/C′c	v/u_o	E/C′c	v/u_o	E/C′c	v/u_o	E/C′ c by eq. 36
1.0000	.500	1.0000	.333	1.0000	.250	1.0000	h′/ π
.9998	.46	.9998	.318	......	....	.9972	143
.99905	.413	.9682	.254	......	....	.9964	155
.9856	.338	......	....	.9858	.212	.9945	174
.9626	.299	......	....	.9607	.1873	.9919	200
.9183	.247	.9290	.227	.9212	.1648	.9881	208
.8742	.209	.8813	.1965	.8363	.1301	.9830	213
.8169	.167	.8265	.1597	......	....	.9670	219
.6977	.855	.7718	.1265	.7598	.0987	.9551	221
......	....	.7167	.0941	.7010	.0752	.9401	216
......	....	......	....	.6422	.0487	.9213	214
......	....	......	....	......	....	.8984	204
......	....	......	....	......	....	.8710	188
......	....	......	....	......	....	.8389	168
......	....	......	....	......	....	.8020	145
......	....	......	....	......	....	.7602	121
......	....	......	....	......	....	.7142	093
......	....	......	....	......	....	.6643	067
......	....	......	....	......	....	.6112	038
......	....	......	....	......	....	.5557	014
......	....	......	....	......	....	.5000	000

Whenever $u/u_o = P_m$ h is independent of x and we have the following table:

TABLE IV.

$P_m = u/u_o$	m	$h = \sqrt{\frac{m^3 D_m u/u_o}{(u/u_o)^3}}$
0	0	$\sqrt{\pi/4}$
.1125	.1	.8863
.2227	.2	.88
.3286	.3	.88
.4284	.4	.88
.5205	.5	.88
.6039	.6	.87
.6778	.7	.87
.7421	.8	.86
.7968	.9	.85
.8427	1.0	.83
.8802	1.1	.80
.9103	1.2	.78
.9340	1.3	.75
.9661	1.4	.70
.9763	1.5	.67
.9838	1.6	.64
.9891	1.7	.55
.9928	1.8	.49
.9953	1.9	.45
1.0000	0.0	.00

APPROXIMATE FORMULÆ.

Now, if we suppose that we can make certain simplifications, which can often be done, we can obtain approximate formulae which are very easy to handle. We will assume that y is equal to y′ and that the fraction $\frac{c-(2y+y')}{c-2y} = 1$. This we may do when the contact zone is relatively small. We will also introduce h′ as before and call $v/u_o = f^2$. Also we will call the expression $(k\,c/a\sqrt{u_o})$ K and we have the following formulæ:

(40). $C' = K/ch'(2v/u_o - 1)^{\frac{3}{2}} = K/ch'(2f^2 - 1)^{\frac{3}{2}}$ from (26 and 27) As $K/c = k/a\sqrt{u_o}$, C′ is independent of the size of the dike or the contact zone.

(41) $A = K/ch(v/u_o)^{\frac{3}{2}}\sqrt{2} = K'/chf^3\sqrt{2}$, from (33).

(42) $B = Ky'/ch(v/u_o)^{\frac{3}{2}}\sqrt{2} = Ky'/chf^3\sqrt{2}$, from (34).

(43) $E = K/\pi\sqrt{v/u_o} = K/\pi f$ from (19).

From (41) and (42) we can determine y′, the effective contact zone, which is B/A.

(44) $y' = \frac{B}{A}$.

If y is less than 1/12 of c, we may be sure that so far as the contact zone is concerned our approximate solutions are pretty close. From (41) and (43) we can find c in the terms of A, E and K—that is, we can sometimes by observation of the grain determine the thickness of the dike when it was not previously known. Moreover, we can find f in the terms of A, E and c, and if f^2 does not come too near to unity we may feel that our approximate formulæ and results from them are not likely to be far out. We can also find K in the terms of A, E and c, or if we have also Eq. (40) we can either check on our observations or get along without c or some other factor. From formulæ (40) and (41) we can find f in the terms of C′ and A, although the equation is a cubic. It is quite rapidly solved by approximation. We can then find K/c.

Then let a certain observed rate of increase of grain, represented by the slope of a certain straight line tangent to the curve of grain be s. We may not know whether it represents C or A as we have said. We shall have either

(45) $f^2 = u/u_o = E/.45\ hsc$ — from equations (43) and (41) letting s = A; or from equations (43) and (40) letting s = c

(46) $2f^2 = 1 + (E/.45\ h'sc)^{\frac{2}{3}}(2f^2)^{\frac{1}{3}}$. This equation is, however, already solved approximately in figure 6. Thus it will be comparatively easy to obtain the alternative values of v/u_o on the two hypotheses. Since $2v/u_o$ is always between 1 and 2 and the cube root varies but slowly, it will be easy to insert an approximate value in (46) and obtain nearer approximations. Moreover having found v/u_o we can go on to find K.

(47) $K = \pi E f = \pi E\sqrt{\frac{u}{u_o}} = Cch'(2v/u_o - 1)^{\frac{3}{2}} = Ach(v/u_o)^{\frac{3}{2}}\sqrt{2}$.

It remains to be seen if K which depends on the diffusivity, the size, the initial temperature, and a constant which represents the facility of crystallization of a particular mineral in a particular magma, will prove constant for one and the same flow as it is assumed to be for the sections from a given point.

An interesting question is where the various tangents meet—that is, where will the zone of marginal grain become equal to that of the center. If we refer to the equations of the three tangents we have the following formulæ:

(48) $x'_{13} = E / C' = ch'(2f^2 - 1)^{\frac{3}{2}} / \pi f$,

where x'_{13} is the abcissa of the meeting point of the two tangents; x'_{13} / c must be less than .282 according to this formula which is the same as that represented by the full line of the figures showing the ratio E / C′c. Similarly

(49) $x'_{23} = (E-B)/A = .45\ hcf^2 - y'$,

where $(x'_{23} - y')/c$ is less than .4; and finally lines g′ and g″, formulæ (40), (41), and (42) will meet at a point given by

(50) $x'_{12} = B/(C'-A) = y \Big/ \frac{h}{2h'}\left(1 - \frac{1}{2f^2}\right)^{-\frac{3}{2}} - 1$.

From formulæ (48) and (49) we shall not be able to find the width of the contact zone, but we may often be able to if we know over what range some of these formulæ are closely applicable. If, for instance, formula (40) holds at least to a value x′, it may be shown that

(51) 2y is not less than $x'/P^{-'}\left[(u/u_o) - 1 \text{ or } 2w\right]$

Also in equation (50), if we know exactly x'_{12}

(52) $y = \left(\frac{(2 \text{ to } \infty)^{\frac{3}{2}} h}{2h'} - 1\right)x'_{12} = \left(1.4 \text{ to } \infty\ \frac{h}{h'} - 1\right)x'_{12}$

C must always be greater than A and between C′ and A there is the relation

(53) $\frac{C'}{A} = \left(\frac{h\sqrt{2}\,(2 - 1/5^2)^{\frac{3}{2}}}{h'}\right)\left(\frac{C-2y}{c-2y-2y'}\right)$

up the values of $f^2 = u/u_o$ of about $.80\frac{h}{h'}$ is nearly 1 but as u/u_o gets above .90 it rapidly diminishes but does not fall below $\frac{1}{\sqrt{2}}$. Thus for small contact zones

(54) $\frac{C'}{A} = \frac{\sqrt{2}}{1 \text{ to } \sqrt{2}}\left(2 - \frac{1}{f^2}\right)^{-\frac{3}{2}}$

Again as $c - 2y = 2w$ we find $\frac{c-2y}{c-2y-2y'} = \frac{2w}{2w-2y'} = \frac{w}{w-y} = 1\Big/\left(1 - \frac{y'}{w}\right)$ so that if $y' = w$ $\frac{C'}{A} \doteq \infty$

$A \doteq 0$

APPLICATION OF THEORY OF GRAIN TO THE LIGHT HOUSE POINT DIKE.

We will now proceed to apply this theory numerically to a typical intrusive—the quartz diabase dike of Light House Point, Marquette, probably genetically belonging to the Keweenawan flows; but we wish to make two preliminary comments.

First: It is likely that the margin of the dike will be somewhat cooled during the act of intrusion before the molten magma has ceased to flow, and in so far as it has done that and crystals have been formed floating during motion, we cannot expect our formulæ to apply. The irregularities will be greater in the earlier formed crystals.

