New Models for a New Michigan DNR Timber Volume Inventory System

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I. Background

This report describes a new timber inventory process and an associated volume modeling system.

The system described here is an updated version of the system detailed in MacFarlane (2015), which describes a new procedure for point sample – timber cruising for the Michigan Department of Natural Resources (DNR). The DNR has chosen to use Variable Radius Sub-Sampling (VRSS), mimicking a USDA Forest Service approach, to reduce tree measurement burden. The tree measurements described herein, which are inputs into the volume models, will only be measured on a subset of trees and the other trees of that species/product are simply tallied as "in".

Successful implementation of the DNR-VRSS approach required a volume modeling system to translate field data into volume estimates. It had previously been determined (MacFarlane 2015) that main stem volume could be computed using a modified version of the Clark, Souter, Schlaegel (CSS) (1991) segmented, form class, taper model, with accompanying models for predicting merchantable wood in the branches.

This report describes the updated modeling system, hereafter referred to as the CSS-M (Clark, Souter, Schlaegel- MacFarlane) model system, addresses several outstanding issues detailed in a previous report by MacFarlane (2017):

1. A method is presented to predict, rather than measure, **D**iameter at **F**orm-class **H**eight (DFH), which is an input variable required by the CSS stem model.
2. The updates presented include CCS-M model coefficients which vary by species and Merchantable Form Types (MFTs). MFTs are assignments of trees into stem and branch merchantability classes, determined by whether a tree has saw or pulp logs in the main stem and / or saw or pulp sized logs in branches or forks.
3. A method is presented for estimating the volume of branch wood, because taper models only describe the contour of a single main stem. Work by MacFarlane (2010, 2011) and MacFarlane and Weiskittel (2016) showed that branch wood volume is directly related to estimation of volume in the main stem. Since MFTs create categories that explicitly consider both branch and stem wood in a tree, an accompanying branch volume model was added to the taper model, to allow for estimation of total volume, including all merchantable wood in all parts of the tree. The branch model volume was developed with the assumption that no additional data would be collected to estimate branch volume.
4. Complete coefficient sets, statistical analyses of model fitting procedures and comparisons to the current DNR methods and validation data are presented.

II. Methods

A. Data used to create and test the model system

New data collection was initiated in January 2018, based on recommendations by MacFarlane (2017), to strengthen available data to fit the CSS taper model and associated DFH and branch volume models. Available data were from destructive sampling of trees in 2004, 2012 and 2018 by MacFarlane and the DNR, that specifically addressed DNR modeling needs. These data were combined with stem taper data from other sources, consisting of measurements of stem diameter at different points along a tree's stem, which were screened for utility for fitting the CSS-M models. As discussed with the Timber Measurements Technical Team (TMTT), the data had to be from Michigan and contain key variables for fitting the 4" top variant of the CSS taper model, in addition to measurements of stem diameter along the stem in a vertical (height) profile; these included stem diameter at breast height (DBH), stem diameter at form-class height (DFH; 17.3 feet above the ground) and height to a 4" top diameter (H4). The final data sets selected included:

1. Michigan DNR volume data sets (2004, 2012, 2018)
2. FIA-MSU biomass tree data (2011-2018)
3. Legacy tree data (historical data covering multiple decades; see http://www.legacytreedata.org/)

The final data are shown in Table 1, below, and consisted of 2,967 usable trees drawn from 667 stands in Michigan. The majority of trees were of commercially important species and predominantly hardwoods. A minimum sample of 30 trees (from an effectively infinite population) was considered as suitable for model fitting, but a number of species had fewer than 30 trees in the data base. So, trees were collapsed into groups where minimum sample sizes were not achieved. In some cases (see tables at the end), error statistics were computed by species, for completeness, but those statistics were not always reliable for judging the quality of the model at the species level, due to small sample sizes.

Table 1. Data for fitting Michigan-wide CSS-M model, by species and number of stands.

Genus Species	No. trees	No. stands
Acer nigrum	3	1
Acer rubrum	269	56
Acer saccharinum	5	4
Acer sacchrum	433	51
Amelanchier spp.	1	1
Betula alleghaniensis	29	17
Betula papyrifera	57	20
Carya cordiformis	6	3
Carya ovata	1	1
Celtis occidentalis	7	1
Fagus grandifolia	89	32
Fraxinus americana	54	1
Fraxinus nigra	9	3
Fraxinus pennsylvanica	1	1
Juglans nigra	1	1
Liriodendron tulipfera	11	1
Nyssa sylvatica	1	1
Ostrya virginiana	3	2
Populus balsamifera	22	11
Populus deltoides	1	1
Populus grandidentata	83	30
Populus spp.	85	5
Populus tremuloides	94	28
Prunus serotina	52	23
Quercus alba	65	24
Quercus ellipsoidalis	73	29
Quercus rubra	140	42
Quercus velutina	76	21
Tilia americana	134	37
Ulmus americana	2	2
Abies balsamea	105	23
Larix laricina	9	4
Picea glauca	57	30
Pinus mariana	62	19
Pinus banksiana	383	33
Pinus resinosa	403	41
Pinus strobus	69	30
Pinus sylvestris	1	1
Thuja occidentalis	55	18
Tsuga canadensis	16	3
Grand total	**2967**	**667**

B. Classifying trees into Merchantable Form Types (MFTs) and Large Branch Types (LBTs)

MacFarlane (2015) and MacFarlane and Weiskittel (2016) showed a significant benefit to assigning trees to a Merchantable Form Type (MFT) category to improve stem volume estimation with a taper model. The MFT has categories that depend on whether or not a tree has *any* merchantable volume of some or all of the following: (1) main stem pulp (MP), (2) main stem saw (MS), (3) branch pulp (BP), or (4) branch saw (BS). A value of "1" is recorded when that part-product is present and "0" when it is not. This produces trees ranging from trees with no merchantable volume (MFT = 0_0_0_0) to trees with all four types (MFT = 1_1_1_1). Recent research and data collection efforts in Michigan have shown that, while all of the possible trees likely exist, some are very common (e.g., MFT = 1_0_0_0) and others are quite rare (e.g., MFT = 0_0_0_1, which is a tree where the main stem is all cull, but the tree contains at least one saw log in a branch, but no pulp wood in a branch). Since branching has a major effect on stem taper, just knowing that a tree has a merchantable branch or not provides useful information. MFTs can be greatly simplified into Large Branch Types (LBTs), which is a binary code with a "0" meaning no merchantable branches and a value of "1" indicated that the tree has merchantable branches.

Data assembled from the three data sources above allowed for about ¾ of the trees to be assigned a MFT; for the rest the MFT was unknown (Table 2). The unknown trees were from the legacy database, because this type of information was not collected historically and, in fact, trees with significant forks and large branches were generally excluded from such data collection efforts (see MacFarlane and Weiskittel 2016).

Table 2. Merchantable Form Type codes for sample trees in Table 2.

MFT	No. trees	LBT	MFT.c*	No. trees
0_1_0_0	14	0	0_1_X_X	22
0_1_0_1	1	1	0_1_X_X	
0_1_1_0	4	1	0_1_X_X	
0_1_1_1	3	1	0_1_X_X	
1_0_0_0	568	0	1_0_0_0	568
1_0_1_0	240	1	1_0_1_0	240
1_1_0_0	623	0	1_1_0_0	623
1_1_1_0	603	1	1_1_1_0	603
1_1_0_1	1	1	1_1_X_1	181
1_1_1_1	180	1	1_1_X_1	
Subtotal	**2237**			**2237**
UnKn	730	UnKn	UnKn	730
Total	**2967**	**-**	**-**	**2967**

* "X" indicates it may be either a "0" or "1".

C. Merchantable Form Types combined (MFT.c)

There were five MFTs with very low sample sizes across all tree species and locations: 0_1_0_0, 0_1_0_1, 0_1_1_0, and 0_1_1_1. These were all tree-types (MFT starting with "0_1") which only had saw-timber in the main stem which break up into branches before tapering to pulp-sized sections in the main stem; some of these branches may have been merchantable. All of these trees were assigned to a combined MFT.c = "0_1_X_X"; with the "X" indicating it can either be a "0" or "1". There was only one tree with a MFT = "1_1_0_1", a large tree, with both saw and pulp logs in the main stem and at least one saw-log sized branch, but no pulp sized branches. This was combined with 180 trees with MFT = "1_1_1_1" to create a MFT.c. = "1_1_X_1".

In order to better understand the trees for which MFT and LBT were unknown (from historical data sets), the relative change in diameter with height was plotted by LBT for both hardwoods and softwoods, along with the trees with an unknown LBT (Fig 1, below). The figures show the expected acceleration in taper of the main stem when a large merchantable branch (or branches) compete(s) with the main stem (trees with an LBT = 1 taper more rapidly than those with LBT = 0). Softwoods with an LBT of "1" showed more rapid taper, lower in the tree, but more shallow tapering higher up in the tree, than hardwoods with LBT = 1. This is probably because the largest branches tend to be lower in the crown in softwoods. Visual assessments of softwoods suggest that the leveling-off of taper higher up could be because softwoods with LBT = 1 were more likely to have fairly symmetrical forks.

The LBT "unknown" trees showed an average taper curve more like that of trees without merchantable branches, in the case of softwoods, which probably reflects a historical bias of removing forking trees or trees of 'unusual' form from softwood data sets (MacFarlane and Weiskittel 2016). Though this historical data bias may also have been present for hardwood taper modeling, large branches are more part of the typical form of hardwoods, so hardwoods show the opposite trend. Because LBTs are fairly-coarse groupings, the next step was to graphically examine major trends associated with combined MFTs (called MFT.c) (Fig. 2). For simplicity, the term MFT is used instead of MFT.c to describe MFTs and “X” is used to replace a “0” or “1” to denote a combined type.

Figure 1. Relative change in diameter with tree height for hardwoods and softwoods with and without branches and for trees with LBT unknown.

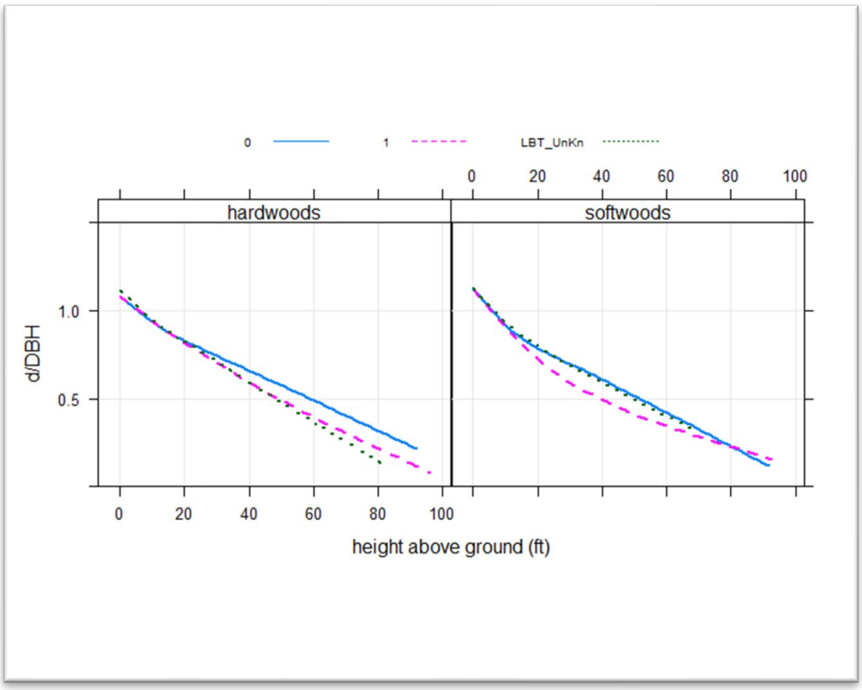
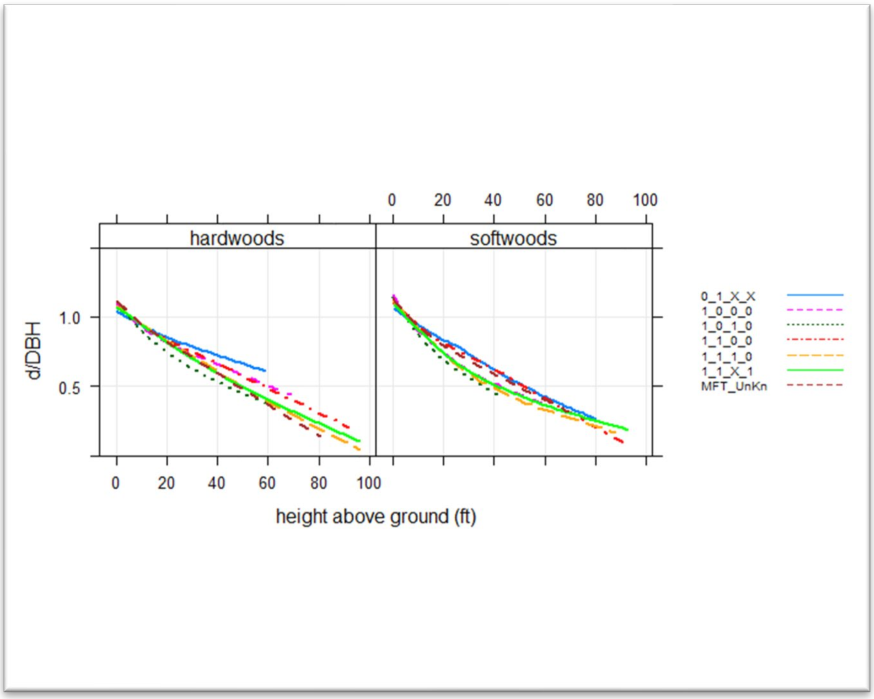


Figure 2. Relative change in diameter with tree height for hardwoods and softwoods with and without branches and for trees with LBT unknown.



