MODEL COMPETENCIES

Mathematics

BIG IDEAS AND PERFORMANCE INDICATORS

CBE-CASS
Competency-Based Education

Competencies and Assessment
Structures/Supports

K–8 BY GRADESPAN

MICHIGAN
Department of
Education

Model Competencies for Mathematics

Introduction

What is a competency?

Competencies are the core ideas or concepts that develop over time. The College and Career Readiness & Success Center at American Institutes for Research (AIR) states that "College and career ready standards are the building blocks that provide a frame of what a student needs to know and do to be successful. Competencies built on these world–class standards set clear expectations for what a student must 'know and show' to demonstrate mastery" (Patrick, 2014).

Why competencies?

Research into how students learn has shown simply possessing knowledge of facts isn't enough. For students to be able to retain, transfer and apply their knowledge, the knowledge must be organized as a set of ideas (National Research Council, 2005). "We know that the longer a student is engaged with content and the more deeply they are invited to think about it, the more likely they will be to retain it for future use" (Boudreau, 2020). By focusing in on core ideas or concepts, educators allow students to develop a deep understanding of these important concepts and avoid superficial coverage of disconnected topics. In other words, the proverbial "mile–wide, inch–deep" curriculum is changed into a curriculum supporting mastery and understanding of fundamental ideas of the various disciplines.

Instruction and curriculum based on competencies recognize and build on student assets. Competencies act as building blocks for coherent learning trajectories, curricula, and assessments that support Tier 1 instruction for students as they move through their education based on what they know and can do, rather than on seat time. As such, competency education models support equitable learning opportunities for all students by recognizing and building on student assets using a variety of assessment and instructional tools.

Michigan Model Competencies for ELA and Mathematics

The Michigan Department of Education (MDE), with support from English language arts (ELA) and mathematics educators from across the state, has identified model student competencies for K–8 ELA and mathematics. These competencies are distilled from the current Michigan K–8 standards and represent the foundational understandings of each grade level. Written as performance indicators, these competencies are organized into cross–grade progressions of related concepts, skills, and procedures; they eventually evolve into the ELA and mathematics understandings students need for success in their high school course taking pathways.

This document and the companion Model Competencies for English Language Arts are developed as models that a district might choose to use as tool for focusing lessons, resources, and supports so that students are provided the opportunity to deeply engage in the important content of each discipline. Districts and educators can also use these models to guide the development of local assessments and competency–based grading and reporting systems.

Questions regarding these model competencies, or about competency–based education in general, should be sent to Steve Nemeckay, nemeckays@michigan.gov.

References

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Structure of Model Competencies

In conversations with educators and administrators interested in competency-based education, it became apparent that the model competencies needed to:

- Be of **appropriate grain size**. If too fine, then there would be too many, blurring the intended focus and coherence. By identifying blocks within grade spans that connect with similar blocks in previous and/or subsequent grade spans, educators can provide the coherence and connections students must have to develop the deep understandings of important concepts necessary for continued learning and ultimately career-and college-readiness.

- Be **descriptive** of the idea and content embedded within the competency so that it is apparent to a variety of audiences, including parents and students, exactly what the competency is and why it is important.

As a result of these criteria, these model competencies are constructed as performance indicators organized around big ideas in each content area that span K–12.

Big Ideas

The big ideas outline the broader, practical purposes of the skills; in other words, they outline the ultimate desired outcomes of K–12 instruction in ELA and mathematics. By organizing these competencies around the big ideas rather than the domains and strands found in the standards documents, the "why" of the skill is more visible which can help with designing lessons that are more relevant and engaging to students.

The big ideas for **mathematics** are:

A. **Understanding and Applying Number Systems**: Students understand that numbers hold value and can choose the appropriate representations and algorithms to reason quantitatively, abstractly, and efficiently.

B. **Operations and Algebraic Thinking**: Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.

C. **Measurement and Data Analysis**: Students can collect and organize data to interpret, model, and investigate issues connected to their communities, lived experiences, and cultural identities.

D. **Geometric Reasoning**: Students can analyze, evaluate, and generate explanations about their world by exploring the properties and relationships of points, lines, shapes, space, and the positions of figures.

Is the student on target with knowing this content?