In the second place microscopic examination shows that olivine is more conspicuous near the margin and intergrowths of quartz and orthoclase near the center. This suggests a chemical change and the analyses of this dike given in connection with the theory of copper deposition show that it really occurs. Apparently the olivine crystallized first and a less magnesian magma was crowded to the center. The variation of grain due to these factors remains to be seen. Initial irregularities of temperature rapidly smooth out in cooling.

AUGITE.

We will begin with the mineral which appears to be the latest formed of the unquestionably primary minerals, the augite. Some observations are found on pages 243 to 245 of Volume VI, Part 1, but I did not consider whether the marginal zone of increase there discussed was limited by x_{13} or x_{23}, a serious omission. The average observations on a set of sections more recently taken, on Michigan Street, Marquette, July 17, 1902, are (accuracy of measurements not over 10 per cent):

Distance.	Difference.	Grain in millimeters.	Difference.	Rate of increase.	Sx[1]	B?
0		.046				
	50		—.028	—.00056		
50		.018			.0275	—.0095
	125		.061	.000485		
175		.079			.0965	—.0175
	231		.137	.000595		
406		.216			.233	—.007
	355		.194	.000518		
761*		.410		Ave.00055	.419	—.009
	861		.259	.000303		
1622		.669				
	6608		.344	.000052		
8230		1.013				

*Augite tends to gather into chondri.

The specimens at 8,230 is from the center of the dike, so that 2w is 16,460 millimeters. The specimen at 1,622 millimeters was taken at what seemed to be the end of the zone of increasing grain, and it is obvious that if the rate of increase which holds from the margin up to 761 millimeters continued the grain of the augite would there be .9 millimeters, or nearly the same as at the center, so that the field estimate was not very far out of the way.

A second set of specimens were collected by Mr. F. E. Wright at another point not far off, where the dike is about 55.9 feet or 17,040 millimeters wide, and the contact with the Mona schist is 20 paces east of the fence at the foot of Circle street, Marquette. We had one section which was made 5 centimeters long, to cover the 50 millimeters next to the contact. This shows that even at the margin the increase of grain is not so much greater as it would have to be if the slope we called C were A.

The result of observations on the augite at different distances are given below. It must be remembered that when the grain is very fine it is difficult to get an accurate idea of it. The average grain for the three center slides agrees well with what we found before. E is 1.07. Up to 670 millimeters the rate of increase C is .00046, which is quite as near to that previously found as we could expect, but the grain of augite at 1,890 millimeters, is certainly much finer, and that at 4,155 mm, perhaps somewhat coarser than we might normally expect. I have made repeated sets of observations on the former section, and can only account for the fineness as due to some initial irregularity of temperature or composition.

Distance.	Difference.	Grain.	Difference.	s.
0		0		
	10		.012	.0012
10		.012		
	10		.005	.0005
20		.017		
	10		.006	.0006
30		.023		
	10		.009	.0009
40		.032		
	10		.010	.0010
50		.042 .041[1]		
	400		.123	.00031
456		.165		
	166		.145	.00087
670		.310		
	1,274		.12	.00009
1,890		.43 to .59[1] .444[1]		
	2,225		.83	.00039
4,115		1.26 1.00[1]		
	1,645		— .52	
5,760		.78 .93[1]		
	1,840		.39	
7,600		1.17 1.1[1]		

[1] Figures obtained upon revision, the second set of five sections, not quite so carefully measured. As there is none of the country rock visible in section 1, a few millimeters should perhaps be added to the distances. A further discussion of this dike will be left till later.

A third set was taken. Dr. F. E. Wright wrote Dec. 2, 1903: "Have just sent ten samples of the Light House Point diabase dike to Voigt and Hochgesang. The numbers of the specimens are 1, 10, 40, 96, 137, 170, 280, 404, 516, 670 and 740. The numbers express the distance in cm. of the specimens from the contact of the dike with the Mona schist. The specimens were taken from the same old place just east of the fence, leading off north from Arch street. You know the thickness of the dike from last year's measurements, I believe. Width —55.9 feet—17,040 mm. Not corrected for dip which is from 5°—10° to N. (measured on jointing pillars. Strike to N 75° W. The walls of the diabase are approximately parallel to the planes of schistosity of Mona schist. Dip of latter 75°—80° N.)"

One object of taking the third set was to see how far accordant results would be obtained; also the first sets did not suffice to determine A.

The results of measurements were:

Distance in mm.	Difference.	Grain.	Difference.	Ratio	
10		.01		.001	
	90		.064	.0007	
100		.074			
	300		.169	.00056	
400		.243			C′=.00061 B=.033
	560		.125	.000224	
960		.368			
	410		.097	.000235	.127
1370		.465			
	330		.015	.0000454	.150
1700		.480			
	1100		.210	.000191	.081
2800[1]		.690			.048
	1240		.34	.000274	
					A=.000177
		zone of constant grain.			
4040		1.030			
	1120		—.07		
5160		.96			
	1540		+.18		
6700		1.14			
	1700		+.34		
7400		1.38			
		Average constant grain 1.13			

[1] This was misnumbered in section: I was enabled to detect the mistake by the grain.
Average gradient margin to 2,800 mm. = .000246.

The thickness of the dike is about the same in all three cases, as also the coarseness of the augite at the middle. It appears to change slowly if at all over quite a broad belt. It agrees very well with the central augite grain in Volume VI, part I, p. 244, and plate XV. We have in this later case, for a distance to the center of the dike about (23 feet) 7,000 mm. to (25 feet) 7,600 mm. dimensions of 1.0, 1, 0, 1.3, or in average 1.1 mm. Dr. Wright's specimens for one set give us 1.13 mm. and for the other set 1.07. My specimen at 80,230 mm. gives us 1.13 mm. It is plain therefore that E can hardly differ 10% from 1.10 mm.

In the Isle Royale report we had only one observation on a section close to the margin at a known distance from it, p. 244. That would give us .1 (mm.) ÷ 76.3 (3 inches = .0013).

The more recent sets both show an irregularity of increase of grain from the margin to the next adjacent section. Then the increase of grain is nearly uniform or shows no systematic tendency to vary from a uniform rate up to 761 mm. in one case, 670 mm. in the other, and 400 mm. in the third set. Though there is considerable variation in the various slides the average rate of increase or slope of the tangent is nearly the same in both. In the one it is

$$\frac{.410 - .018 = .392}{761 - 50 = 711} = .00055, \text{ in the other,}$$

$$\frac{.310 - .012 = .298}{670 - 10 = 660} = .00046, \text{ in the third}$$

$$\frac{.243 - .01 = .233}{400 - 10 = 390} = .0006.$$

These observations agree within 10% of their average and, considering that the sections are taken at different points where the dike had different thicknesses and probably not exactly the same initial temperature and pressure, as well perhaps as we could expect, and this constancy in the rate of increase rather points to its being C' instead of A. We will, however, work out both suppositions to show the method and how we may distinguish whether it is C' or A.

If the slope .0005 ± is C':

Then we may find $E / C'c = 1.13 / (.0005 \pm)(17{,}040 +) = .132-$.

Accordingly by Fig. 6, following the curve of $E / C'c$ for $y = o$, (because unless we do this we ought to take $c > 17{,}040$ which would make $E / C'c$ proportionately less since $c = 17{,}040 + 2y$), $v / u_o = .77-$.

Or we may assume $\left(\frac{2v}{u_o}\right)^{\frac{1}{3}} = 1.2$ for a first approximation and solve in the equation (46).

To get some idea of y we note that apparently the grain is as coarse at 4,000 mm. as at the center but not as coarse at 2,800 mm.

Therefore by equation (49) $x'_{23} \overset{< 4{,}000}{\underset{> 2{,}800}{}} = .45\, h\, c\, f^2 - y'$.

Or $x'_{23} = (.293-)\, c - y'$

Also $17{,}040 = c - 2y$.

Assuming $y = y'$ as a first approximation: $2(4{,}000 - \text{a part of } 1{,}200) = (.586-)\, c - 2y'$, whence subtracting and eliminating c

$$0.414\, c = 9{,}040 + \text{part of } 1{,}200$$
$$c = 21{,}800 + \text{part of } 2{,}900$$

$$\text{and } y = \frac{21{,}800 + \text{part of } 2{,}900 - 17{,}040}{2} = 2{,}380 + \text{part of } 1{,}450$$

$y / c = c / 8$ very nearly. And if we substitute the value of c in the expression $E / C'c$ it is about 1.04 or $v / u_o =$ about .73.