In general, MFT showed a similar pattern across both hardwoods and softwoods, though large merchantable branches in saw-timber-sized trees (MFT = 1_1_1_0 and 1_1_X_1), appeared to exert a more significant effect on stem taper in softwoods than hardwoods (Fig. 2). Trees which only had saw-timber in the main stem (MFT = 0_1_X_X) were unusual and tended to taper less with merchantable height than other tree types, likely because these are trees that break up into branches before tapering to pulp-sized sections in the main stem. There were only 22 trees of this MFT, so it is a somewhat small sample population for drawing inference to the large one they were drawn from, despite the rarity of these types. Trees with both saw logs and pulp logs in the main stem (MFT = 1_1_0_0) had good form (meaning low taper) for both hardwoods and softwoods (pink dashed line and red dot-dash lines in Fig. 2). Of the smaller tree types, pole-sized trees with pulp logs, with merchantable branch wood (MFT = 1_0_1_0), showed more rapid tapering than those without merchantable branches (MFT = 1_0_0_0), for both softwoods and hardwood; as with larger trees the branch-induced tapering occurred lower on the stem and was more dramatic in softwoods than in hardwoods (Fig. 2). The data set also includes 730 trees for which the MFT is not known. For conifers, the overall taper curve (Fig. 2) for these trees was most similar to MFT = 1_1_0_0 and for hardwoods the curve was most similar to MFT = 1_1_1_0.

Overall, the results suggested that trees could be grouped by MFT to capture important differences in stem taper within species and that softwood and hardwood species should be grouped separately for purposes of taper model development. This approach was adopted for model development and trees with unknown MFT were reserved for models where the effect of form type could not be assessed with the data.

D. Model fitting

a. Model quality

The goal was to get the best taper models for the Michigan DNR. The goal was to achieve a mean absolute percentage error, MAPE < 10% (which indicates a reliable model; see Sileshi 2014) of the diameter - height relationship and a minimum R^2 value of 0.9, with a majority of errors within ½ an inch of the true diameter at a given height, and no strong trends of bias in diameter error across the height profile of the tree.

b. Sample size limitations

The goal was to have sample sizes of at least 30 trees for any model, drawn from multiple locations across the state, representing a theoretically "large" data set from an effective infinite population of trees which the models represent. Sample size exceptions were made for rare cases, such as the case of trees with MFT = 0_1_X_X, for which there are only 22 trees from all species and locations available (see below).

c. Species and Species groups

Species were combined with other species following conventional guidelines, assuming taxonomic similarity (e.g., same genus), indicating an underlying similarity in form dictated by genetics. Variation in MFT within species was considered where data allowed. MFT-only models were applied to combined-species groups were appropriate.

d. Mixed-effects modeling framework

Models were developed in a mixed-effects modeling framework. There are five coefficients for the CSS model: c, e, r, p and q. Coefficients c, e, and r control the shape of the lower stem below DBH, mostly affecting the flare of the trunk as it meets the root stock at ground level. Coefficients p and q control the shape of the lower bole (from DBH to form class height (FH = 17.3' above ground)) and the upper stem (above FH), respectively.

Various mixed-effects models were used for trees of a given MFT and species, while controlling for random effects for c, e, r, p and q, or some subset of these coefficients, associated with different sample locations (which are basically "stands"). This way random effects associated with differences between stands were accounted for, when estimating the fixed effects. In practice, these random effects are set to zero, when using the models, as they represent arbitrary stand/ locations. Statistical modeling was conducted in the R statistical language.

e. Model statistics

Model statistics are included in the appendices to this report and cited where necessary in the next section, which describes the modeling system which resulted from the methods described above.

III. New DNR Point-Taper Modeling System

This timber inventory system uses point sampling to collect data (see section II below), with CCS-M, a new variant of the CSS (1991) taper model (please see the original publication for additional detail- www.srs.fs.usda.gov/pubs/rp/rp_se282.pdf), to describe tree form and predict main stem merchantable wood volume (See Sections I and III, below). There is an associated model system to estimate merchantable branch volume (see III, C below). Differences between trees are characterized by basic tree measurements (see below) along with tree group (= 'hardwood' or 'softwood'), tree species (Gen.spp) and merchantable form type (MFT, described below). *Note: The volume system is designed for trees ≥ 4.6" DBH < 26" and will be most accurate for species and MFTs which have specific, rather than generic coefficients. Models can be extrapolated beyond the scope of this population, but this could introduce large errors into model estimates*.

I. Taper model

A Michigan variant of the CSS (1991) taper model, which predicts outside bark stem volume to a 4" top diameter outside bark (dob, inches) is the basis for timber volume calculations. The model computes dob at any height (h, feet) above ground along the stem, up to H4 (the height at a 4" top, feet) from stem diameter at breast height (DBH, inches) and stem diameter at form-class height (DFH, inches)(form class height, FH = 17.3' above ground along the main stem). The model is specified for trees with DBH >= 4.6" and H4 > 17.3 as:

Eq. 1

dob = (

I_S*(DBH^2*((1+ (c+e/ DBH^3)*((1-h/H4)^r - (1-4.5/H4)^r)) /(1-(1-4.5/H4)^r)))

+I_B*(DBH^2 - (((DBH^2-DFH^2)*((1-4.5/H4)^p-(1-h/H4)^p))/((1-4.5/H4)^p-(1-17.3/H4)^p)))

+I_T*(DFH^2 - (DFH^2-Dx^2)*(1-((H4-h)/(H4-17.3))^q))

)^0.5

where:

Dx = 4 (this is the diameter at H4, by definition)

I_S = ifelse(h < 4.5, 1, 0)

I_B = ifelse(4.5 <= h & h <= 17.3, 1, 0)

I_T = ifelse(h > 17.3, 1, 0)

Coefficients c, e, r, p and q vary by species and MFT or by broader groupings (See Table A.1, below).

II. Field data collection at sample points

The steps below are modified from "A new procedure for point sample – timber cruising with the CSS taper model for the Michigan Department of Natural Resources" (MacFarlane 2015).

▪Record BAF chosen for the stand (see recommendations in Table 5, in MacFarlane 2015).

▪Record any trees at the point that are "in", given the BAF; check limiting distance if needed, after DBH is measured (below).

▪Measure DBH-**must be measured to compute volume**.

▪Record tree species or assign to 'hardwood' or 'softwood' tree group, if unknown.

▪Record SH, on any tree which deviates significantly from the 1' stump rule.

▪Measure DFH- **must be known to compute volume, but may be predicted from DBH** (see below).

▪Measure H4-**must be measured to compute volume**. However, there may be trees where H4 does not exist on the tree. The most likely case is when there is a fork in the main stem, where the stem is > 4" dob below the fork and < 4" dob above the fork. In this case a model, can be used to estimate H4 from measured Hp (below).

▪Measure Hp on any tree with a pulp stopper below H4:

If 4" top DOB is a stopper, Hp = H4; otherwise measure Hp.

If tree has no pulp, record Hp = "NA".

▪Record length deduction for pulp cull (LDPC) on any tree without saw volume. This is the total length of all cull sections from stump to Hp.

▪Record length deduction for upper pulp cull (LDUPC) on any tree with saw volume. This is the total length of all cull sections from Hs to Hp.

▪Determine whether there is any saw timber in the main stem of the tree.

If no, Hs = 0; the model system computes no saw timber volume for the tree.

If yes, then, determine if there is a saw stopper other than d_S, which specifies the minimum upper stem diameter outside bark for a saw stopper.

If yes, measure and record Hs, the height to the saw stopper, then volume is computed between heights L and U = Hs (measured).

If there is no saw topper other than d_S, record Hs = 999 (or some other, arbitrary and impossible value). This means that the taper model computes how high d_S is on the tree and assigns that height to Hs (case 1, above). Then volume is computed between heights L and U = Hs (predicted).

▪Record length deduction for saw cull (LDSC) on any applicable tree. This is the total length of cull sections from stump to Hs.

▪Determine whether there is any 'large pulp' in the main stem of the tree. This is a saw-log-*sized* section of the main stem, which is downgraded to pulp. To make the taper model more accurate the 'large pulp' portion of the tree is modeled as a larger (saw-sized) piece.

If no, nothing more needs to be done related to large pulp.

If yes and Hs > 0, record length deduction for large pulp (LDLP) on any applicable tree. This is the total length of large pulp sections from stump to Hs.

If yes and Hs = 0, assign the tree a value of "1" for MS, for determining the MFT of the tree. The model computes only main stem pulp volume for this tree (i.e., the volume to Hp).

▪Record whether the tree has at least one pulp log in a branch (PB = 1, 0 otherwise) or at least one saw log in a branch (SB = 1, 0 otherwise), so that tree can be assigned to an MFT.

Trees are assigned to an MFT based on whether or not a value of "1" or "0" is assigned for MP (main stem pulp), MS (main stem saw), BP (branch pulp) and BS (branch saw). *Note: for a tree with large pulp MS = "1"*. So, a MFT = 1_0_1_0 has pulp volume in the main stem and at least one pulp size-branch. MFTs need not be assigned in the field, they can be assigned using all the field data collected above.

Note: Due to low sample sizes for some of the more unusual MFTs, MFTs were combined into logical groupings to meet minimum sample sizes for model fitting. The new MFT groupings give a value of "X" for some tree components, meaning it can be a value of either 1 or 0.

III. Volume models for estimating volume from field data

A. Compute merchantable cubic-foot volume outside bark (vob) of main (dominant) stem.

A1. Small pulp trees: DBH >= 4.6 & H4 <= 17.3 & MFT = 1_0_0_0 or 1_0_1_0, use the following ratio estimator to compute the total outside bark pulp volume to Hp (dom.Hp.vob):

Eq. 2

dom.Hp.vob = 0.0048546*(DBH^2*Hp)

A2. For all other trees, with merchantable volume in the main stem: MFT = 1_X_X_X or 1_1_X_X or 0_1_X_X & DBH >= 4.6 & H4 > 17.3, use the CSS - 4" top dimeter outside-bark volume model to predict volume to Hp (dom.Hp.vob) or Hs (dom.Hs.vob) or both (Eq. 3 below).

There are two main portions of the main stem that are computed with the volume model (Eq. 3, below):

1. Potential merchantable volume outside bark = dom.Hp.vob
2. Potential saw volume outside bark = dom.Hs.vob

The CSS-M computes components 1 and 2 above using:

Eq. 3

```
dom.U.vob =
0.005454154*(
  I1*(DBH)^2*((1-G*W)*(U1-L1)+ W*((1-L1/(H4))^r*((H4)-L1)-(1-U1/(H4))^r*((H4)-U1))/(r+1))
 +I2*I3*(Tt*(U2-L2)+Z*((1-L2/(H4))^p*((H4)-L2)-(1-U2/(H4))^p*((H4)-U2))/(p+1))
 +I4*(N*(U3-L3)+R*((1-L3/(H4))^q*((H4)-L3)-(1-U3/(H4))^q*((H4)-U3))/(q+1))
)
```

where U is the upper limit on the stem to which volume will be calculated.

U = either Hp or H4 (they will often be equal) for dom.Hp.vob

U = Hs for dom.Hs.vob

▪For trees with Hs = “999” the taper model will be used to compute Hs from DBH, DFH and H4 and a specified top diameter (d) that specifies the minimum upper stem diameter outside bark for a saw stopper.

L is the lower limit on the stem, which will either be L = 1 for standard one-foot stump or L= SH, if recorded.

Once L and U are known for each tree they are used to create the following combined and indicator variables:

L1 = max(L,0)

U1 = min(U,4.5)

L2 = max(L,4.5)

U2 = min(U,17.3)

L3 = max(L,17.3)

U3 = min(U,(H4))

I1 = ifelse(L < 4.5, 1, 0)

I2 = ifelse(L < 17.3, 1, 0)

I3 = ifelse(U > 4.5, 1, 0)

I4 = ifelse(U > 17.3, 1, 0)

G = (1-4.5/(H4))^r

W = (c+e/(DBH)^3)/(1-G)

X = (1-4.5/(H4))^p

Y = (1-17.3/(H4))^p

Z = ((DBH)^2-DFH^2)/(X-Y)

Tt = ((DBH)^2-Z*X)

J = (1-17.3/(H4))^q

Dx = 4, this is the diameter at H4 and it is fixed.

R = (DFH^2-(Dx)^2)/J

N = (DFH^2-R*J)

Coefficients c, e, r, p and q vary by species and MFT or by broader groupings (see Table A.1).