- **Identify where the student is in the progression (1) relative to grade level performance indicators (2) using formative and diagnostic assessments.**

- **Target support or enrichment as needed so that the student continues on track toward the big idea (3).**

Performance Indicators

The competencies are worded as performance indicators so that they more clearly identify what the students need to know and be able to do. They are written to be measurable with multiple and varied types of assessment. The performance indicators originated from the Michigan Mathematics Standards and The Mathematics Crosswalks: Claims, Targets and Standards.

Progressions

The performance indicators are organized so that progressions of related concepts, skills, and procedures are clearly visible across the grades. These progressions facilitate the identification where a student is in their learning trajectory so that instruction can be personalized based on what a student knows and can do, rather than on age–based grade level.

Grades K–2	Grades 6–8	**A. Understanding and Applying Number Systems**	C. Measurement and Data Analysis
Grades 3–5		B. Operations and Algebraic Thinking	D. Geometric Reasoning

A	**Understanding and Applying Number Systems:** Students understand that numbers hold value and can choose the appropriate representations and algorithms to reason quantitatively, abstractly, and efficiently.		
Progression	**K**	**1**	**2**
Counting	Students can count to 100 by ones and tens. Students can use written numerals from 0–20 to represent quantities. Students can classify objects into given categories and count the number of objects in each category.	Students can count, and read and write numerals, to 120.	Students can count within 1000; skip-count by 5s, 10s, and 100s. Students can read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
Comparing	Students can compare two numbers between 1 and 10 presented as written numerals. Students can compose and decompose numbers from 11 to 19 into ten ones and some further ones.	Students can record the results of comparisons with the symbols >, =, or <. Students can compare two two-digit numbers based on meanings of the tens and ones digits.	Students can compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits.
Strategies for Addition and Subtraction		Students use efficient, accurate, and generalizable models and strategies to add within 100 and subtract multiples of 10.	Students use efficient, accurate, and generalizable models and strategies to add and subtract within 1000.
Fluently Add and Subtract	Students can fluently add and subtract within 5.	Students can fluently add and subtract within 10.	Students can fluently add and subtract within 20.

| Grades K–2 | Grades 6–8 | A. Understanding and Applying Number Systems | C. Measurement and Data Analysis |
| Grades 3–5 | | **B. Operations and Algebraic Thinking** | D. Geometric Reasoning |

B	Operations and Algebraic Thinking: Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.		
Progression	K	1	2
Modeling Addition and Subtraction	Students use numbers to model simple joining and separating situations with sets of objects.	Students use a variety of models, including equations, to represent and solve problems involving addition and subtraction within 20, demonstrating an understanding that the operations must reference the same unit.	Students use a variety of models to represent and solve problems involving addition and subtraction within 100, including on a number line diagram. Students can represent the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns as an equation that expresses the total as a sum of equal addends.
Strategies for Adding and Subtracting		Students can apply properties of operations and the relationship between addition and subtraction to solve addition and subtraction problems within 20. Students can determine the unknown whole number in an addition or subtraction equation relating three whole numbers.	

| Grades K–2 | Grades 6–8 | A. Understanding and Applying Number Systems | C. Measurement and Data Analysis |
| Grades 3–5 | | B. Operations and Algebraic Thinking | D. Geometric Reasoning |

C	Measurement and Data Analysis: Students can collect and organize data to interpret, model, and investigate issues connected to their communities, lived experiences, and cultural identities.		
Progression	K	1	2
Comparing Lengths	Students can describe the difference between two objects with a measurable attribute in common by directly comparing to see which object has "more of"/"less of" the attribute.	Students can compare the lengths of two objects indirectly by using a third object. Students can express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end.	Students can measure two objects with the same standard–length unit to determine how much longer one object is than another and express the length difference in terms of the unit used to measure. Students use length units of different lengths to measure the same object and describe how the two measurements relate to the size of the unit chosen.