But there is another way of getting the contact zone. In the third set of observations between 960 mm. and 2,800 mm. the rate of increase is fairly linear as a diagram shows and corresponds to the line $Ax' + B$.

$$A = \frac{.690 - .368}{2{,}800 - 960} = \frac{322}{1{,}840} = .000177 = \frac{1}{5{,}650}.$$

Also B will have a value between .048 and .150 more likely the larger. Thus $B / A = y' \frac{17{,}040}{17{,}040 - y'} \doteq 850$; whence $y' = 900$.

These two values do not agree very closely but they do agree in indicating so small a contact zone that we are justified in assuming $y = y'$.

Again we have remarked that the change from the slope C' to that of A, comes somewhere about 760 mm., between that and 400 mm. Then from equation (50) we have $y / \frac{h}{2h'}\left(1 - \frac{1}{2f^2}\right)^{\frac{3}{2}} - 1$ if $f^2 = .77 = \frac{y}{1.46}$.

Hence $y = 1.46$ (400 to 760).

This value of the contact zone is in harmony with that already found. Hence we are safe in assuming that the contact zone of appreciably affected rock of the Light House Point quartz diabase is not far from a meter at the point in question. We then should use a value of c of 17,040 + 2(1,000) say 19,000 mm.

$E / .45\ hAc = 1.13 / (.45)(.86)(.000177)(19,000).$

$= .87$ or $v / u_0 = .70.$

$E / C^1c = 1.13 / (.0005)(19,000).$

$= 1.20$ or $v / u_0 = 75-.$

Or as an average $v / u_0 = .72$.

Greater refinement might be gained by manipulating the data, but would hardly correspond to a real increase in accuracy.

Drawing all the observations of grain to scale in one diagram we seem to find:

For C′ .00072 and less especially for set 1.

For A .00022 and more for set 1.

For B .150 and for

$B / A = y^1 = y$ nearly 660 (mm.) and almost precisely the same for set 1.

Thus set $c = 2w + 2y = 17,040 + 1,320 = 18,360$.

$E = 1.14$ and possibly less for set (1), i. e., 1.03.

From $E / C'c = (1.14) / (.00072-)(18,360) = .87$ we have by the diagram Fig. 6, $v / u_0 = .80$.

From eq. (45) $E / .45\ h\ A\ c = v / u_0 = (1.14) / (.45)(.86)(00022)(1,836)$ we have $v / u_0 = 74$.

We will therefore assume these values as the most likely: $v / u_0 = .72$, $y = y_1 = 660$, but we know that the ratio v / u_0 may be ± .02 out.

According to this supposition, then, the augite, latest formed of the minerals crystallized from the magma, had hardly formed at all when the magma came to rest,—not over a very few mm. from the contact. It was formed at an average temperature about three times as near the initial temperature of the magma as that of the country rock. So that if the temperature of the country rock was about 40° C. and that of the formation of the augite 1,090° C. the initial temperature of the magma would be 1,440°. Moreover there would be a contact zone appreciably warmed on each side of about a meter.

This is undoubtedly the correct supposition. For if we assume that this slope of .0005 to 7, instead of representing C′, represents A, multiplying it by (x′), the distance of the various sections from the margin, and subtracting it from the grain we ought to get B, and the B/A would by Eq. (44) give us (y) the contact zone. But as a matter of fact if we take the slope from 50 mm. to 761 mm. or 60 mm. to 670 mm. we get as in the last column on p. 226, negative values of B, and hence impossible values of the contact zone, except in one case. And even if we suppose we assume the slope taken for A to be very slightly in error, though we can find slopes within 10% which will not give us negative values of B, they will still be *very small* and the contact zone accordingly also exceedingly small. Practically we shall have to suppose that owing to the wetness and convection of the country rock or some other cause the country rock was kept at a nearly constant temperature. This is the erroneous supposition I made in the Isle Royale report. But in that case v/u_0 would be by Eq. (45)

$$\frac{v}{u_0} = f^2 = \frac{E}{.45\ h\ Ac} = \frac{1.13}{(.45)\ (.88)\ (.0005)\ (17,040+)}$$
$$= .300$$

I got a similar value in the Isle Royale report, simply by observing the curve. If the country rock temperature was 30° C. and the augite formation temperature was 1,090° the initial temperature would have to be 3,350° C. We should have to assume a very much hotter magma initially and get at the same time practically no contact zone, a much less likely state of affairs. Not only this but the rate of increase .0005 should continue practically up to[1] 1,622 mm and beyond, which it evidently does not. So we have here a direct argumeut that the marginal rate of increase of grain can not be A. We have therefore the cumulative effect of an improbably large error in determining the rate of increase, and an improbably high temperature, especially as coupled with an improbably insignificant contact zone against assuming that the rate of increase .0005 represents the slope A rather than C′.

Moreover if it is C′ other minerals formed before and at a higher temperature would be much more liable to be formed in the earlier stages of cooling, or even before flowage ceased than in the latter case. We shall proceed then to discuss the observations on them, taking them in the inverse order of solidification, and shall find that they show distinct indications of having been formed in an earlier and very early stage of the cooling.

MAGNETITE.

The magnetite is in part at least formed after this labradorite, laths of which penetrate it[2] in the sections taken in 1,890, 4,115, and 7,600 mm. Furthermore there is almost no sign of a porphyritic texture with flow crystals which the feldspar has. It, however, distinctly and sharply precedes the augite formation. The rate of increase close to the margin is certainly between .0008 and .0003, and apparently very close to .00036. At the center, however, comparing the observations in the Isle Royale report and our sections it seems as if values differed greatly. For one thing we can not distinguish aggregates and it also seems as if there may have been some absorption.

The intermediate observations vary quite largely as to the intermediate grain, but it is notable that they point to a $y = y'$ of from 600 to 1,500 mm. Thus it will be safe to assume the same width of contact zone and c as for the augite. The value of A lies between .000117 and .00017.

---

[1] If $x' = 1622$, $m = \frac{x'}{c} m_0 = .095 m_0$, and $24\frac{x'}{c}$ has a value between 2 and 3 of table I and if $u/u_0 = .315$ it is easy to see from that table that the value of g will be close to .80 and of m_0 close to 3.34. That implies that u/u_0 is nearly equal to P_m, i. e., by table II, that is, near .29 which implies that $m_0 = 3.05$ but for any such value of m and m_0 h is practically invariable. $x' = .88$ and $g' = Ax' = .8$, or in other words the linear increase of the grain should continue up to 1,622 and beyond.

[2] In working up the grain observations I forgot this fact, which I had noted not only in these slides but also in the Isle Royale report p 246, and started in on the hypothesis that it was older than the feldspar. The curves of grain quickly apprized me of my error.

From $E/C'c = .78+/(.000365+)\ (18360)$ we obtain

$E/C'c = 1{\cdot}16$ a greater value than for augite, i. e., $v/u_0 = .75$.

From the equation (45)—

$v/u_0 = E/.45\ h'\ A\ c = .78+/.383\ (.000117 \text{ to } .00017)\ 18{,}360$ we see that to have a value of v/u_0 as low as augite, if $E = .78+$ A must be $.00022+$, so that v/u_0 is pretty surely greater than for augite. If $v/u_0 = .75$ $A = .0002$.

The indications are therefore that the magnetite is formed at a temperature $v = .765\ u_0$ or greater. Taking E larger will increase v/u_0 rapidly. It seems quite possible that the contact zone and grain may vary at different parts.

Very possibly the more or less ready absorption or diffusion of the gases of the magma into the country rock and consequent oxidation was a determining factor. It seems to me likely that with the loss of temperature and mineralizers and the crystallization of labradorite, when the time came that the olivine already formed was corroded (as microscopic examination shows it was) absorbing silica and turning into the augite molecule, there was a release of some of the Fe–O– which, absorbing a little O from the H–O–, allowed that to become H–O–H or water and became itself magnetite Fe_3O_4. This is a good illustration of the fact that v and u_0 of our expression do not stand for heat merely but for that and any other conditions of crystallization that follow similar laws of diffusion.