A.2.1. Predicting H4 from Hp

When H4 cannot be recorded on a tree, predict H4 from Hp, as follows:

Eq. 4

$$H4 = b + m*Hp$$

Coefficients b and m vary by tree group and MFT (see table A.2).

A.2.2. Predicting DFH when it is not available.

When DFH is not measured, DFH can be predicted from DBH, using the model:

Eq. 5

$$DFH = a+b*DBH + w*H4$$

Coefficients a, b, and w vary by species and MFT or by broader groupings (see table A.3).

A.2.3. Predicting Hs from the taper model.

When Hs is determined by the minimum (outside bark) top diameter for saw logs (d), and there is no other stopper (e.g., for large branches) the CSS taper model can predict Hs given d, DBH, DFH and H4.

For trees where Hs is determined by the taper model (Hs = h):

Eq. 6

$$h = (I_S Hx \{1-((d^2/DBH^2-1)/W+G)^{1/r}\}$$

$$+I_B Hx \{1-(X-(DBH^2-d^2)/Z)^{1/p}\}$$

$$+I_T Hx \{1-(J-(DFH^2-d^2)/R)^{1/q}\}$$

$$)$$

Where:

$I_S = 1$ if $d^2 \geq DBH^2$, else = 0

$I_B = 1$ if $DBH^2 > d^2 \geq DFH^2$, else = 0

$I_T = 1$ if $DFH^2 > d^2$, else = 0

$G = (1-4.5/H4)^r$

$W = (c+e/ DBH^3)/(1-G)$

X = (1-4.5/H4)^p

Y = (1-17.3/H4)^p

Z = (DBH^2-DFH^2)/(X-Y)

T = DBH^2-ZX

J = (1-17.3/Hx)^q

R = (DFH^2-(Dx)^2)/J

N = DFH^2-RJ

Coefficients c, e, r, p and q vary by species and MFT or by broader groupings (see Table A.1).

A.3. Computation of volume product components

A.3.1. saw volume and deductions

Potential saw volume outside bark = dom.saw.vob = dom.Hs.vob, before deductions (if any) are accounted for.

A.3.1.1. Computing the cull portion of stem saw volume.

Eq. 3 is run again twice for any tree where (length deduction for saw cull) LDSC >0

In the first run, the maximum cull deduction is determined to be where all the cull volume is in the lower part of saw length:

Eq. 7a

C.max.saw = dom.U.vob. ;where U = SH + LDSC & L = SH

In the second run, the minimum cull deduction is determined to be Eq.3, where all the cull volume is in the upper part of saw length:

Eq. 7b

C.min.saw = dom.U.vob. ;where U = Hs & L = Hs - LDSC

The average cull deduction is:

Eq. 7c

C.avg.saw = (C.min.saw+ C.max.saw)/2

So, adjusted saw volume with cull removed is:

Eq. 7d

dom.saw.vob.c = dom.saw.vob - C.avg.saw

A.3.1.2. Removing “large pulp” from stem saw volume.

Eq. 3 is run again twice for any tree where (length deduction for large pulp) LDLP>0

Large pulp can occur either above or below Hs, or both, but the model only needs to remove large pulp from saw timber volume which is below Hs; all volume between Hs and Hp is automatically assigned to pulp. In the first run, the maximum large pulp deduction is determined to be where all the pulp volume is in the lower part of saw length (same as cull):

Eq. 8a

P.max.saw = dom.U.vob. ; where U = SH + LDLP & L = SH

In the second run, the minimum pulp deduction is determined to be where all the pulp volume is in the upper part of saw length:

Eq. 8b

P.min.saw = dom.U.vob. ;where U = Hs & L = Hs - LDLP

The average large pulp deduction is:

Eq. 8c

P.avg.saw = (P.min.saw+ P.max.saw)/2

So, adjusted saw volume with large pulp removed is:

Eq. 8d

dom.saw.vob.p = dom.saw.vob - P.avg.saw

A.3.2. pulp volume and deductions

▪For trees with only pulp volume in the main stem, MFT = 1_0_X_X, potential pulp volume outside bark = dom.pulp.vob = dom.Hp.vob

▪For trees with saw and pulp volume in the main stem, MFT = 1_1_X_X, potential pulp volume outside bark = dom.pulp.vob = dom.Hp.vob - dom.Hs.vob

A.3.2.1. Computing the cull portion of stem pulp volume.

▪For trees with only pulp volume in the main stem, MFT = 1_0_X_X, Eq. 3 is run again twice for any tree where length deduction for pulp cull (LDPC) > 0.

In the first run, the maximum cull deduction is determined to be where all the cull volume is in the lower part of pulp length:

Eq. 9a

C.max.pulp = dom.U.vob. where U = SH + LDPC & L = SH

In the second run, the minimum cull deduction is determined to be where all the cull volume is in the upper part of pulp length:

Eq. 9b

C.min.pulp = dom.U.vob. where U = Hp & L = Hp - LDPC

The average cull deduction is:

Eq. 9c

C.avg.pulp = (C.min+ C.max)/2

So, adjusted pulp volume with cull removed is:

Eq. 9d

dom.pulp.vob.c = dom.pulp.vob - C.avg.pulp

▪For trees with saw and pulp volume in the main stem, MFT = 1_1_X_X, Eq. 3 is run again twice for any tree where length deduction for upper pulp cull (LDUPC) > 0.

In the first run, the maximum cull deduction is determined to be where all the cull volume is in the lower part of pulp length:

Eq. 10a

C.max.pulp.u = dom.U.vob. ; where U = Hs + LDUPC & L = Hs

In the second run, the minimum cull deduction is determined to be where all the cull volume is in the upper part of pulp length:

Eq. 10b

C.min.pulp.u = dom.U.vob. ; where U = Hp & L = Hp - LDUPC

The average cull deduction is:

Eq. 10c

C.avg.pulp.u = (C.min.pulp.u+ C.max.pulp.u)/2

So, adjusted pulp volume with cull removed is:

Eq. 10d

dom.pulp.vob.c = dom.pulp.vob - C.avg.pulp.u

A.4. Final outside bark volume

Final outside bark volume is computed for the two main volume components: saw.vob and pulp.vob, depending on the MFT.

A.4.1. Final outside bark saw volume

Final saw volume outside bark of the tree is:

Eq. 11

dom.saw.vob.final = dom.Hs.vob - P.avg.saw - C.avg.saw

A.4.2. Final outside bark pulp volume

▪For trees with only pulp volume in the main stem, MFT = 1_X_X_X, final pulp volume outside bark of the tree is:

Eq. 12a

dom.pulp.vob.final = dom.pulp.vob - C.avg.pulp

▪For trees with saw and pulp volume in the main stem, MFT = 1_1_X_X, final pulp volume outside bark of the tree is:

Eq. 12b

dom.pulp.vob.final = dom.pulp.vob - C.avg.pulp.u + P.avg.saw

Note that P.avg.saw is the "large" pulp volume (if any) that was deducted from the saw-sized section of the tree (see above).

B. Compute inside bark volume of components.

Cubic wood volume or volume inside bark (vib) of the main stem is strongly correlated with volume outside bark (vob). Once final outside bark volume are computed for pulp or saw timber inside bark volume are computed using the formula:

Eq. 13a

dom.pulp.vib.final = k* dom.pulp.vob.final^z

Eq. 13b

dom.saw.vib.final = k* dom.saw.vob.final^z

where coefficient k varies by species and z is constant (see Table B, below).

Note the fact that z > 1 means that the volume inside bark increases as the vob of the stem increases, i.e., larger outside-bark volumes contain relatively more wood and less bark.

C. Compute branch merchantable wood volumes.

Branch wood bark is not computed as part of this system, only merchantable branch wood volume (= vib = volume inside bark).

There are two branch wood components: branch.pulp.vib & branch.saw.vib, which together equal total branch volume:

Eq. 14

branch.total.vib = branch.pulp.vib + branch.saw.vib

C.1. Computing branch pulp wood volume for pole-sized trees; MFT = 1_0_1_0.

For softwoods:

Eq. 15a

branch.pulp.vib = -2.92086 + 0.50167*DBH

For hardwoods:

Eq. 15b

branch.pulp.vib = -2.19652 + 0.39137*DBH

C.2. Computing branch pulp wood volume for saw-sized trees without saw-sized branches; MFT = 1_1_1_0 or X_1_1_0.

Eq. 16

branch.pulp.vib = λ*(DBH^2)

where coefficient λ varies by species group (see Table A.8 below).

C.3. Computing saw and pulp volume for saw-sized trees with saw-sized branches; MFT = 1_1_X_1 or 0_1_X_1.

For softwoods,

Eq. 17a

branch.pulp.vib = 2.981058 + 0.007333*[(Hp - Hs)*(DBH)]

where Hp = H4, if not otherwise specified.

Eq. 17b

branch.saw.vib = 0.0138898*(Hs*DBH)

For hardwoods,

Eq. 18a

branch.pulp.vib = λ + μ*(DBH^2*H4)

where coefficients λ and μ varies by species group (see Table A.9 below).

Eq. 18b

branch.saw.vib = exp(0.9063+0.0628*DBH)

C.4. Computing saw and pulp volume for saw-sized trees with saw-sized branches; MFT = 1_1_X_1 or X_1_X_1.