Grades K–2   Grades 6–8   A. Understanding and Applying Number Systems   C. Measurement and Data Analysis

Grades 3–5   B. Operations and Algebraic Thinking   **D. Geometric Reasoning**

D	Geometric Reasoning: Students can analyze, evaluate and generate explanations about their world by exploring the properties and relationships of points, lines, shapes, space, and the positions of figures.		
Progression	K	1	2
Classifying 2–D Shapes	Students can recognize, name and sort two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners) and other attributes (e.g., having sides of equal length). Students combine two-dimensional shapes and solve problems such as deciding which piece will fit into a space in a puzzle, intuitively using geometric motions (slides, flips, and turns, the informal names for translations, reflections, and rotations, respectively).	Students can name, describe and classify shapes, including drawings, manipulatives, and physical-world objects, in terms of their geometric attributes. Students can combine shapes to create composite shapes that are conceptualized and described as independent entities.	Students can recognize and draw 2-dimensional shapes having a given number of angles, sides or faces.
Equal Shares		Students can show that the more equal shares a circle or rectangle is divided into, the smaller the share.	Students can partition a rectangle into rows and columns of same-size squares and count to find the total number of them. Students can partition circles and rectangles in two, three or four equal shares and describe these shares in terms of halves, thirds, and fourths.

Grades K–2   Grades 6–8

Grades 3–5

A. Understanding and Applying Number Systems

B. Operations and Algebraic Thinking

C. Measurement and Data Analysis

D. Geometric Reasoning

A	Understanding and Applying Number Systems: Students understand that numbers hold value and can choose the appropriate representations and algorithms to reason quantitatively, abstractly, and efficiently.		
Progression	3	4	5
Decimal Notation		Students can use decimal notation for fractions with denominators 10 or 100.	Students can read and write decimals to thousandths using base-ten numerals, number names, and expanded form.
Place Value	Students can use place value understanding to round whole numbers to the nearest 10 or 100.	Students can use place value understanding to round multi-digit whole numbers to any place.	Students can use place value understanding to round decimals to any place.
Strategies for Multiplication	Students can multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations. Students can use area models to represent the distributive property in mathematical reasoning.	Students can multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.	Students can find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using models or equations and strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
Strategies for Adding and Subtracting		Students can add and subtract mixed numbers with like denominators, using strategies such as equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Students can add, subtract, multiply, and divide decimals to hundredths, using models or equations and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
Fluently Add and Subtract	Students can fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Students can fluently add and subtract multi-digit whole numbers using the standard algorithm.	

A	Understanding and Applying Number Systems: Students understand that numbers hold value and can choose the appropriate representations and algorithms to reason quantitatively, abstractly, and efficiently.		
Progression	3	4	5
Fluently Multiply	Students can fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations.		Students can fluently multiply multi–digit whole numbers using the standard algorithm.
Notation	Students can use a symbol to model an unknown number in equations representing problems with one or more of the four operations.		Students can use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. Students can use whole–number exponents to denote powers of 10.
Equivalent Fractions	Students can generate simple equivalent fractions and explain why the fractions are equivalent, using number lines and other models.		
Strategies for Multiplying Fractions		Students can use number lines and other models to express a whole number multiple of a/b as a whole number multiple of 1/b and use this understanding to multiply a fraction by a whole number.	Students can apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Students can divide unit fractions by whole numbers and whole numbers by unit fractions.

A	Understanding and Applying Number Systems: Students understand that numbers hold value and can choose the appropriate representations and algorithms to reason quantitatively, abstractly, and efficiently.		
Progression	3	4	5
Comparing Fractions	Students can compare two fractions with the same numerator or the same denominator, and justify their conclusions, using number lines and other models.	Students can compare two multi–digit whole numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Students can compare two fractions with different numerators and different denominators and justify their conclusions by creating common numerators or common denominators; comparing to benchmark fractions; or using number lines and other models.	
Comparing Decimals		Students can compare two decimals to hundredths and justify their conclusions by reasoning about their size.	Students can compare two decimals to thousandths and justify their conclusions based on meanings of the digits in each place.

Grades K–2   Grades 6–8

Grades 3–5

A. Understanding and Applying Number Systems

B. Operations and Algebraic Thinking

C. Measurement and Data Analysis

D. Geometric Reasoning

B	**Operations and Algebraic Thinking:** Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.		
Progression	**3**	**4**	**5**
Solve word problems with whole numbers	Students can represent and solve two–step word problems using equations and the order of operations with any of the four operations using whole numbers and having whole number answers. Students can use drawings and equations to model and solve word problems in situations involving equal groups, arrays, measurement, quantities, and scaled bar graphs requiring multiplication or division within 100. Students can represent and solve one–step word problems operations involving time, mass, or volume within a single unit with any of the four operations using whole numbers and having whole number answers.	Students can represent and solve multistep word problems posed with whole numbers and having whole–number answers using the four operations, including problems in which remainders must be interpreted.	