GRAIN OF MAGNETITE OF QUARTZ DIABASE DIKE, LIGHT HOUSE POINT, MARQUETTE.

Observations on set (1).

1 Distance from margin.	2 Difference.	3 Grain in mm.	4 Difference	5 4 ÷ 3 quotient 3
0		.005 .01±		
	50			
50		.002 dust		
	125		.008	.00055
175		.07		
	231	.131	.061	.0002
406		.064 to .092[3]	.15	.00042
	355	.28	.17 to .143	.00048 .0004
761		.235	.129	.00015
	861	.269	.280	.000326
1622		.515	.56	.00008
	6608	.93[1]	.40	.00006
8230		.92[2]		

Observations on set (2).

1 Distance from margin.	2 Difference.	3 Grain in mm.	4 Difference	5 4 ÷ 3 quotient 3
0		.0		
	10		.02	.0007+
10		.0		
	10			
20				
	10			
30		.02		
	10		.01	.001
40		.03		
	10		.01	.001
50		.04		
	400		.09	.00022
450		.13		
	220		.11	.0005
670		.24		
	1220		.34	.0003
1890		.58		
	2225		.38	.0002
4115		.96		
	1645		.04	.00001
5760		1.00		
	1840		.46	.0002+
7600		1.46		

---

[1] Upper numbers obs. October 31, on Kuntz's sections.

[2] December 24.

[3] One extra large porphyritic grain.

Observations on set (3) in mm.

Distance.	Grain.
10	? (dust)
100	.02 (dust)
400	.128
960	.238
1370	.209
1700	.335
2800	.437
4040	.454
5160	.53
6700	.65
7400	.78

GRAIN OF LABRADORITE IN LIGHTHOUSE POINT QUARTZ DIABASE.

The feldspar at the extreme margin is more slender and trichitic than when it is coarser and that complicates the measurement of the grain. Near the margin these feldspars show flow arrangement and lines of coarser and finer. At 50 mm. there is a distinctly porphyritic texture and even at 175 mm. and 450 mm. and 616 there is a quasi-porphyritic texture there being a good deal of range in the size of the feldspars, many large but some quite small. Farther in the porphyritic appearance disappears. This plainly indicates that at the time of cessation of motion the temperature had dropped so much for a belt at least 7 and 16 inches wide from the margin that feldspar formation had begun. This being a broader belt than for the augite a temperature and time of formation of feldspar in advance of the augite is indicated, and we shall expect to find more evidence of initial irregularities of temperature in the feldspar grain. This we do as appears from the observations. It is more difficult to measure the grain of the feldspar owing to its pronounced tabular shape, and the fact that at the margin the latter seem to arrange themselves parallel to the walls. This may be due merely to the flow and pressure but I strongly suspect that it is a thermal phenomenon.

The method of determining the grain, which is fairly successful for the augite and magnetite fails entirely for the feldspar for it is entirely impossible to exclude the rhyocrystals (or crystals formed while the magma) was still in motion from those formed after it came to rest. Moreover their presence as crystallization centers (German "Impfen", buds or grafts) would tend to produce a finer grain, by interference. At a general look it seems quite plain that the feldspar is coarser at 7,400 mm. than at 6,700 mm., but I have been able to devise no system of measurements to put this into clear mathematical and tangible shape. As a matter of fact, the feldspar began forming among the very earliest constituents, even before the magnetite and olivine. But there are indications that it continued growing while the augite was forming, even though as far as the M (010) face is concerned it is idiomorphic against it. Where the feldspar is imbedded in a patch of augite composed of one or more individuals it is often noticeable that the feldspar laths near the center of the patch are smaller than those in the margin or grouped by themselves outside.

In fact not only did the feldspar continue to grow during the formation of the augite but, changing in composition and inter-grown with quartz, it fills the last interstices. In this case, however, it is apparently orthoclase, and there appears to be a rather sharp line between that and the previous soda lime feldspar.

It is thus indicated that the conditions of formation of the feldspar are very largely chemical.

A few of the porphyritic laths of feldspar are as long at the margin as at the center but the average length is certainly much greater, and they are also broader in proportion. Near the margin the rate of increase C′ for the smaller crystals, not formed while in motion is not less than .00075 in any case.

Then C′c will be say 20,000 mm. x .00075=15 mm. E is for these surely not greater than 1.65 x .34=.56 mm. for some of the porphyritic feldspar formed in motion might be mixed (or $\frac{\sqrt{50 \text{ to } 55 \times 18}}{53}$) from third set of observations). This would give for E / C′c a value of .038, so low as to point to consolidation immediately at the temperature of the margin, which is not in harmony with what we have said.[1]

LABRADORITE.—FIRST SET.

No.	Distance.	Difference.	Grain in mm.	Difference.	Quotient.
1	0		.0131 .49×		
		50		.0093 —.65	.00019 --.013
	50		.0224 .425×		
		125		—.007 .22×	—.00005 .00191×.3
2	175		.0152 .64×		
		231		.320 .21×(.3)	(.0008) to 406 ave. .0010 .001×.3=.0003
5	406		.334 .345		
		355	.85× 1.28 to 1.48× $\sqrt{.1}$	.094 .050 .26×(.3)	.000264 .00014 .00073×.3=.00021
3	761		.428 .395 1.11×	.214 .395 .08×(.3)	.000248 .000218 .0001×.3=.00003
		861			
4	1622		.642[1] .79[2] 1.19×[3] 1.64× $\sqrt{.1}$[4]	.025 —.236 .67×3	.000004 —.00004 .0001×.3=.00003
		6608			
12	8230		.667 .554 1.87× 1.7 to 2.15×$\sqrt{.1}$		

[1] Observations of Ohm sections, Jan. 13.
[2] Observations of Wright sections, Oct 31.
[3] Observations of Wright sections, Oct.
[4] Observations of Ohm's sections, Dec. 24.

The lower two sets are rough incomplete observations.

The magma appears to have secreted first a little magnesia silicate (olivine). Then as the temperature dropped there seems to have been no tendency to separate any lime salt etc., until it could crystallize out say the Labradorite $Ab_2\,An_3 = 2\,(Na_2O\ Al_2O_3\ 6\ SiO_2)+3\ (2\ CaO,\ 2\ Al_2O_3\ 4\ SiO_2)$ which has the ratio $Na_2O : SiO_2 :: 1 : 12$. Beginning with this the amount of soda gradually increased as the variation in extinction shows. The augite may have begun to crystallize when the ratio of Ab: An in the Labradorite was $Ab_2 : An_1$ for that has the same ratio of bases to SiO_2 as augite. Thenceforward the augite and the labradorite may have grown pari passu as a eutectic mixture as above noted, until the lime was practically exhausted. Previously, however, a certain amount of iron (which may at high temperatures have been ferrous iron Fe–O–having perhaps taken O from H–O) crystallized as magnetite.

After the crystallization of the augite and labradorite the remainder was probably capable of undergoing a considerable drop without crystallizing, being probably a hydrous solution of silica, potash and alumina, calcium and other chlorides.

---

[1] While this is in press, I have seen Day's paper on the fusibility of the feldspars. The higher temperature of formations which he finds for the feldspars and their tendency to undercooling fit in very well and explain much that had puzzled me.

The olivine which very early crystallized out, yet only in very small porphyritic crystals at the margin, did therefore not begin to crystallize much if any before the dike came into position. This inference agrees with the facts: first that the grain varies from center to margin: second that the marginal crystals are not corroded while the central are: third with the fact that Barus found the temperature of thorough liquidity or fluidity of a very similar diabase to be 1,360°; and fourth with the fact that the curve of grain of the augite indicates that the initial temperature of the magma was some 1,400° or a little above that of fusion of olivine as we have already mentioned. Observations on the grain of olivine indicate, therefore, much less.

Grain of Mt. Bohemia Gabbro-aplites.

It will be remembered I stated that Mr. F. E. Wright, the Assistant State Geologist, is studying the Mt. Bohemia gabbro. Its composition is much like that of the Duluth gabbro described by Winchell and U. S. Grant as will be seen by comparison of analyses. In it there occur seams or aplite veins and it itself sends out aplitic seams of the red rock into the contact zone of altered ophites. These rocks are mentioned in Volume VI, part 2, pp. 72–73, and the presence of these red stringers from the gabbro in the contact zones confirms the conclusions I drew there as to the intrusive character of the gabbro. They are formed only a few feet from the gabbro in the contact zone, and correspond to the red rock described by U. S. Grant and Bayley, and by Irving.