For softwoods,

Eq. 18a

branch.saw.vib = 0.1810706*branch.total.vib^1.4407065

Eq. 18b

branch.pulp.vib = branch.total.vib - branch.saw.vib

For hardwoods,

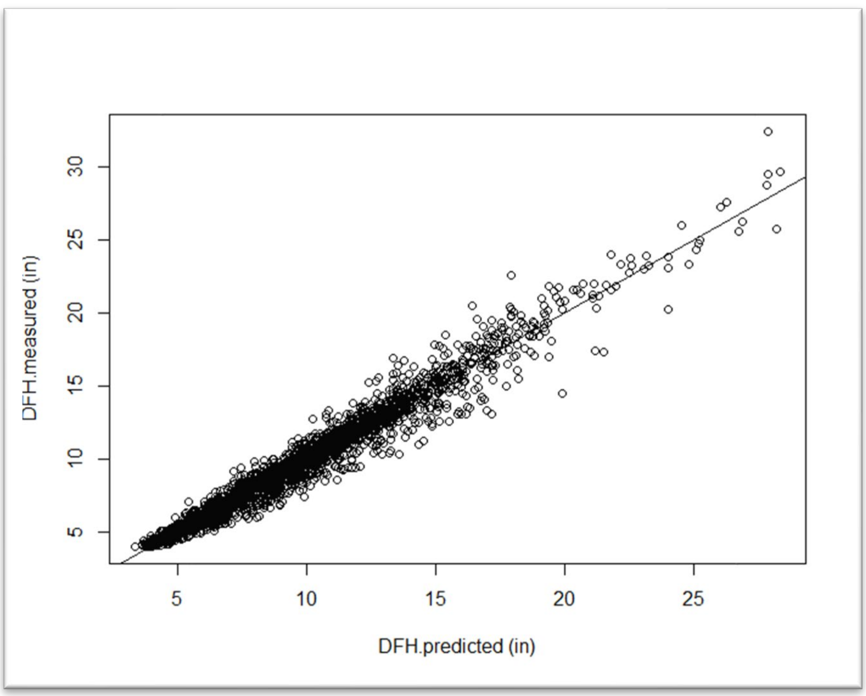
IV. Model diagnostics

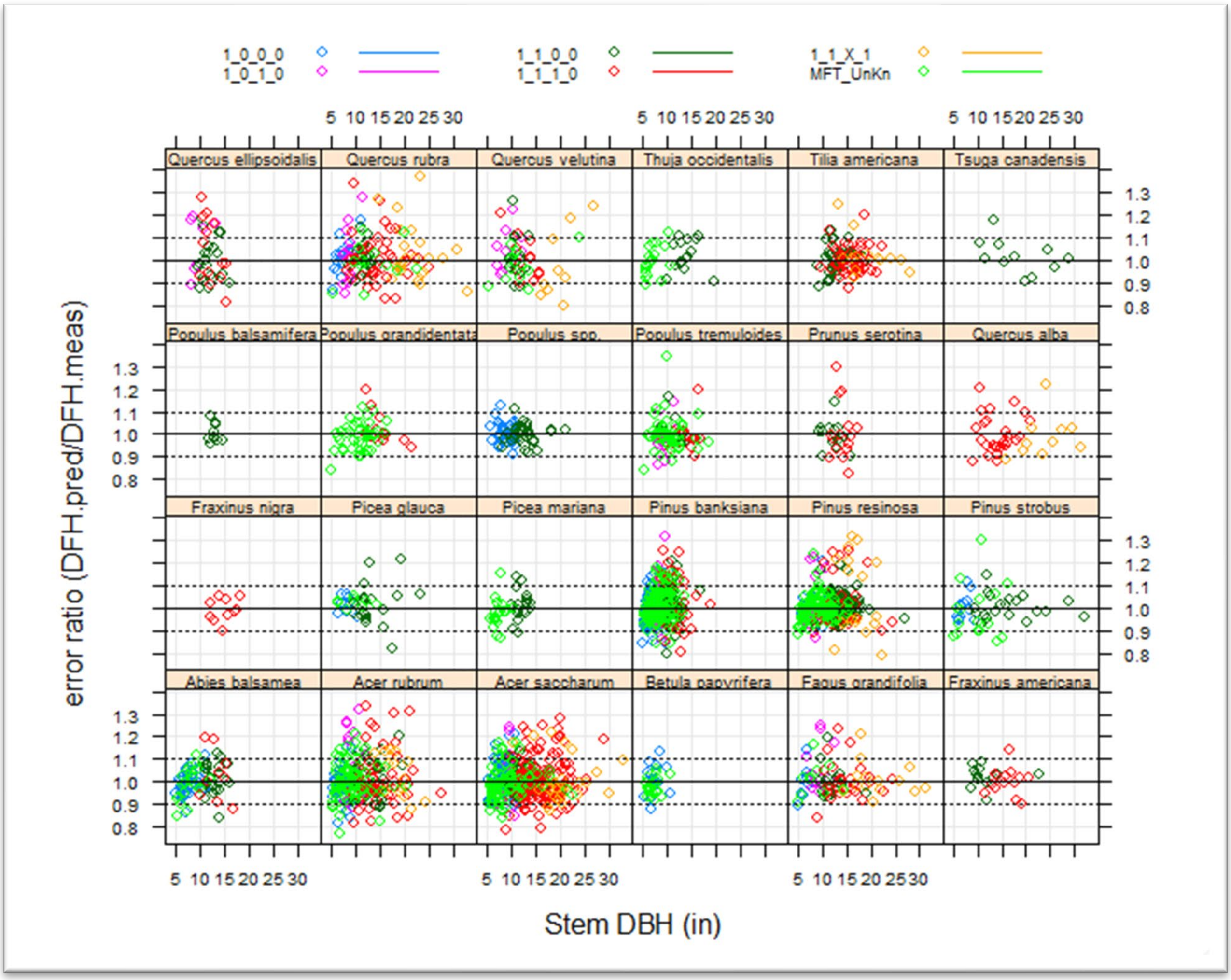
A. Taper model fit statistics

Model fit statistics for the CSS – M taper model are shown in Table A.1.1. The model generally fit the data well ($R^2 > 0.9$) and was reliable (MAPE ≤ 0.10) across the range of species and MFTs examined. The model proved less reliable for trees with MFT = 0_1_X_X, likely because the model was fit to a sample population that was a small number of mixed species of this MFT. This also represents an unusual type of timber tree, with unusual form.

B. DFH prediction

The model which predicts DFH from DBH and H4 (eq. 5) adjusting for both species and MFT (Table A.3) appears to predict DFH well over all trees (see top figure below), with no obvious bias in terms of form type or species across different size classes (see bottom figure below), though significant overprediction of DFH was evident for "branchy" MFTs of some species. The overall model had an R^2 = 0.96 and a MAPE = 0.056. The dotted lines in the bottom figure represent errors in predicting DFH of ± 10% (1.1 and 0.9 respectively, for + and -).



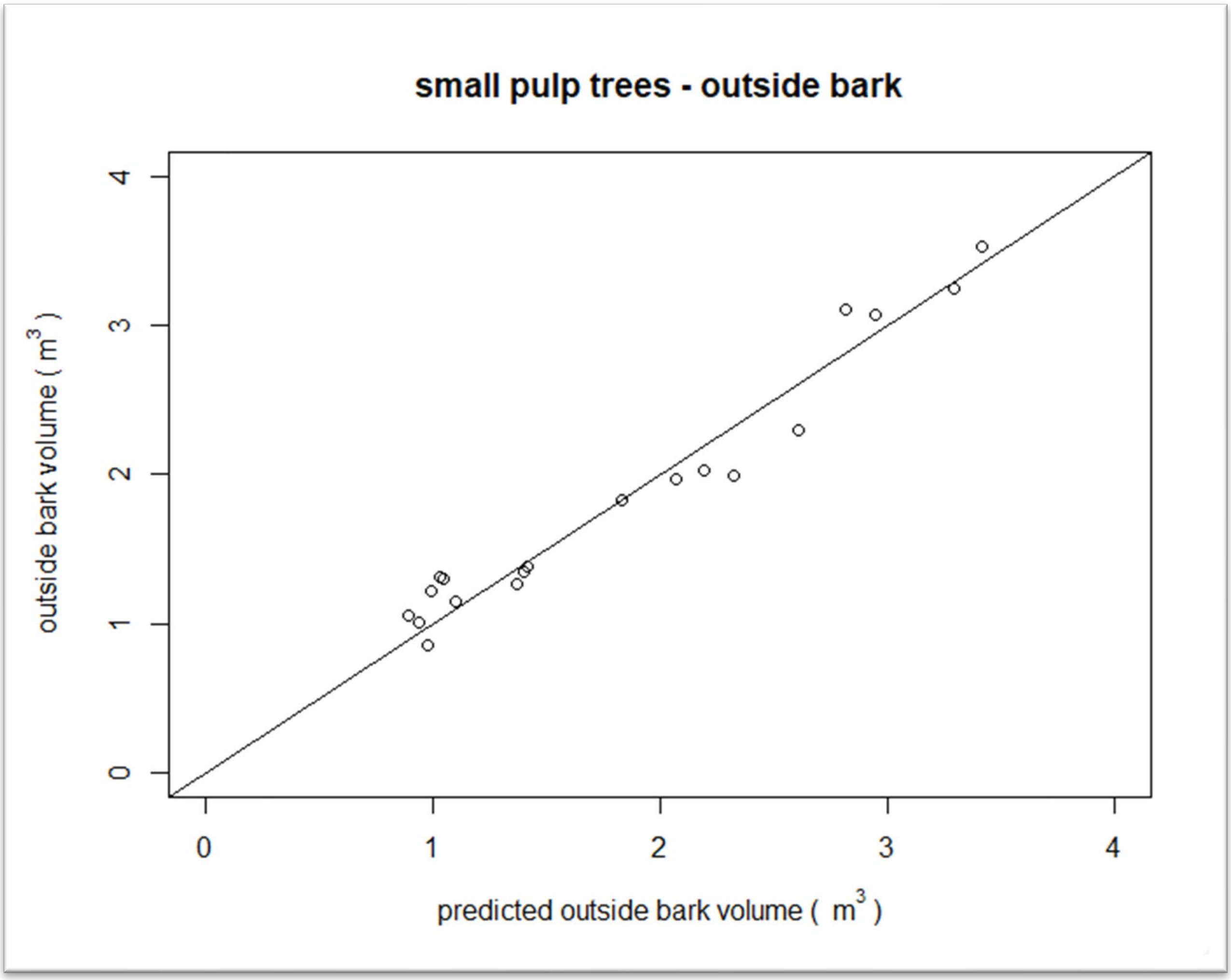


Model fits statistics are shown in Table A.3.1. The statistics show that the precision (as captured in R^2) is generally lower for MFTs where the trees contain merchantable branches. This likely reflects distortion of the stem near the top of the first log for trees with large branches or forks, which the DFH prediction model could not capture. This was worst for MFT = 1_0_1_0, which are pole-sized trees with pulp wood in branches, generally indicating a significant fork. The prediction model is least reliable for these species (see Table A.3.1). The species and MFTs with the least reliable models are the ones that it would be most efficient to measure DFH directly on.

C. Main Stem Volume

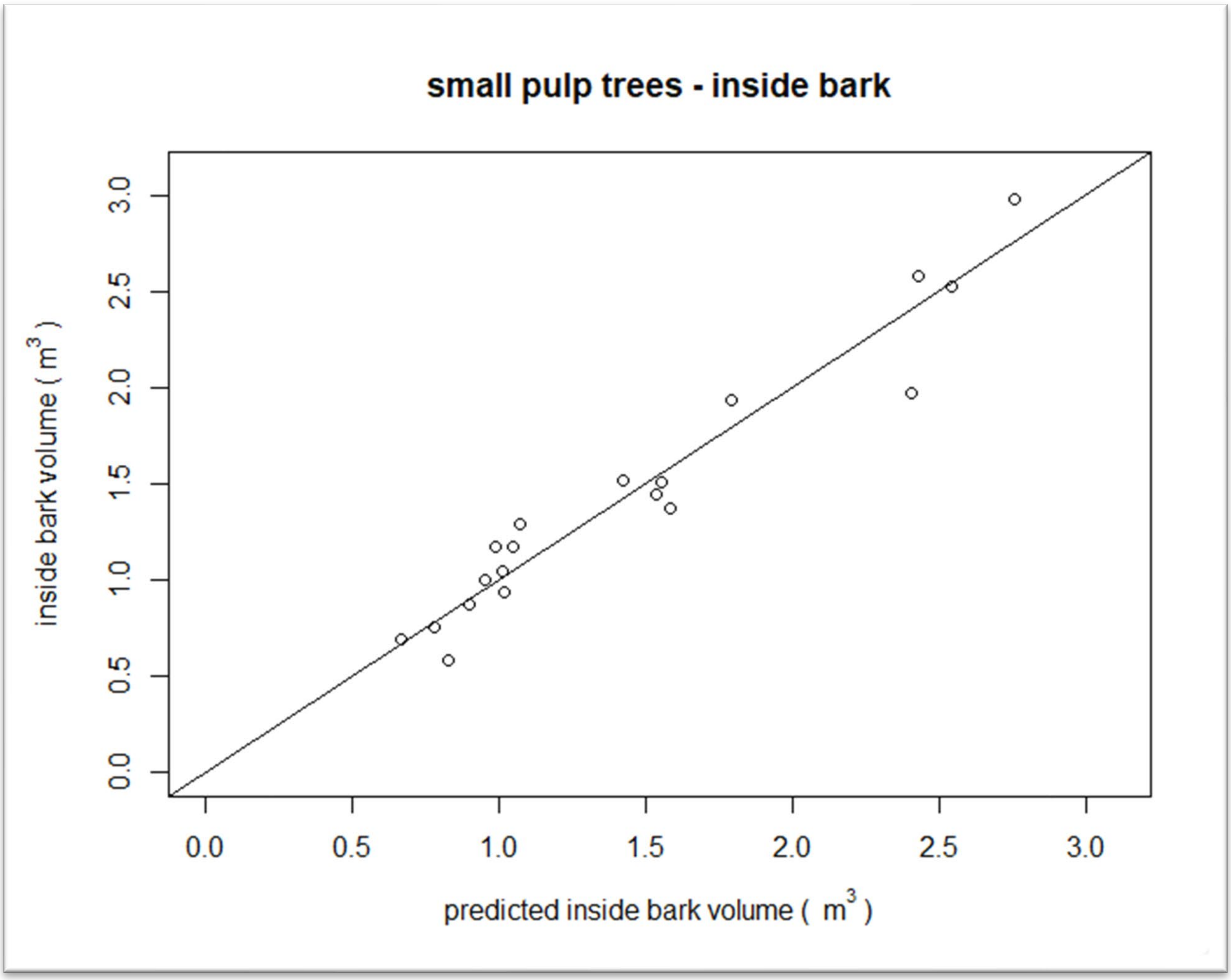
C.1. Small pulp trees- outside bark volume

There were very few trees in the data base with measured volume which qualified as ‘small pulp trees’, under the definition of the CSS model. This occurred because sample trees were mostly selected using variable radius (point) sampling.



The simple model (Eq, 2) listed under III.A.1, above, performed adequately for the 19 trees available (see figure below), with an $R^2 = 0.952$ and a MAPE = 0.092 indicating a strong correlation and reliable model.