B	Operations and Algebraic Thinking: Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.		
Progression	3	4	5
Solve word problems with fractions		Students can represent and solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, using models and equations to represent the problem, and including problems involving distances, time, volume, mass, and money. Students can represent and solve word problems involving multiplication of a fraction by a whole number by using models and equations to represent the problem.	Students can represent and solve word problems involving addition and subtraction of fractions, including cases of unlike denominators, using models or equations. Students can represent and solve word problems involving the multiplication of fractions and mixed numbers using models or equations, including area models. Students can represent and solve word problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions using models or equations.

Grades K–2   Grades 6–8   A. Understanding and Applying Number Systems   **C. Measurement and Data Analysis**

Grades 3–5   B. Operations and Algebraic Thinking   D. Geometric Reasoning

C	Measurement and Data Analysis: Students can collect and organize data to interpret, model, and investigate issues connected to their communities, lived experiences, and cultural identities.		
Progression	3	4	5
Measuring Length	Students can generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch, and represent this data using line plots.		
Converting Units		Students can solve problems that require expressing measurements given in a larger unit in terms of a smaller unit within a single system of measurement.	Students can convert among different–sized standard measurement units within a given measurement system and use these conversions in solving multi–step, real world problems.
Solving measurement problems	Students can solve real world and mathematical problems involving areas and perimeters of rectangles with whole number side lengths. Students can solve real world problems involving areas of rectilinear figures by decomposing them into non–overlapping rectangles and adding the areas of the non–overlapping parts.	Students can apply the area and perimeter formulas for rectangles in real world and mathematical problems. Students can measure angles in whole–number degrees using a protractor and sketch angles of specified measure. Students can solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.	Students can apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. Students can solve real world problems involving volumes of solid figures composed of two non–overlapping right rectangular prisms by adding the volumes of the non–overlapping parts.

Grades K–2   Grades 6–8   A. Understanding and Applying Number Systems   C. Measurement and Data Analysis

Grades 3–5   B. Operations and Algebraic Thinking   **D. Geometric Reasoning**

D	Geometric Reasoning: Students can analyze, evaluate and generate explanations about their world by exploring the properties and relationships of points, lines, shapes, space, and the positions of figures.		
Progression	3	4	5
Classifying 2–D Shapes	Students can recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	Students can classify two–dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size, including right triangles.	
Symmetry		Students can identify line–symmetric figures and draw lines of symmetry.	
Graphing			Students can represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.

Grades K–2   **Grades 6–8**   **A. Understanding and Applying Number Systems**   C. Measurement and Data Analysis

Grades 3–5   B. Operations and Algebraic Thinking   D. Geometric Reasoning

A	Understanding and Applying Number Systems: Students understand that numbers hold value and can choose the appropriate representations and algorithms to reason quantitatively, abstractly, and efficiently.		
Progression	**6**	**7**	**8**
Notation		Students can convert a rational number to a decimal, and a decimal to a rational number.	
Rational Numbers	Students can use positive and negative numbers to represent quantities in real–world contexts, explaining the meaning of 0 in each situation.	Students can show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real–world contexts.	Students can use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.
Properties of Operations	Students can use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. Students can apply the properties of operation to generate or identify equivalent expressions.	Students can apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	Students can apply the properties of integer exponents to generate equivalent numerical expressions, including numbers expressed in scientific notation.
Fluently Divide	Students can fluently divide multi–digit numbers using the standard algorithm. Students can fluently add, subtract, multiply, and divide multi–digit decimals using the standard algorithm for each operation.		

Grades K–2   **Grades 6–8**   A. Understanding and Applying Number Systems   C. Measurement and Data Analysis

Grades 3–5   **B. Operations and Algebraic Thinking**   D. Geometric Reasoning

B	Operations and Algebraic Thinking: Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.		
Progression	6	7	8
Compute with rational numbers	Students can represent and solve word problems involving division of fractions by fractions using models and equations. Students can evaluate expressions at specific values of their variables including expressions that arise from formulas used in real–world problems and those involving whole number exponents.	Students can solve real–world and mathematical problems involving the four operations with rational numbers, using tools strategically.	Students can perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
Solve linear equations	Students can solve real–world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. Students can analyze the quantitative relationship between dependent and independent variables using graphs, tables, and equations.	Students can solve word problems leading to equations of the form $px + q = p(x + q) = r$, where p, q, and r are rational numbers.	Students can solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Students can use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. Students can solve real–world and mathematical problems leading to two linear equations in two variables.