They are only a few mm. thick but they are uniform in grain from side to side and while fine grained as will appear by no means aphanitic. The fact that they are crystalline though so small and that the grain is uniform indicates (comparing curve 40 of Fig. 5) that the country rock was hot at the time of their injection. Of course the country rock may have been heating first and cooling later as the result of the injection of the gabbro as a whole. But the average effect on the grain is as though the temperature of consolidation was considerably nearer that of the country rock than the initial temperature of the magma, and that we see agrees nicely with the field conditions. The grain is then wholly determined by the formula for $E=\frac{kc}{\pi a\sqrt{u}}$ and u represents the difference between the country rock and the consolidation temperature. As c is small u must also be relatively small that E may have much value. E, the grain should also depend on c, which we may assume to be proportional to the breadth of the little dike. As we see from the table of the sections below, which gives the average length of the rectangles of orthoclase and the breadth of the dike, such is the case.

Letter of specimen.	Breadth.	Average dimensions of feldspar.	Calculated.
B	10 mm.	0.305	.324
E	14	0.486	.455
D	22	.73	.71
C	31	1.23	1.01
H	32	.89	1.04

We see that the grain does appear to vary quite directly with the size of these feeders. The observations on the grain check with the field observations and distribution and close welded contacts in proclaiming these either contemporaneous with or closely subsequent to the main intrusion.

THE THEORY OF COPPER DEPOSITION.

ALFRED C. LANE, STATE GEOLOGIST.

During the past few years there has been a lively interest in the theory of the origin of ore deposits, and recently two works have been published by the Institute of Mining Engineers and by the *Engineering and Mining Journal,* which give a very good account of the present state of the controversy, and references enough to start one pretty well over the whole field of the latter.[1] In these discussions our deposits of iron ore and copper of Lake Superior have been frequently used as illustrations of the various theories by those who take part in the discussion. In view of these facts, it seems proper to give a review of what is known concerning the copper of Lake Superior and of the theories regarding the same with reference to recent developments. There is also a practical interest involved in the discussion. As we shall shortly see, all the best authorities at present agree that the copper has been deposited by water, but there is some difference of opinion as to whether the water current is a descending one and the copper was deposited and a circulation produced by gravity, or ascending, and the circulation due to one or more principal causes, which we may call, as a common name, volcanic, meaning thereby, that they are connected with the interior heat of the earth. Now, it is a common notion among the practical Cornish miners of the copper country, although I do not remember to have seen the statement in print, that the copper is liable to occur under high ground.

To understand what is meant by the expression "high ground," we must remember that at the present day the bulk of the copper is deposited in bedded lodes. It would be perhaps more correct to say that it comes from lodes whose strike is the same as that of the beds of the Keweenaw formation. It is commonly accumulated in the originally more porous parts of these beds. Sometimes these porous parts are sandstones and conglomerates, but more often they are the porous upper parts of lava flows. It is, I believe, true that in many cases there are faults parallel to the bedding planes, or so nearly so that the difference has not been detected, which have had an important influence on the production of copper. In some

---

[1] Genesis of ore deposits. Reprinted papers from Volumes XXIII, XXIV, XXX and XXXI of the Transactions of the American Institute of Mining Engineers. Published by the Institute at the office of the Secretary, New York, 1902.

Ore Deposits, a discussion republished from the *Engineering and Mining Journal*, New York, 1903.

See also Geological Survey of Michigan, Vol. I, Part II, p. 43. Vol. VI, Part I, p. 216.

Yet more recent: Trans. Am. Inst. Min. Eng., October, 1902. Igneous Rocks and Circulating Waters as Factors in Ore Deposition, by J. F. Kemp; Ore Deposits near Igneous Contacts, by W. H. Weed, and discussion of same. Annual Report of the State Geologist (of New Jersey), 1902. Copper Deposits of New Jersey, by William Harvey Weed. The Chemistry of Ore Deposition, by Walter P. Jenney.

cases we know there are such faults, which generally have a somewhat steeper dip than that of dips generally.[1]

Nevertheless, in a practical way, the most characteristic feature of these lodes is the porous beds. Any one of these porous beds may contain copper, and there are few of them, which are decomposed, that do not show some trace of copper. But the parts which are relatively rich, rich enough to be the sole object of interest to the miner, are rare, and the meaning of the idea that copper occurs along high ground is, as I understand it, that in following the outcrop of such lode, chutes of copper are liable to occur where the outcrop of the lode is extra high. Now, there is some ground for this idea. If we take the Baltic lode, just developed, we find that in the Baltic, Trimountain and Champion mines, this is rich, while just northeast, on section 16,[2] the Atlantic mine has done a good deal of exploring without being able to find the lode because the lode when encountered was not rich. Rising once more on the high land, we find the Isle Royale mine close to the deep trough of Portage Lake, where, on the other side, is the Quincy mine, on high land again. The Sheldon and Columbia and Hancock mines, more down in the Portage Lake valley, do not appear to have been so successful. Going farther north, we find the Calumet & Hecla, Tamarack, Kearsarge and Wolverine mines, not very far from the Allouez gap on the southwestern side; on the northeastern side is the Mohawk mine. Nearer the gap is the Ahmeek property, which is just being opened up and whose true value has not been determined. If the "high ground" notion has any substantial basis, its prospects would not be so good as those of the Mohawk and Wolverine mines,[3] although it lies on the same lode and between them. The Phœnix mine and the Cliff lie on higher land, not far from the gap of Eagle river, and turning to the other end of the range we find the Minnesota and the National on one side and the Victoria on the other of the gap made by the Ontonagon river, while the Mass and Adventure lie on the high land between the Flint and the Fire Steel rivers.

Now, this grouping of mines in accordance with this notion that the copper occurs on the high ground may be due to the fact that the porous beds are usually covered by drift, and therefore, not exposed, and not easily exploited or developed, except on high ground. It might also be suggested that the alterations which produce the copper has cemented these beds more firmly and had thus given a greater resistance to erosion, either by ice or water. The copper itself, however, even in the richest mines, is only a small fraction of the rock, and is easily decomposed chemically, and so are some of the associated minerals, and, although at times, copper bearing amygdaloids, as the igneous porous beds are called, are more or less saturated with silica and epidote, I do not think those minerals are so characteristic of the copper-bearing lodes as to lead to a

---

[1] The top of the Calumet and Hecla is markedly slickensided. See also Volume VI, Part II, pp. 86–94; the slide fault in the Central mine appears to be nearly parallel to the Kearsarge conglomerate. The accumulation of copper was in the vein above this slide, and on reaching the conglomerate they worked on top of it finding good copper ground.

The annual report of the Phœnix mine for 1901 shows in the section by Dunbar D. Scott, the steeper fault slide in that mine, in the St. Clair vein. The old Minnesota, now Michigan mine, had its largest deposit of copper where a steeper fissure intersected a lode. See report Commissioner of Mineral Statistics for 1880, p. 76. Copper Handbook, 1902, p. 195.

See also Van Hise, Iron Ore Deposits of the Lake Superior Region, pp. 344–348, for account of the same system on the Gogebic Range.

[2] Vol. VI, Part II.

[3] Per contra the good rock obtained at the Ahmeek, and as Agent Denton informs me in the south end of the Winona under the swamp, is against this idea.

relatively greater elevation of such parts of the lode. However, there is room here for inquiry.

I leave to the last another possible explanation which has a more direct connection with the theory of the deposition of the copper. If the copper is deposited by descending waters, as Pumpelly, who has done by far the most work upon the subject, suggested,[1] and the motion of these descending waters is determined by gravity, descending along the lodes at one branch of the inverted siphon and rising either in the same lode at a lower point of its outcrop, or in some cross fissure, which might very well be the cause of the gap in the range, then we can readily see that the greatest activity of circulation and the greatest deposition of the copper consequently, should be beneath salient points of the outcrop of the lode. Take for instance, the Calumet & Hecla. That deposit outcrops 600 or 700 feet above Lake Superior, and the chute of the richer streaks in the deposit is northward, and we may imagine the waters working down in that direction to reappear over in the Allouez gap or up some fissure which may possibly have determined the gap. We see, therefore, that the question as to whether the copper was deposited by waters circulating in one fashion or another has a practical interest in guiding the search for the richest parts of the lode. Van Hise has suggested not only this, but that the richer parts of the lode—called chutes—will be found beneath upward bends if the waters of deposition are ascending, beneath downward bends if the waters of deposition are descending. If he is right, which I doubt, in saying that the copper of the Michigan lodes are deposited by ascending waters, the southern end of the Ahmeek and the northern part of the Kearsarge properties is more likely to be extra productive according to Hubbard's map of the Allouez gap area (Volume VI., Part II., Plate VII.), but if the waters are descending, the same area is more likely to be lean.