C.2. Small pulp trees inside bark volume

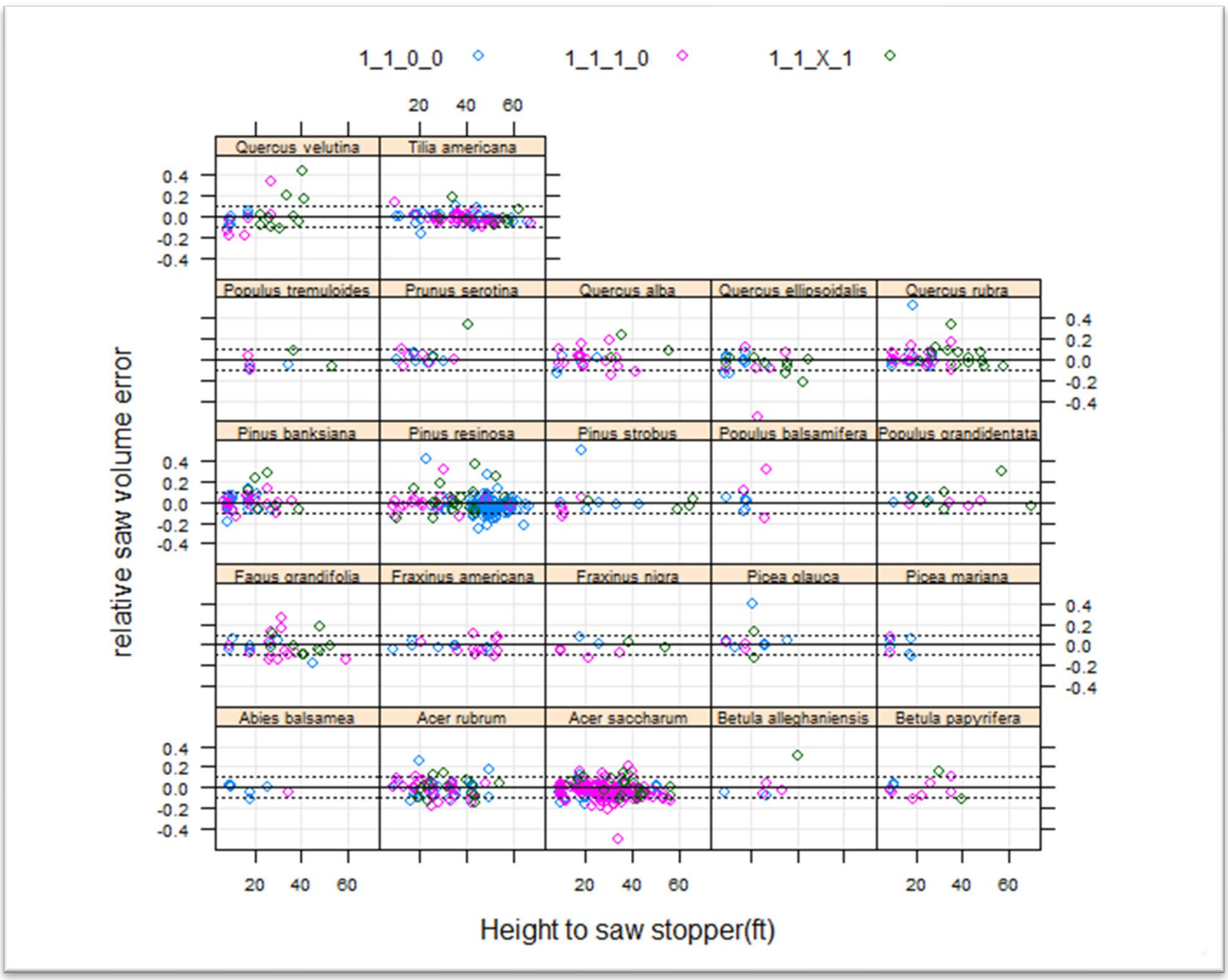


Small pulp trees proved to have relatively thicker bark, which is not unexpected for smaller trees, so they have separately estimated coefficients for predicting inside bark volume (Table B), regardless of species, because there were not enough observations to fit the small tree vib-vob relationship by species. The model worked well with an R^2 = 0.94 and no detectable bias (slope = 0.997)(see figure above). For 'one-pulp-stick' small trees, this model produces very similar results to that described by Gevorkiantz and Olsen (1955, their table 6) for 5 to 8" trees with one bolt.

C.3. Trees with stem volume estimated with the CSS-M taper model

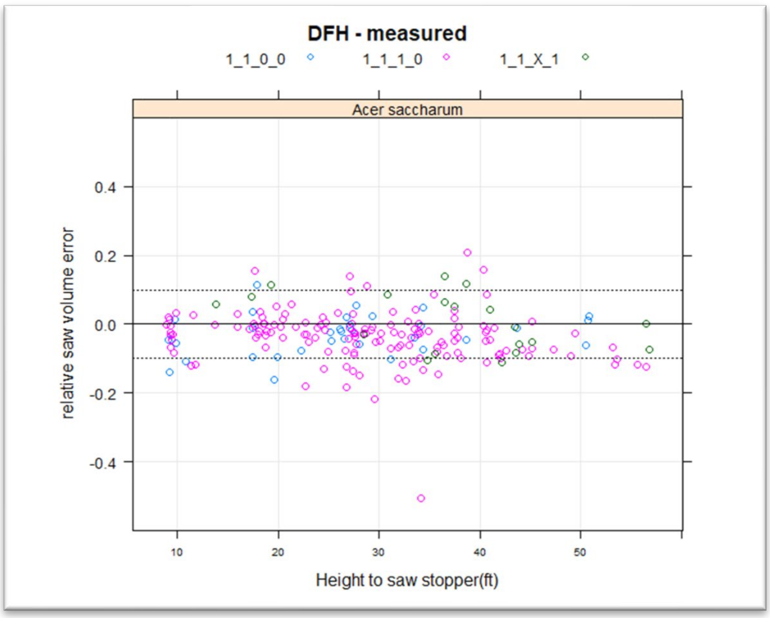
C.3.1. Saw timber-outside bark

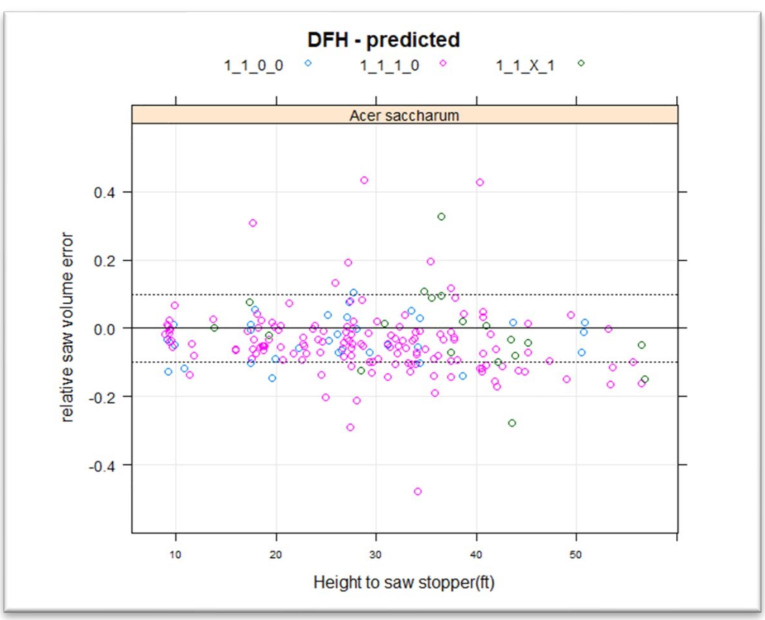
Sawtimber volume outside bark predicted from the CSS-M models were accurately predicted for the vast majority of destructively sampled trees, and robust across species and MFTs (see figure below) as indicted by fit statistics (see Table A.4.). As expected the models were most reliable for estimating saw timber volume in the main stems of trees without branches (MFT = 1_1_0_0) and had a lower accuracy for trees with pulp (MFT = 1_1_1_0) and saw sized (MFT = 1_1_X_1) branches, respectively (see MAPE values in Table A.4). Over all trees the R^2 = 0.97 and the MAPE = 0.063, the latter being somewhat inflated do to some highly unusually-formed trees (see outliers in figure below).



C.3.1.1. Using predicted rather than measured DFH for saw timber volume estimation

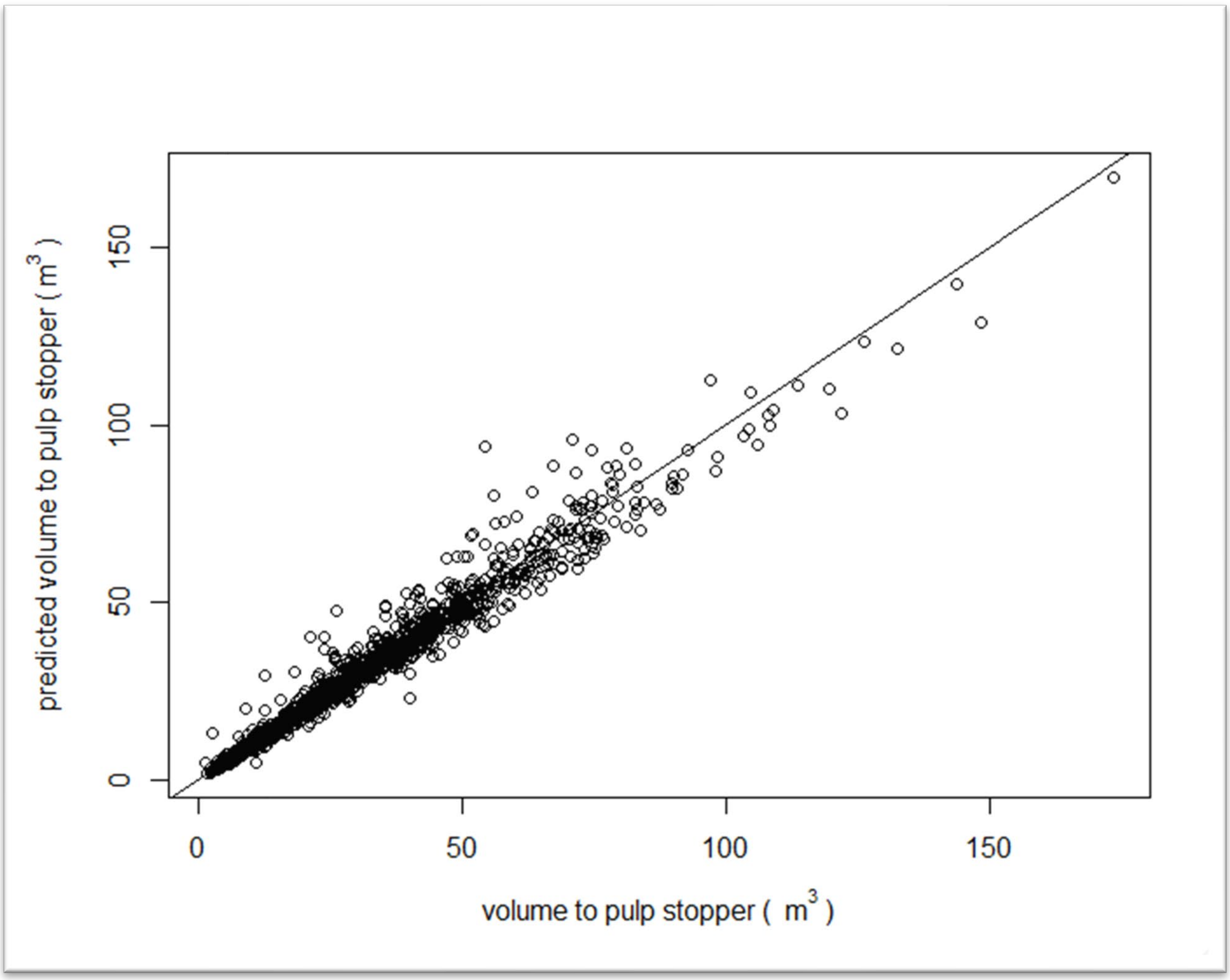
Using Eq. 5 to predict DFH caused a small reduction in precision and a modest decrease in reliability (R^2 = 0.95 decreasing from of 0.97 and MAPE increasing from 0.063 to 0.077, respectively) of stem saw volume prediction from the taper model. The top figure below shows the saw volume error (dotted lines ± 10%) for sugar maple with DFH measured and the bottom figure with DFH predicted. For most trees, the difference isn't large, but some trees with merchantable branches (MFT = 1_1_1_0 or 1_1_X_1) can have fairly-large errors, likely due to distortion of stem form near the top of the first (16') log.