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B	Operations and Algebraic Thinking: Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.		
Progression	6	7	8
Solve inequalities	Students can write an inequality of the form x > c or x < c to represent a constraint or condition in a real–world or mathematical problem. Students can represent solutions of inequalities on number line diagrams.	Students can solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are rational numbers. Students can graph the solution set of an inequality and interpret it in the context of the original problem.	
Solve measurement problems	Students can apply volume formulas to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real–world and mathematical problems.	Students know the formulas for the area and circumference of a circle and use them to solve problems. Students use facts about supplementary, complementary, vertical, and adjacent angles in a multi–step problem to write and solve simple equations for an unknown angle in a figure.	

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B	Operations and Algebraic Thinking: Students can use mathematics to analyze and evaluate historical, political, economic, scientific, and social problems and make conjectures about possible solutions.		
Progression	6	7	8
Ratios and Proportional Relationships	Students can solve unit rate problems using ratios of whole numbers, including those involving unit pricing and constant speed. Students can solve problems involving finding the whole, given a part and the percent. Students can use ratio reasoning to convert measurement units.	Students can solve unit rate problems using ratios of fractions, including lengths, areas and other quantities measured in like or different units. Students can recognize and represent proportional relationships between quantities using graphs, tables and equations. Students can use proportional relationships to solve multistep ratio and percent problems. Students can solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale using technology.	Students can graph and compare proportional relationships, interpreting the unit rate as the slope of the graph.
Functions			Students can construct a function to model a linear relationship between two quantities. Students can Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Grades K–2 **Grades 6–8**   A. Understanding and Applying Number Systems   **C. Measurement and Data Analysis**

Grades 3–5   B. Operations and Algebraic Thinking   D. Geometric Reasoning

C	**Measurement and Data Analysis:** Students can collect and organize data to interpret, model, and investigate issues connected to their communities, lived experiences, and cultural identities.		
Progression	**6**	**7**	**8**
Data Displays	Students can display numerical data in plots on a number line, including dot plots, histograms, and box plots.		
Using Data	Students can calculate and use quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), to describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	Students can use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	Students can construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Students can use relative frequencies calculated for rows or columns to describe possible association between two variables.
Probability		Students can develop a probability model (which may or may not be uniform) by observing frequencies in data generated from a chance process. Students can find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.	

Grades K−2   **Grades 6−8**   A. Understanding and Applying Number Systems   C. Measurement and Data Analysis

Grades 3–5   B. Operations and Algebraic Thinking   **D. Geometric Reasoning**

D	Geometric Reasoning: Students can analyze, evaluate and generate explanations about their world by exploring the properties and relationships of points, lines, shapes, space, and the positions of figures.		
Progression	6	7	8
Transformations			Students can describe a sequence of rotations, reflections, and translations that exhibits the congruence between two congruent figures. Students can describe a sequence of rotations, reflections, and translations that exhibits the similarity between two similar figures.
Distance between two points	Students can find the horizontal and vertical distances between two points on a graph. Students can solve real–world and mathematical problems requiring them to draw polygons in the coordinate plane given coordinates for the vertices and using coordinates to find the length of a side joining points on the horizontal or vertical axis.		Students can apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real–world and mathematical problems in two and three dimensions. Students can apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Classifying 2–D Shapes		Students can construct (freehand, with ruler and protractor, and with technology) triangles from three measures of angles or sides, and identify when the conditions determine a unique triangle, more than one triangle, or no triangle. Students can describe the two–dimensional figures that result from slicing right rectangular prisms and right rectangular pyramids.	

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D	Geometric Reasoning: Students can analyze, evaluate and generate explanations about their world by exploring the properties and relationships of points, lines, shapes, space, and the positions of figures.		
Progression	6	7	8
Solving measurement problems	Students can solve real–world and mathematical problems using the nets to find the surface area of three–dimensional figures.	Students solve real–world and mathematical problems involving area, volume, and surface area of two– and three–dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Students can solve real–world and mathematical problems involving volume of cylinders, cones, and spheres.