In the first place, we may premise that it is a settled question that the copper was deposited by water. All kinds of authority agree in this, although at one time a few geologists thought of its being inserted in a molten state. But native copper and native silver occur together, as they could not if they were melted. They would at once be alloyed. Jewelry is often made of sections of nuggets of copper and silver, popularly known as halfbreeds, where the sharp and irregular line between the copper and silver is beautifully displayed. We also find copper grown upon minerals, like analcite and prehnite, which one can fuse in a candle flame. It is not very rare to find a sharp crystal of dog-tooth spar entirely plated over with copper, and then the growth taken up again.[2] Pumpelly has given in Volume I. of our reports, a most thorough discussion of the way in which the copper occurs.

It is worth noting that most of the Michigan geologists from Douglass Houghton down, have recognized that the copper was intimately associated with the traps in its origin, and that Dr. Wadsworth agrees with Foster and Whitney, Mueller and Pumpelly, in attributing the deposition of the copper to descending waters.

In the "Notes on the Geology of the Iron and Copper Districts of Lake Superior," which he issued in 1880,[3] he says:—

---

[1] Vol. I of these reports, Part II, p. 40.

[2] See Volume I, Part II, Chapter 3; also Volume VI, Part II, pp. 163 to 165 of our reports.

[3] Bulletin of the Museum of Comparative Zoölogy, Vol. VII, No. 1, pp. 86 to 130. See especially the resumé on p. 109.

"Whatever may be our view respecting the sources of the copper, it is evident that it was deposited by aqueous agencies, and that the general course of the solution was downward," p. 110, and again on page 123, he refers to the decomposition brought about by the medium of the "percolating waters, whose course seems, in many cases at least to have been downwards." On page 126 he writes, "The copper has been found throughout the district underlying heavy beds of trap or a series of smaller ones; in fact, in the parts visited by us, experience has shown that copper in abundance was only found where trap in large amounts overlaid the vein or bed" [this would not apply to the Nonesuch]. "The copper was found filling, at the Calumet and Hecla mine, the joints of the overlying trap, and extending as a continuous sheet through fissures at right angles to one another. At Copper Falls spikes of copper extend downward out of the overhanging trap into the ashbed. These are generally large at the upper and pointed at the lower end. Like them are the secondary depositions of calcite, in this trap, in long spike-like forms, that here and elsewhere have been taken for fossils. These features prove that the copper comes from fossils above downward,[1] and that it was deposited after the jointing of the trap."

A very interesting specimen, owned by Dr. Hubbard, shows a crystal of quartz which has been corroded and mainly replaced by native copper. Moreover, in the deeper part of the Quincy mines, Dr. Koenig has found a water which is now depositing copper and contains 9 grams to the metric ton of the same, and Mr. Geo. L. Heath, chemist of the Calumet and Hecla has also found copper in their mine water (p. 166).

I have shown in my United States Geological Survey water supply paper No. 31, on the different waters of Lower Michigan, that while each porous bed varies in the character of water from point to point, yet there is little intercommunication between them and it is difficult to see how there could be much, except upward along fissures or drill holes. Beds of clay or shale are known to be so impervious to water and to oil, they may be taken to be, even in a geological sense, impervious layers, permanently guiding and separating the different flows of water. The same statement applies to clayey belts of decomposed rock, paint rock and fluccan, as Van Hise himself has pointed out in ably discussing chutes and formation of the Galena lead deposits. Thus, it must be remembered, that Van Hise's figures of underground flow apply only to a homogeneous medium. His figure 5, for instance, might represent the flow of water in one single porous bed, say of conglomerate, sandstone, or amygdaloid, but not the formation at random. It is by no means practically true, therefore, that the zone of fracture "will be searched to its base by moving waters," unless beside being not only potentially but really fractured, so as to make it practically pervious as a whole, it is also covered by a surface topography so rough as to stimulate circulation. These two conditions will be best fulfilled in those mountainous districts, which, as Van Hise remarks, are most liable to contain ore deposits, page 416.

Now, the difficulty in supposing that the copper deposits are due to such a general circulation of water taken in at the surface, as Van Hise imagines, is very great. The following is a sample of water from the

Arcadian shaft, a relatively shallow shaft, analyzed by Dr. Koenig, August 23, 1898:

	Grams per ton.
$CaCO_3$	32.7
{ $Fe_2\ O_3$	13.7
{ Kaolin	100.0
$Fe\ CO_3$	24.5
$Mg\ CO_3$	25.6
$Mg\ SO_4$	60.2
$K_2\ CO_3$	10.9
$Na_2\ SiO_3$	101.3
Na Cl	trace
$Na_3\ P_2\ O_5$	2.2
$Na_2\ CO_3$	42.3
Organic matter, $C_{50}\ H_{54}\ O_{27}$	8.2
Total	421.6

Elsewhere in this report a number of similar analyses are given. The specific gravity is practically 1.000.

On the other hand a deep mine water coming in at the 46th level of the Quincy, was analyzed by Dr. Koenig, with the following result:

Sp. gr.	1.1898
$Ca\ Cl_2$	17.91
Na Cl	2.96
$Mg\ Cl_2$	—
SO_3	0
Iron	0.004
Copper	0.009
CO_2	0.00

Now these two analyses are typical.

Analyses at Freda and in the Hecla mine, given above, indicate a similar deep water. It will require considerable head to counterbalance such a difference of gravity.

The deep waters are strong solutions of earthy chlorides. A water with nearly 1 per cent. of bromine oozes in the 45th level of the Tamarack. The shallow waters are high in alkalies, and so low in chlorine that the alkalies have to be combined with other acids. It is no wonder that, as Pumpelly remarks, alkaline zeolites occur in the upper levels. One might explain the loss of carbonates if the upper water was descending by a precipitation of the same, such as we know has taken place, but I do not see that we can so explain the presence and absence of chlorine. That must, it seems to me, have been an original constituent of the deeper rock moisture, either from the sea in which the rocks were laid down, or of the igneous magma. Prof. Moore in his presidential address before the Liverpool Geological Society (1903, p. 269) has shown that at the top of a 96 foot thick intrusive sheet there is a 10 to 15 foot belt, corresponding to the amygdaloids of the Keweenawan series, which contains a little over 4 per cent. carbonic oxide and 2.6 per cent. water which are, as he believes, probably primary, although now combined as ferro-dolomite.

---

[1] Smyth and Van Hise are the only prominent geologists who look for the origin of the copper outside the formation. The association of copper with traps in New Jersey and elsewhere points to a genetic connection.

Moreover, around volcanic centers the escape of vapors containing chlorine and carbonic oxide and the formation of crusts of iron chlorides are common.

Now, if they escape they must be present in proportion to their vapor pressure in the lava and can hardly wholly escape, but must be present more or less, in the rock moisture of the acid interstices which I have so fully described for the intrusive rocks. But even in an effusive as the rock above figured, we see that between the crystal of augite and that of feldspar, each having its own shape, is an angular space which must have been a pore originally, filled probably with gas. In a thoroughly crystallized trap, doloritic melaphyre or diabase, like that of Lighthouse Point, whose analysis and porosity we give, the porosity is not over 1 per cent. But in the case of an amygdaloid the original amount of vesicular space may have been very considerable, and this space must have been filled either with the original gases or possibly in the case of submarine flows with sea water more or less contaminated with such

ANALYSES OF STONE FROM LIGHTHOUSE POINT,

Under direction of E. D. Campbell, by E. E. Ware, June 30, 1903.