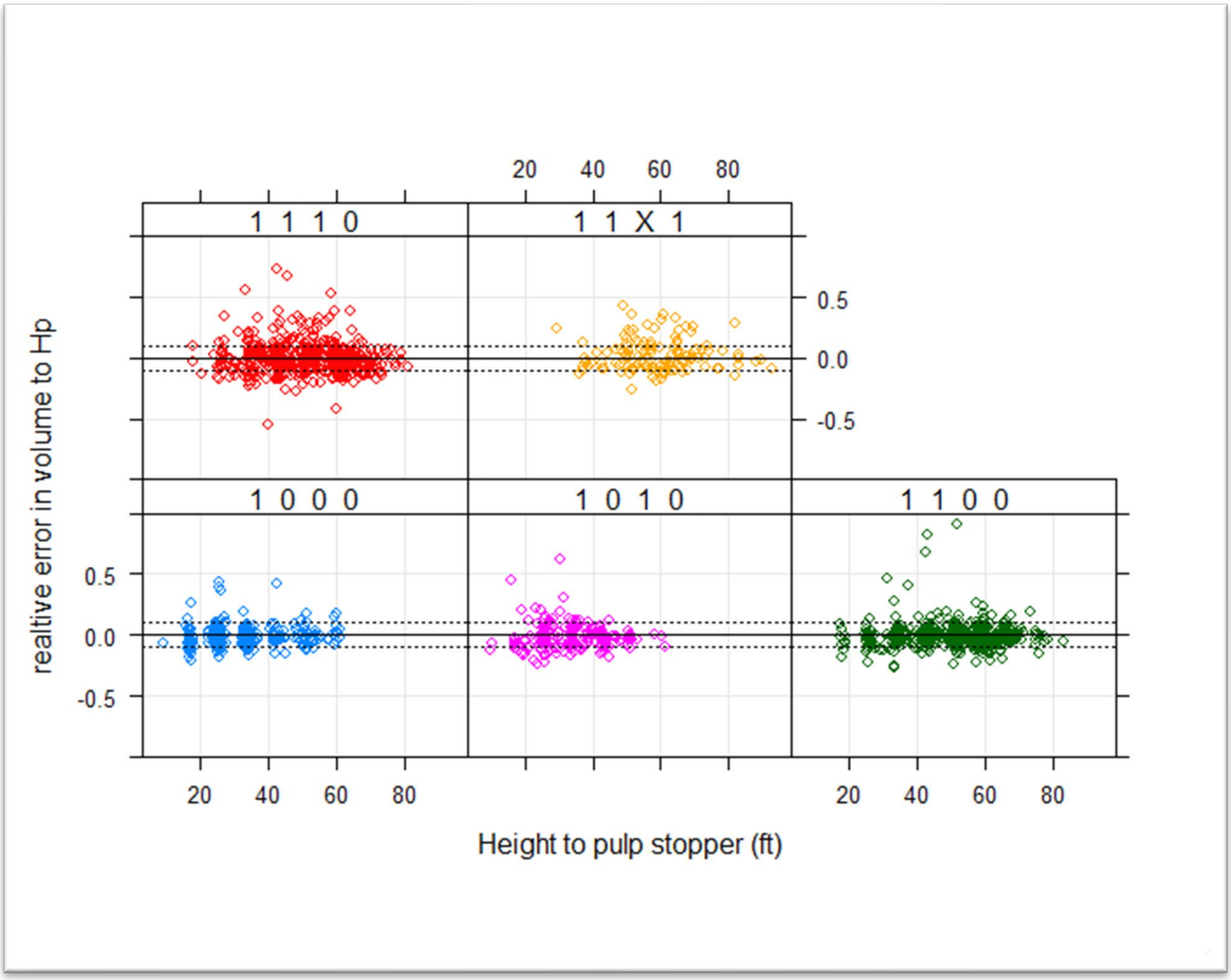




C.3.2. Pulp volume estimation with the taper model

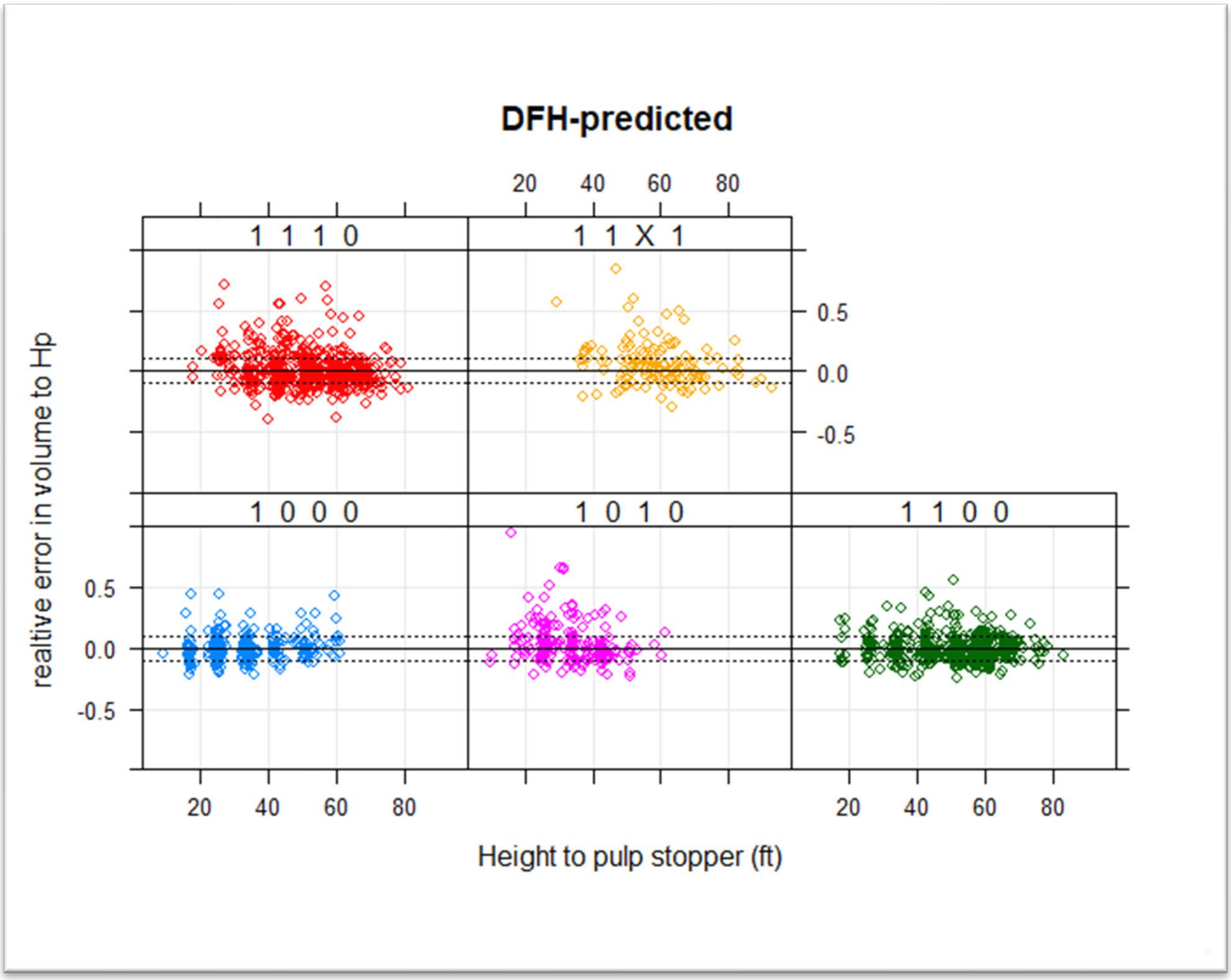
The CSS-M taper model (eq. 3) provided very good estimates of volume from stump to Hp for trees of a wide variety of sizes and species (top figure below), with no apparent bias across the different form classes or merchantable heights (see bottom figure below). The overall model had an $R^2 = 0.97$ and a MAPE = 0.078, indicating a reliable model. Model fit statistics by MFT and species are shown in Table A.5, showing the greatest reliability for MFTs without merchantable branches.





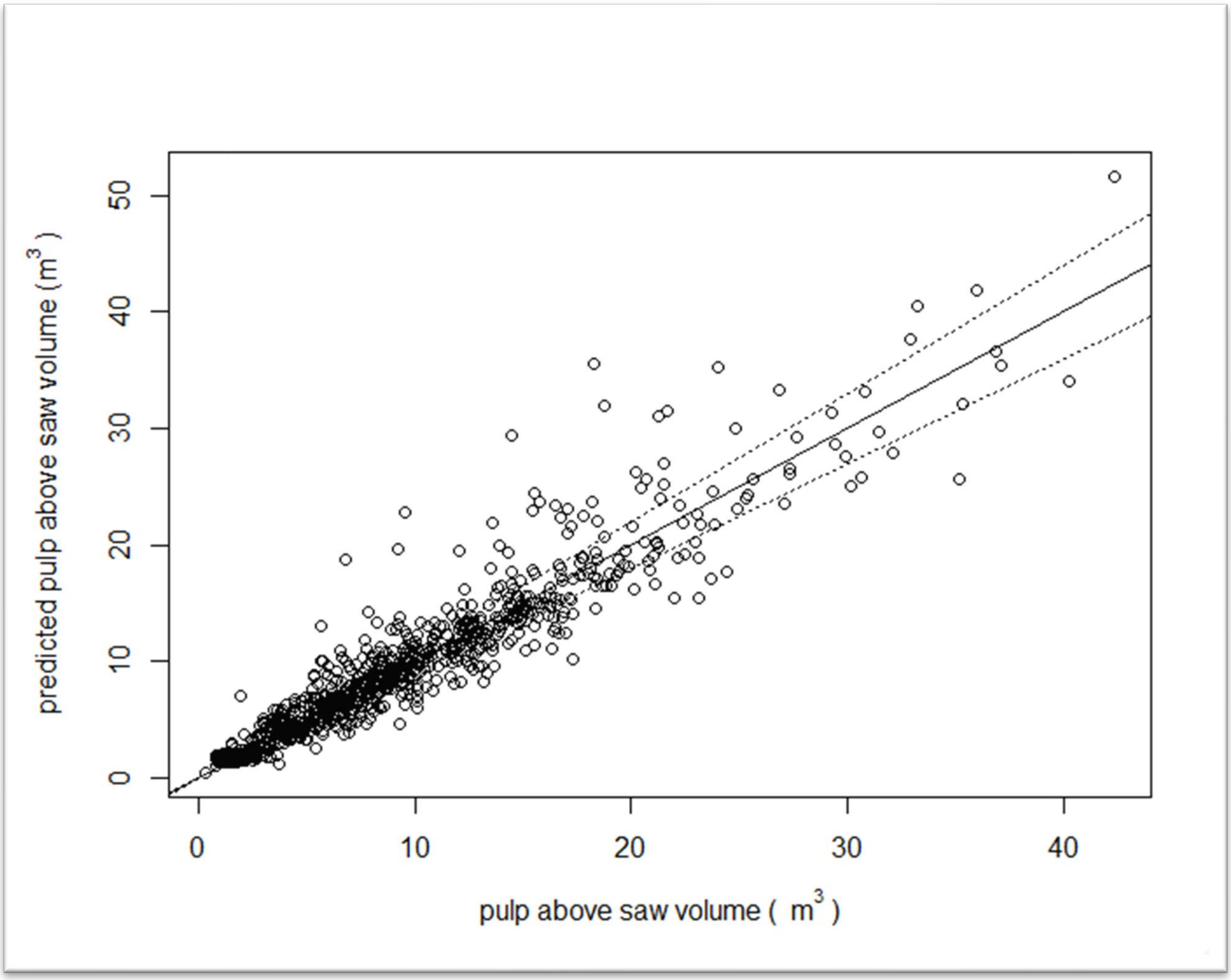
C.3.2.1. Using predicted rather than measured DFH for pulp volume estimation

Using predicted DFH instead of measured DFH for computing volume to Hp, caused a small change in the overall relationship between measured and predicted pulp volume, with R^2 = 0.957 from R^2 = 0.970, though with a relatively larger decrease in model reliability, from a MAPE = 0.078 to a MAPE = 0.100. This manifests as a relatively greater scatter around the zero-error line, with more trees outside of the ±10% relative volume error line (dotted line in figure above and below). On average, this means about a 2.2 % increase in pulp volume estimation error, when using predicted DFH, though more trees have much higher error margins than 10% with DFH- predicted (compare figs. above and below)).