Distance from margin.	Contact.		616 mm. 2.2 ft.		4,115 mm. 13.5 ft.		7,600 mm. 24.9 ft.	
	No. 1.	Molecule.	No. 3.	Molecule.	No. 6.	Molecule.	No. 8.	Molecule.
SiO_2	46.98	.783	47.67	.795	47.25	.787	47.10	.785
Al_2O_3	17.85	.175	17.55	.172	18.00	.176	17.47	.172
Fe_2O_3	3.13	.019	2.51	.016	2.21	.014	2.66	.017
FeO	10.30	.143	12.59	.175	12.42	.172	12.93	.179
MgO	(a) 7.10	.177	5.65	.141	6.35	.159	6.88	.172
CaO	(a) 8.47	.152	10.75	.192	11.45	.204	10.27	.183
Sodium oxide	2.04	.033	2.21	.035	1.96	.031	1.91	.031
Potassium oxide	.60	.006	.65	.007	.66	.007	.59	.006
H_2O at above 800° C	(b) 1.97	.168	.35	.112				
H_2O at 110° C	(a) 1.55	.036	.40	.028				
CO_2	.20	.005	.18	.004				
P_2O_5	.143	.001	.169	.001	.158	.001	.161	.001
S	.097	.003	.183	.0057	.086	.0027	.111	.003
Cl	.07	.002	.05	.0014	.02	.0006	.09	.0025
MnO	.26	.003	.19	.003	.18	.003	.15	.002
	100.700		101.102		100.744		100.322	
Ratio, alkalies: SiO_2	.048		.053		.0483		.0472	
Ratio, pore space: solid space, in gasolene	.0473		.0012		.0032		.0013	
In water	.0429		.0008		.0028		.0015	
Sp. Gr. in gasolene	2.83		3.02		3.01		3.02	
In water	2.89		3.06		3.05		3.07	

(a) Checked later in another sample.
(b) Determined on new sample, first method incorrect.

gases. Such an origin would readily account for the saline character of the waters, and it is worth noting that such saline waters attack copper, as is shown by the fixtures around the salt baths of Lower Michigan.

Pumpelly furthermore concludes that the water which deposited the copper was descending. One of the arguments which he used is that the alkaline silicates abound in the upper levels and are (page 40) rare in depth; "in other words, they are abundant in that zone of the veins which lies between walls of those portions of the beds of the melaphyre in which we should look for the most advanced stages of alteration in the components of melaphyre, supposing such alteration to be due to the action of descending solution." By alkaline silicates he means analcite, apophyllite, orthoclase (and datolite of the same age). Copper occurs of similar age in some of these deposits. In studying the alteration of the lava flows which form so large a proportion of the Keweenawan series, I find that the olivine is first to alter, then the augite, and lastly the feldspar.

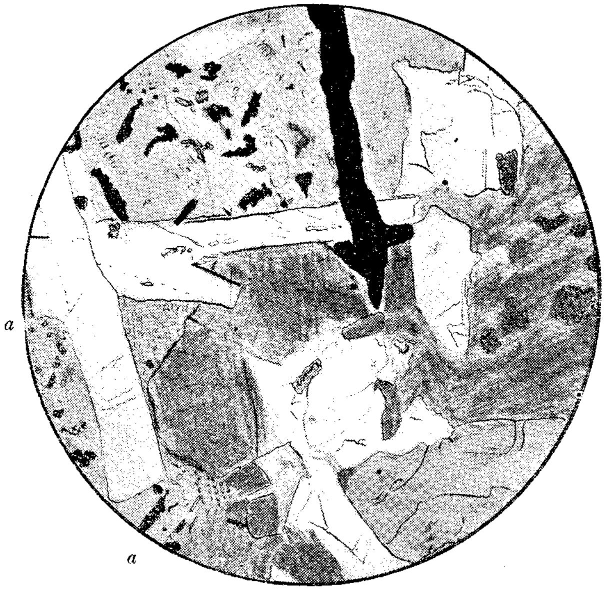


Fig. 7. Illustrates original cavities, in a rock of the copper bearing series such as may have been originally filled with chlorine gases, as at a, wedged in between feldspar laths and an octagonal augite grain.

There are other arguments, like those mentioned by Wadsworth, which may be used to support this theory with regard to the origin of copper. I wish also to add a few new considerations. As has been said, down to say 500 or 600 feet, the water of the mines is quite fresh. In the deeper mines, while there is very little water, it is an extremely strong solution of chlorides. The line between the two classes of water is reported to be very sharp, and there is a chance for a very interesting investigation right here. It would seem quite difficult to suppose a circulation of this heavy water up into a light fresh water, especially under high ground, and to imagine that there could be a sharp line between them. One would expect to find brackish waters clear to the surface, and that even if the heavy waters rising were diluted by affluents, they would retain the same general character, whereas the surface waters and the deep waters are chemically entirely different. If there was a tendency for the waters to descend, however, the rocks might naturally draw in fresh water of entirely different character from the outcrop.

Van Hise might, however, suggest that the present distribution of waters is a recent phenomenon, the present circulation being indeed downward, but much later than the origin of the copper.

We may also suggest another factor which might stimulate the circulation of water. We have mentioned the fact that there has been faulting. Now, if by faulting, the upper light sodium silicate water is brought on one side of the fault plane, and the heavier earthy chloride water on the other, there would surely be circulation, and precipitation, too, unless the fault plane were coated with a very perfect impervious sheet of clay flucan.

Suppose, for illustration's sake, a sliding of the upper beds down, such as that suggested by Hubbard at the Central mine,[1] relatively light, and superficial waters would be brought above the deep waters and the heavy earthy chloride waters would tend to work up through the fault plane, depositing as they went. If, on the other hand, the fault was a reversed fault, the reverse phenomenon would take place.

Another most weighty argument is the occurrence of copper native in the iron ores near Crystal Falls.[2] One can hardly imagine this other than produced by descending waters, since the iron ore is universally allowed to have been formed by descending waters. Moreover, it occurs in the upper parts of iron ore bodies and is not known to have any connection with lower deposits. It may easily be conceived to have been derived from an over-lying extension of the Keweenawan, now eroded away.

Pumpelly supposed that the copper may have been originally deposited with the strata, as sulphurets under submarine conditions. He was slightly inclined to call the old lavas altered (metamorphic) sediments.

Irving apparently agreed with Pumpelly[3] and Wadsworth, speaking of the copper having been arrested in its descent. More recent writers on ore deposits, however, differ from the Michigan Geological Survey and seem inclined to refer the origin of the copper deposits to the upward rising waters. For instance, Posepny writes as follows[4]:—

"Some of the attempted explanations assume, in my opinion correctly, as the cause of the first ore depositions, the action of hot springs—in which connection it is only to be emphasized that these thermal effects occurred long after the intrusion of the eruptive flows between the sedimentary strata, so the ores were brought, not by or in the eruptives themselves, but by the later springs, from great depths and perhaps from considerable distances. This explanation, applicable to all deposits, suits also the exceptional case cited by R. D. Irving, namely, the Nonesuch copper bed in the sandstone of Porcupine mountain, far from an eruptive outflow." Posepny seems to have been influenced in the first place by a strong prepossession as to the role of ascending solutions, and in the second place by the occurrence of the ore as a mineral or rarely sulfide and not as carbonate. The connection of the copper with the Keweenawan trap formation is much too close to be ignored, and, in fact, widely in the world native copper and traps (hessoses) are associated, as recently described by Weed in New Jersey. The Nonesuch bed is dark and very largely made up of basic trap fragments.

Prof. Van Hise, in his very interesting article on "Some Principles Controlling the Deposition of Ores," uses our metallic copper deposits as a conspicuous illustration of ore deposits where the concentration by ascending waters has been sufficient without secondary concentration by descending waters, writing as follows:—

"In some cases the deposits thus produced are sufficiently rich, so that they are of economic importance. In these cases, which undoubtedly exist, but which perhaps are less numerous than one might at first think, a concentration of ascending waters has been sufficient."

"A conspicuous illustration of ore deposits of this class which may be mentioned are the metallic copper deposits of the Lake Superior region. The copper was in all probability reduced and precipitated directly as metallic copper from upward moving cupriferous solutions. The reducing agents were the ferrous compounds in the solid form, in part as magnetite and as solutions derived from the iron bearing silicates. When the copper was precipitated, the iron was changed into the ferric condition. It is well known that metallic copper once formed is but slowly affected by the oxidizing action. Oxidation has, in fact, occurred in the Lake Superior region, but from the facts now to be observed, not to an important extent. An oxidized belt may have formed in pre-glacial times, but if so, it was swept away by glacial erosion, and sufficient time has not yet elapsed to form another. The ore deposits now worked have apparently remained practically unchanged since the time of their concentration. In this fact we have the explanation of the great richness of these deposits to extraordinary depths[1]."