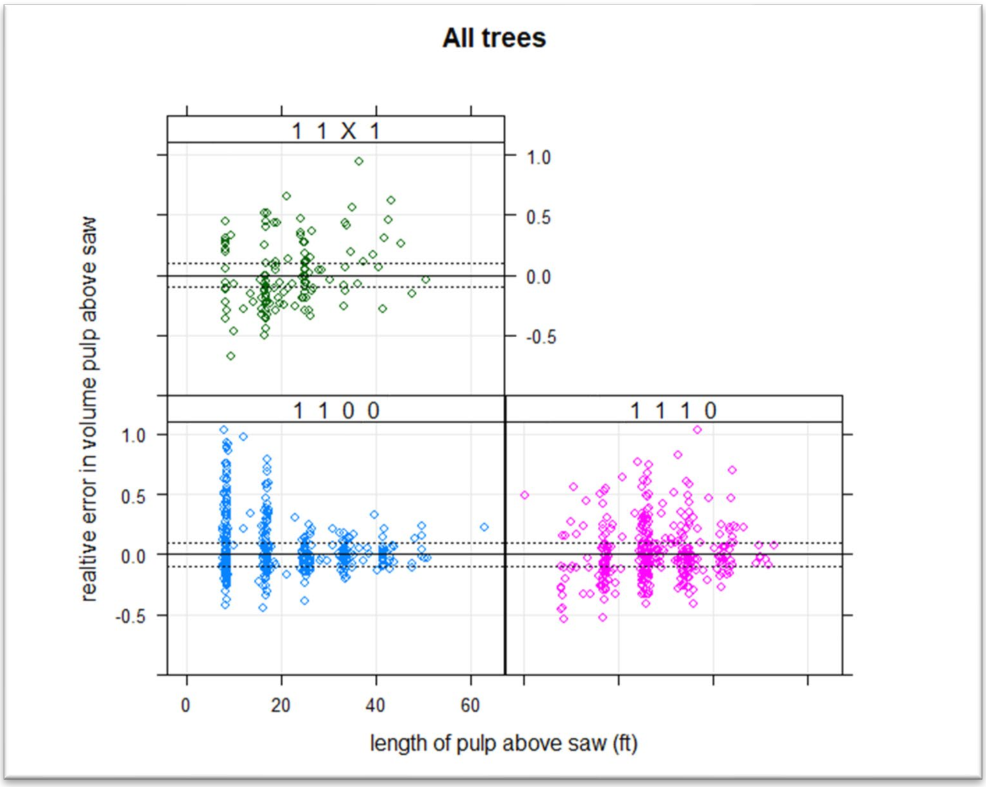


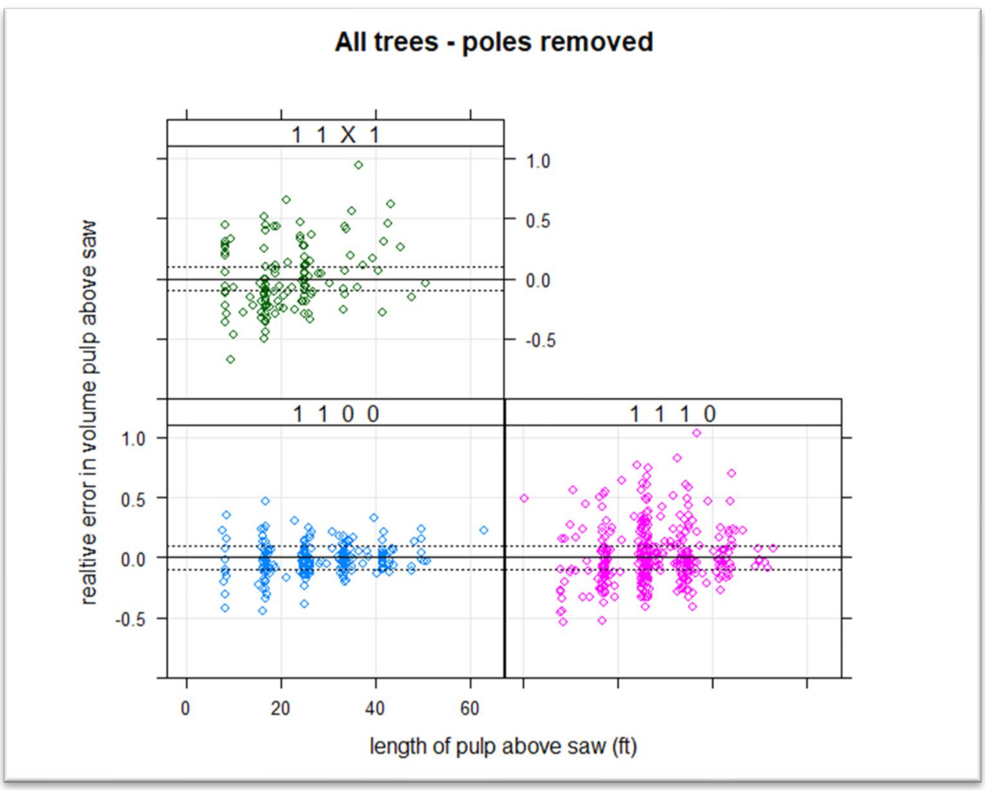
C.3.3. Pulp volume above saw volume

For trees with saw and pulp volume in the main stem (MFT = 1_1_X_X), potential pulp volume outside bark = dom.Hp.vob - dom.Hs.vob. When the eq. 3 was used to predicted pulp volume above saw volume (i.e., where L = Hs and U = Hp), the model performed less -well than it did for predicting either dom.Hp.vob or dom.Hs.vob, individually, across trees of differing MFTs (figure below) with an overall R^2 = 0.878 and MAPE = 0.190. The best results were for trees without branches (see Table A.6). The figure below shows that many trees are predicted outside of a ± 10% error margin (dotted linew, in fig below); the error margin is closer to ±20%. These results are not poor, however, and likely adequate for the timber inventory system (later described in comparisons in section E).



In relative terms, a fairly wide range of volume errors can be seen in the figures below, indicting imprecision in estimating pulp volume above saw volume. Analysis showed an overestimation bias (above 0 line in figures below) for some trees with an 8.3' or 16.3' pulp log, above the saw section of the main stem (top fig below). However, on closer investigation these were red pine trees from a destructive sample site where the predominant saw timber product were poles- the taper model had difficulty capturing the unusual upper stem form of trees of this type, which observations of trees of this type suggest tapered more rapidly above the "saw" section tops. As noted in a previous report based on these data (MacFarlane 2013), these red pine sample trees were not measured in the same manner as other sample trees used in this study. If these red pine "pole" trees are removed from the validation data, then this apparent bias goes away (bottom figure below). The CSS 4" top variant model does not have an additional parameter which might accommodate this upper stem taper. So, the model should be used with caution for estimating top pulpwood volume in red pine trees with long saw log lengths and short pulp log lengths above them, such as typically observed with trees used for pole products.

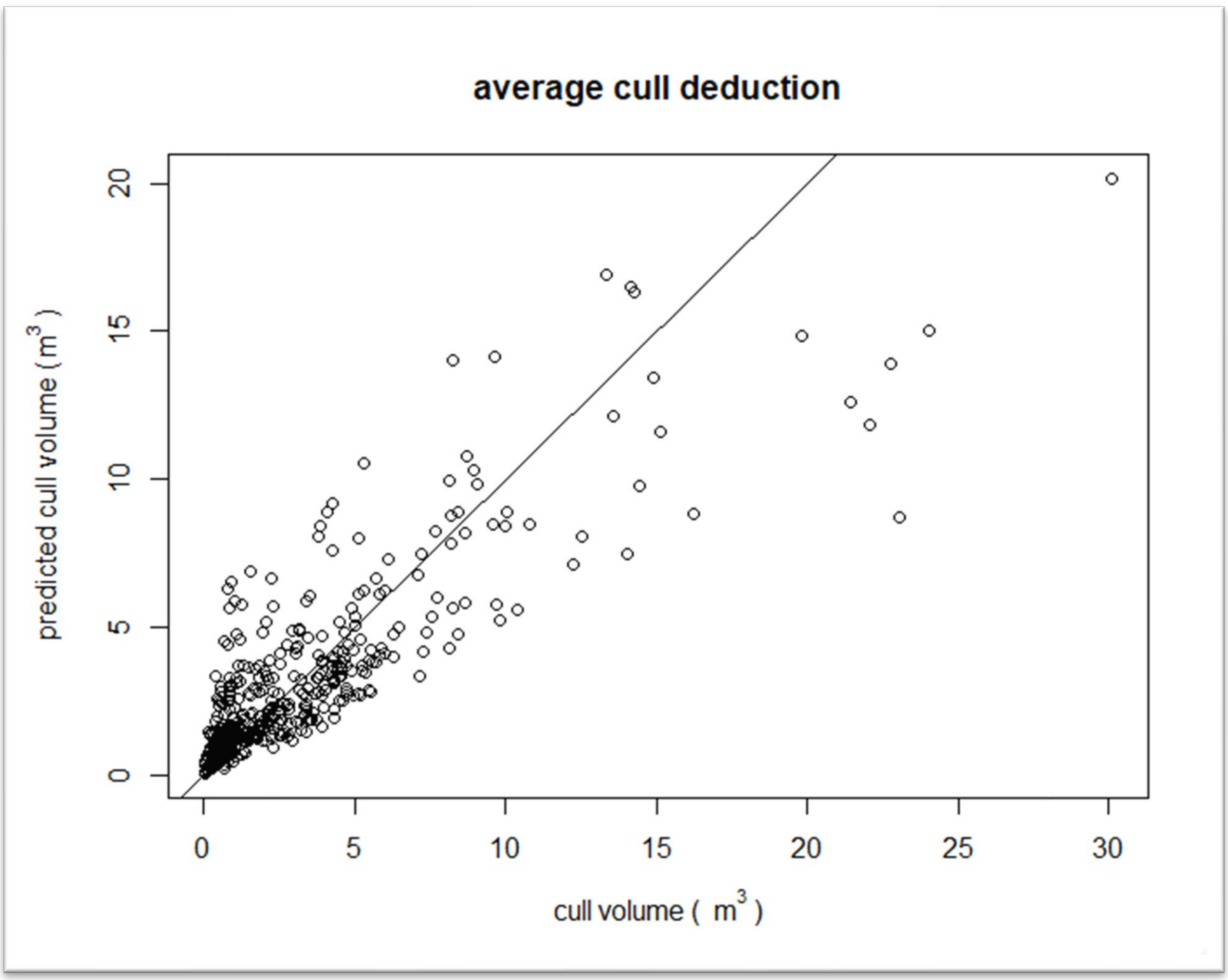

All trees
1 1 X 1
1 1 0 0
1 1 1 0
1.0
0.5
0.0
-0.5
0
20
40
60
realtive error in volume pulp above saw
length of pulp above saw (ft)

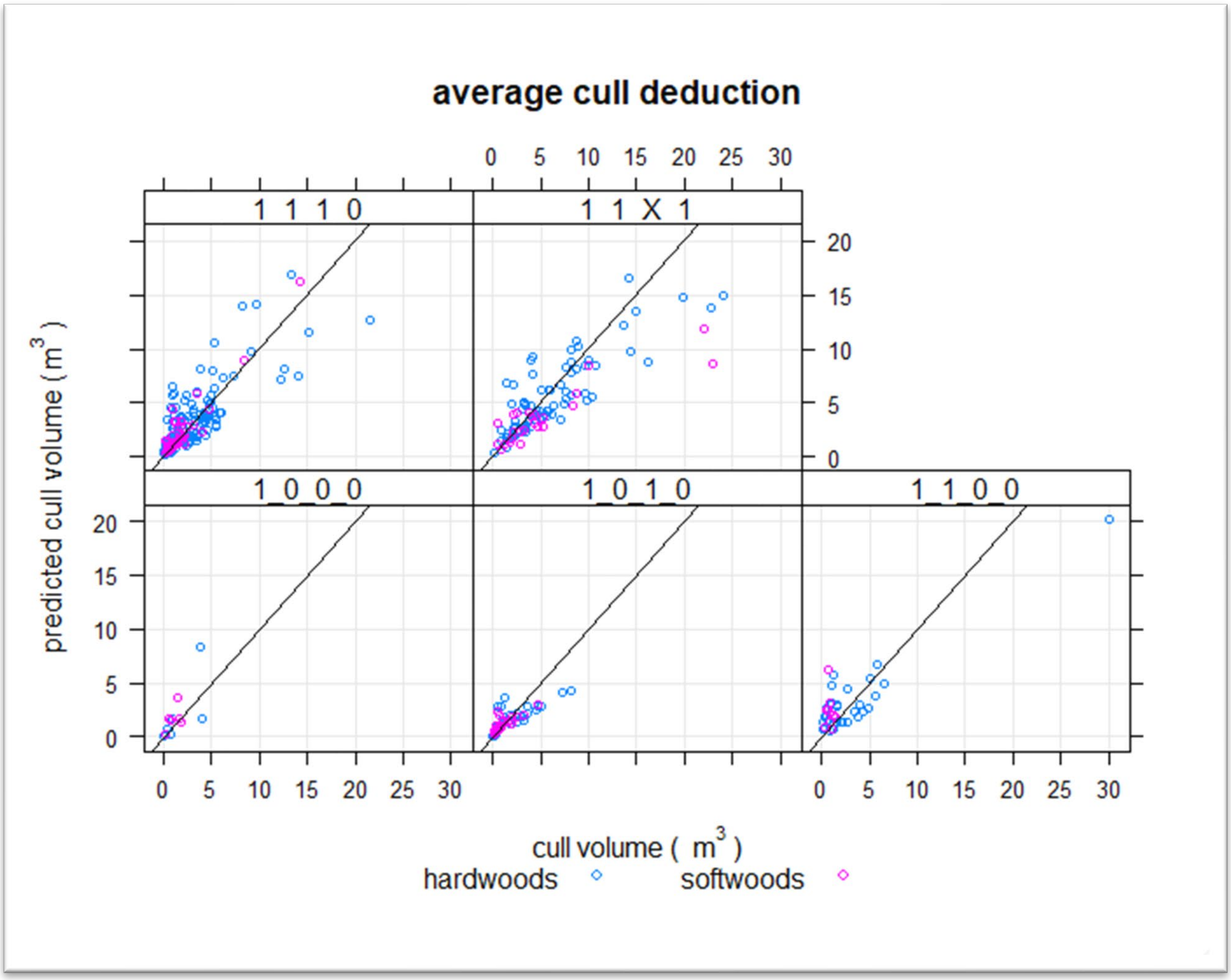

All trees - poles removed
1 1 X 1
1 1 0 0
1 1 1 0
1.0
0.5
0.0
-0.5
0
20
40
60
realtive error in volume pulp above saw
length of pulp above saw (ft)

C.3.4. Cull Volume Deduction

In this system, cull volume is computed where the total cull length is recorded and then used to compute a liberal (eqs. 7a & 9a) and conservative (eqs. 7b and 9b) estimate of cull volume for trees, from the taper model. These are averaged to get an answer in between (eqs. 7c and 9c).

This method of estimating cull volume from the taper model proved to be reasonably precise (R^2 = 0.72), but unreliable, especially for tree with merchantable branches (Table A.7; MFTs = X_X_1_1), where cull distribution was less predictable and the taper model itself less reliable for describing the stem form of these trees. Looking at all tree types, predicted versus observed cull volumes looked reasonably good (top figure below), but there were some noticeable biases for trees of certain form types (bottom figure below). It is also worth noticing that the majority of trees with cull were trees with merchantable branches.