Prof. H. L. Smyth, of Harvard, had also adopted the same belief and I have already discussed it in Vol. VI. of our reports. Prof. Smyth believes that the various flows were surface weathered and the earlier nonalkaline minerals produced thereby. The later alkaline minerals he believes to have been associated with the northerly and northwesterly tilting, and the formation and the filling of the fissures and the impregnation and partial replacement of amygdaloids and conglomerates with copper, the copper not being derived from overlying sandstones nor from traps, but probably by ascending solutions from deep-seated sources.

Returning once more to Prof. Van Hise's paper, we find that however his theories may apply to the other deposits, they apply very largely to copper-bearing rocks. His first premise is that the greater number of ore deposits are the result of work of underground water. His second is that the material of ore deposits is derived from rocks within the zone of fracture. This would seem to be true, and there are many reasons for believing that the copper is derived from the associated igneous rocks. His third premise is that by far the major part of the depositing water is meteoric. By this he means that it is derived from the air, rain water which has worked down into the ground. In view of the composition of the water at considerable depths above given on the Keweenawan Range,

---

[1] Vol. VI, Part II, p. 91.

[2] Prof. A. E. Seaman writes that he has native copper in iron ores from the Cliff mine at Iron Mountain, also with ferruginous chert from the tenth level of the Great Western mine, Crystal Falls, also from the Montana mine, Tower, Minn., where it occurs in the iron ore.

[3] Copper-bearing Rocks, pp. 426, 427.

[4] Page 145.

---

[1] Loc. cit., p. 353. See also pp. 414, 429.

it seems probable that this is not true, but that the largest part of the water may either have been buried originally with the sediments (he tells me he would class this as meteoric), or occluded in the original magma, as he suggests. It is a subject for further investigation, just how much of these three classes of water we have involved.

His fourth premise is that the flowage of the underground water is caused chiefly by gravitative stress. If this is true, I believe it may be, then it follows, as Van Hise himself has remarked (p. 417), that if the copper is most concentrated along the higher parts of the outcrop it must be formed by descending waters; moreover, as he also calls attention (p. 412) in case of the minor flexures and pitching folds in the bed, if the waters are descending the richest parts should be in the troughs of these folds, or possibly on lines leading from an anticline down to the trough of the folds. Referring once more to Plate 10, of Vol. VI., Part II, it will be seen that in such a case the copper of the Baltic and Trimountain may be expected to chute to the north when followed down. So should the mines around Calumet, while the Quincy mine should chute southwestward. The Winona mine also seems to chute southward under the swamp. And yet, as the flowage of water is under gravitative stress, it must be remembered that it will take a considerable difference in head for a fresh water to move or balance water with a specific gravity of (1.1898) a fifth more. We have already referred to the possible effect of faulting in promoting circulation. Moreover, the Keweenawan series consists mainly of a great series of lava flows, many of them over 100 feet thick. (See as an illustration of this, the section of Tamarack shaft No. 5, and correlated beds next following.) They are not likely to have lost heat for a long time after their effusion, in fact very likely not before their burial under succeeding flows,[1] so that for thousands of years the remnant heat of the effusions and the heat of the later intrusions may have aided the circulation, and particularly the solvent action of the water, as Van Hise (pp. 300, 346, 774), but more particularly J. F. Kemp and others have insisted. And yet the accumulation of copper in the Nonesuch belt of sandy shales, made up of lava sand, would indicate that it is the chemical character of the lavas rather than their heat which is most of importance. The source of the copper, Pumpelly considers to be sulphides originally deposited and leached out and reduced by the ferrous iron. This may be so, and yet it is strange that we see so little of sulphides in the original rock or of sulphates in the secondary minerals. I have seen some fine selenite from the National mine, but in general sulphates are rare. The arsenides and sulphides that do occur are very peculiar, occurring mainly in the veins, and perhaps rather more frequently, as at Mount Bohemia, associated with the acid rocks. There are signs that at least at times they are secondary after the native copper. It has occurred to me that possibly a ferrous or ferric chloride containing a trace of copper was an early volcanic emanation. It is, however, also true that olivine, which is one of the earliest minerals to develop, contains ferrous silicate with which is likely to be associated a trace of copper and nickel. Furthermore, under the microscope the olivine, an early formed mineral, appears to gather at the sides of this dike and the top of the flow. Analyses (Vol. VI. and here) seem to indicate the same thing in the variation of the magnesia and iron.

Thus the copper may have been concentrated, first, with the olivine in amygdaloid tops or residual chlorides; secondly, by leaching out of the olivine, which decomposed either by atmospheric action and meteoric waters, or immediately after the outflow of the lava in presence of the waters (mineralized and perhaps hot), buried with this formation; thirdly, by reactions due to the circulation downward of this water set up by the uplifting of the edge of the great Lake Superior synclinal and attendant faulting. It must also be remembered that according to the earlier geologists there has been enormous erosion, which, according to L. L. Hubbard's theory (VI., p. 94), may be in part replaced by a sliding of the upper beds on the lower for miles. In either case there may have been a considerable migration downward, in the porous belts of the formation, of the material of the strata and the original water thereof.

There is yet much to be learned, but three things appear to me to be extremely probable: the copper was associated with the original lava flows; that originally deposited water or gas has been an important factor, possibly merely in bringing the copper into solution, and that the water circulation which finally precipitated the copper was downward.

It is chemically reasonable and accords with the analyses from the Quincy, Calumet, and Tamarack[1] mines to suppose that the copper is in solution in the strong solutions of earthy chlorides.

That it should be precipitated by solutions of alkaline carbonates is quite natural, and entirely in accord with its occurrence in association with calcite, and also in accord with the fact that silica has evidently been dissolved by the same waters. Electrochemical action undoubtedly attended the deposition of the copper. Whether it would commonly be called the cause, or more ordinary chemical reactions such as those indicated were the cause, remains to be seen. Faulting might cause alkaline carbonate solutions to be brought against earthy chloride solutions.

It is apparent, however, that we need still to test the rival theories. We need to trace some one horizon, some one conglomerate or flow continuously through and survey it carefully and accurately to determine the minor flexures. Dr. L. L. Hubbard has done this in part, but the work is not complete.

---

[1] In the succession of flows noted in the Isle Royale drill cores of Vol. VI and the Tamarack shaft, and other sections studied if there had been a long interval between flows and they had been exposed to air, the amygdaloids would have decayed to red clays and iron ores, and if they had been long enough under water there would have been more or less deposition. As is obvious from the Tamarack section, there is but very little deposition, and while there may have been some contemporary decomposition, of the amygdaloids—in fact probably has been, and it may have helped in the copper concentration, yet in very many cases, it is clear that it did not progress far before the next flow came. In fact in some cases an effect on the marginal grain of the underlying flow is indicated. Now, just for illustration's sake, if (p. 245 of the Isle Royale report, Fouqué and Lévy's observations) an ophite cooling in about six days has augite grains 0.03 square millimeters in area, then one which has them about 50 square mm. in area, like the Greenstone 120 feet from the wall, would take about (6 × 50 ÷ .03) 10,000 days before it had actually consolidated, that is, it would be between twenty and thirty years before the center of a sheet 240 feet thick had fully consolidated, and it would still be red hot. But the increase of the grain of the augite clean to the center shows that it must have been during a very early stage of cooling, and a glance at figures 1 and 2 on pages 205 and 206, shows that after more than ten times that lapse of time, say 200 to 300 years, the temperature at the center would still retain something like an eighth of its original excess of temperature over the country rock. The temperature toward the margin decreases of course, and the total amount of calories yet left in the flow will be readily found by integrating equation (11) or (12) of the Isle Royale report. Of course these figures make no pretense to accuracy. We have no right to apply Fouqué and Lévy's observtions on the grain of a rock of one composition off hand to another. Yet the order of figures is likely to be the same, and it is plain that if the Tamarack cross section has some fifty flows, and this section only represents a third or less of the whole pile of flows thus rapidly piled on each other there may have been temperatures near boiling tens of thousands of years after the formation of the pile, during all of which time the zeolites we now see may have been forming. Obviously, too, there will be a large amount of energy to promote aqueous circulation.

[1] Pages 143 to 166, 243.