Alternative approaches for cull estimation with the taper model were examined, which instead assigns all cull to the mid-point of the stem:

Eq. 7d where U = Hs - (Hs - SH)/2 + LDSC/2 and L = SH + - (Hs - SH)/2 - LDSC/2

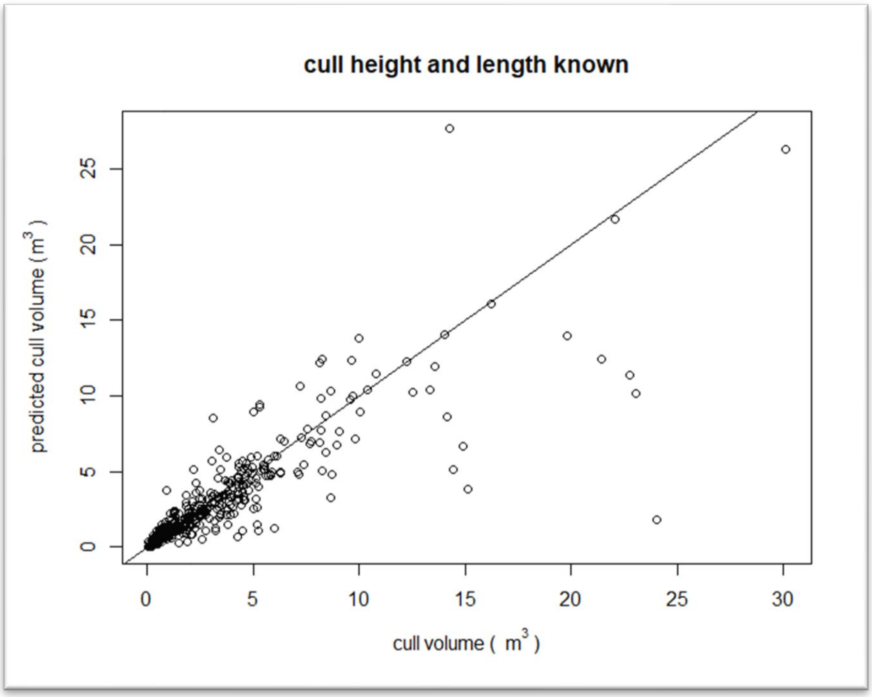
Eq. 9d where U = Hp - (Hp - SH)/2 + LDPC/2 and L = SH + - (Hp - SH)/2 - LDPC/2

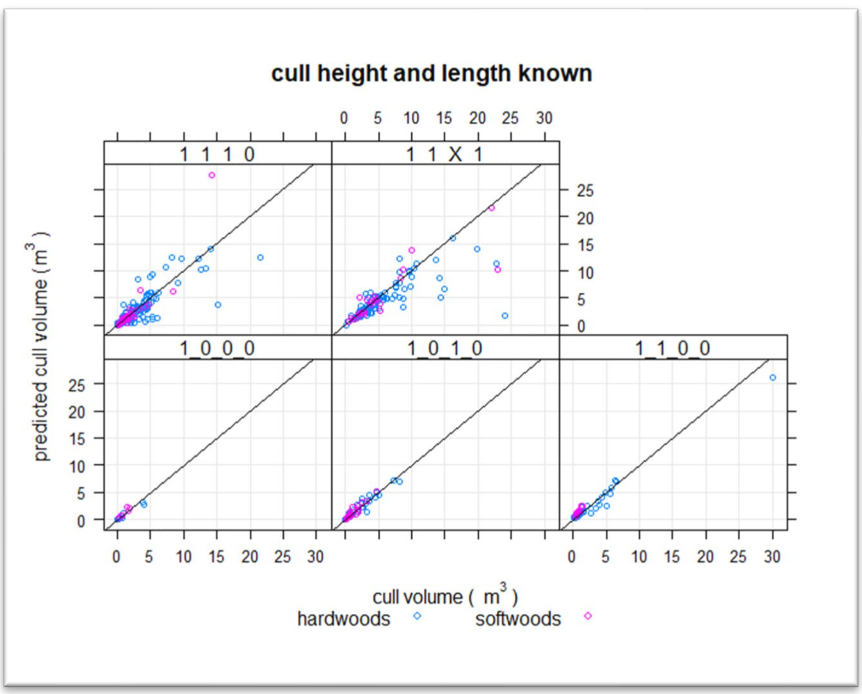
These approaches provided no important difference in predictive power, so another set of alternatives were examined, but these require the measurer to record the height to the top of the highest cull (H.cull) along with the total length of cull:

Eq. 7e where U = H.cull and L = H.cull – LDSC

Eq. 9e where U = H.cull and L = H.cull – LDPC

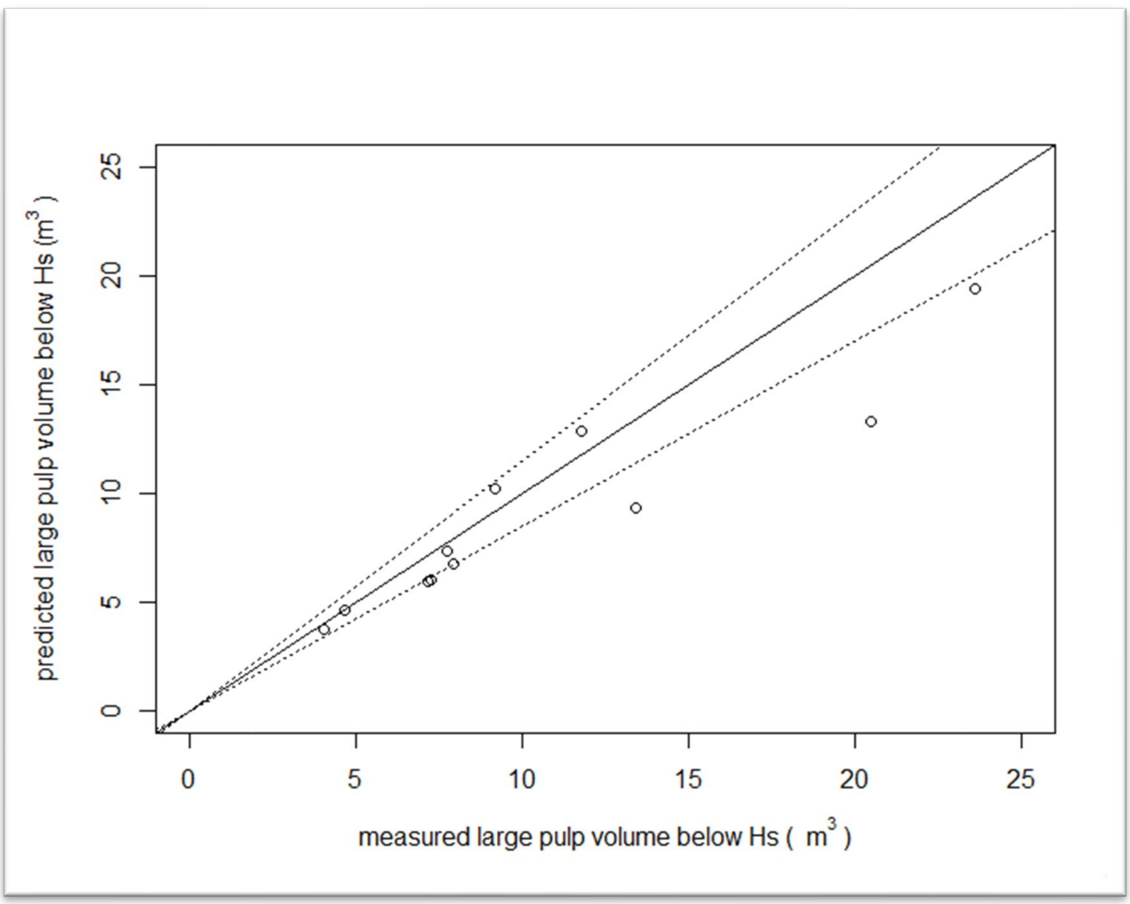
The results show that if the height to the top of the cull and cull length are known, the accuracy of cull volume prediction increases dramatically, except for trees with very large cull volumes (see top figure below). However, this method shows very good results across different MFTs, with most of the error for trees with MFT = 1_1_X_1. (see bottom figure, below). So, if the DNR wants to improve cull estimation accuracy, it should add H.cull as an additional measurement.





C.3.5. Large Pulp Volume Deduction

In this system, "large" pulp volume, which are saw log-sized sections that have been downgraded to pulp can occur both above and below saw logs in the main stem. However, saw logs above Hs are computed as "pulp volume above saw volume" (see C.3.2, above), so the model only needs specific equations to deduct the large pulp below Hs (Eqs. 8a, 8b and 8c). The validation data set contained 627 trees with at least one log determine to be a "large" pulp log, of which 595 had large pulp above the last saw log (Hs), but only 12 had large pulp volume only below Hs. For these 12 validation trees, the large pulp model performed reasonably well, except for the three largest trees, where it tended to underestimate large pulp volume (see figure below). These underestimates were caused because there was only one pulp log in the saw portion of their stems (i.e., below Hs) and there was at least one saw log above it, so the conservative estimate (Eq. 8b) was too conservative. These three trees were all MFT = 1_1_X_1 and had considerable cull in the lower stem and large branches. Examining the differences between observed and measured values the R^2 = 0.79 and the MAPE = 0.159, indicting a model that could get within 15% of the true value for most trees (dotted line, figure below).

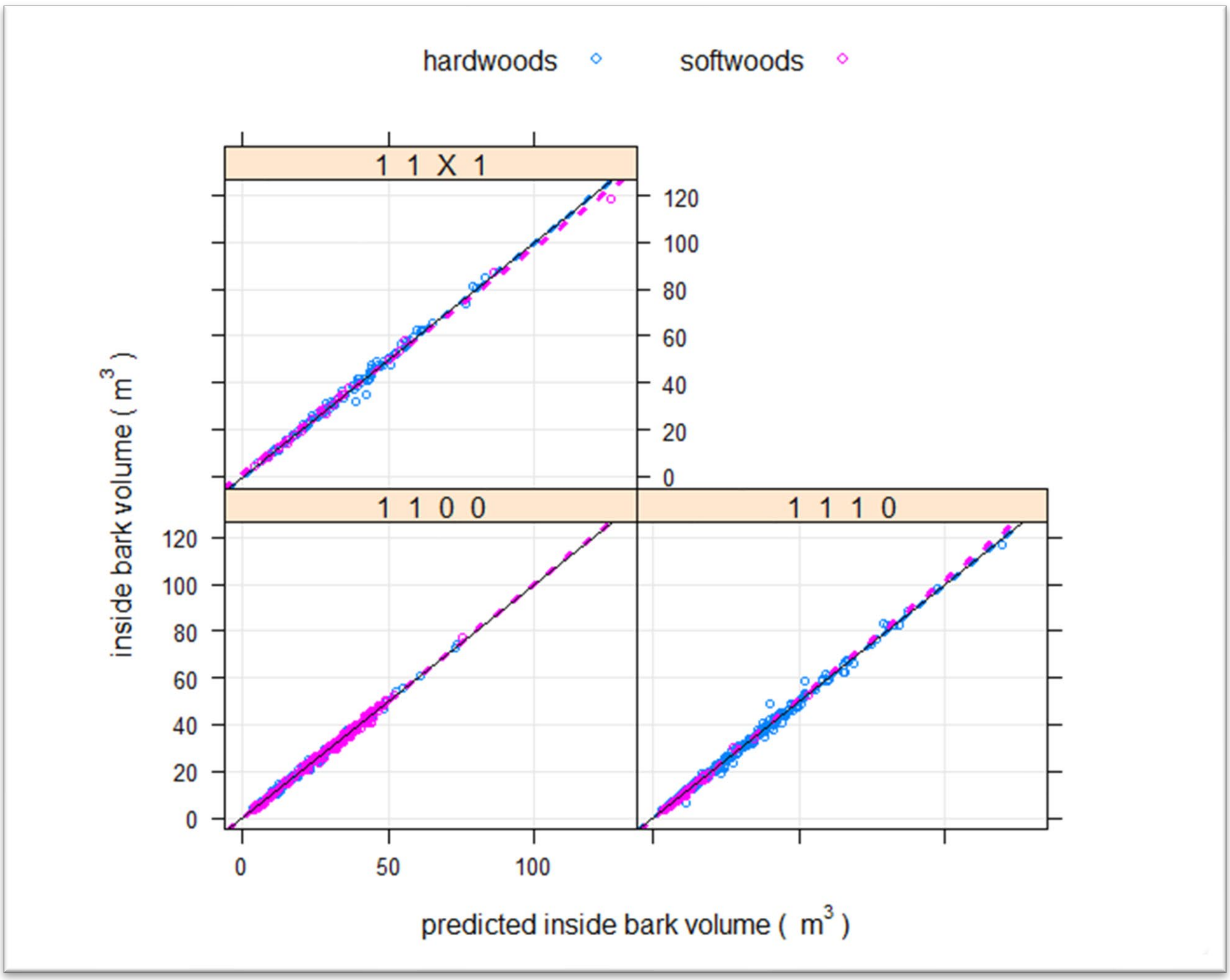


C.3.6. Volume Inside Bark

After outside bark volumes are predicted for various portions of the tree these are converted to inside bark volumes by the inside to outside bark equations (eqs. 13a & 13b).

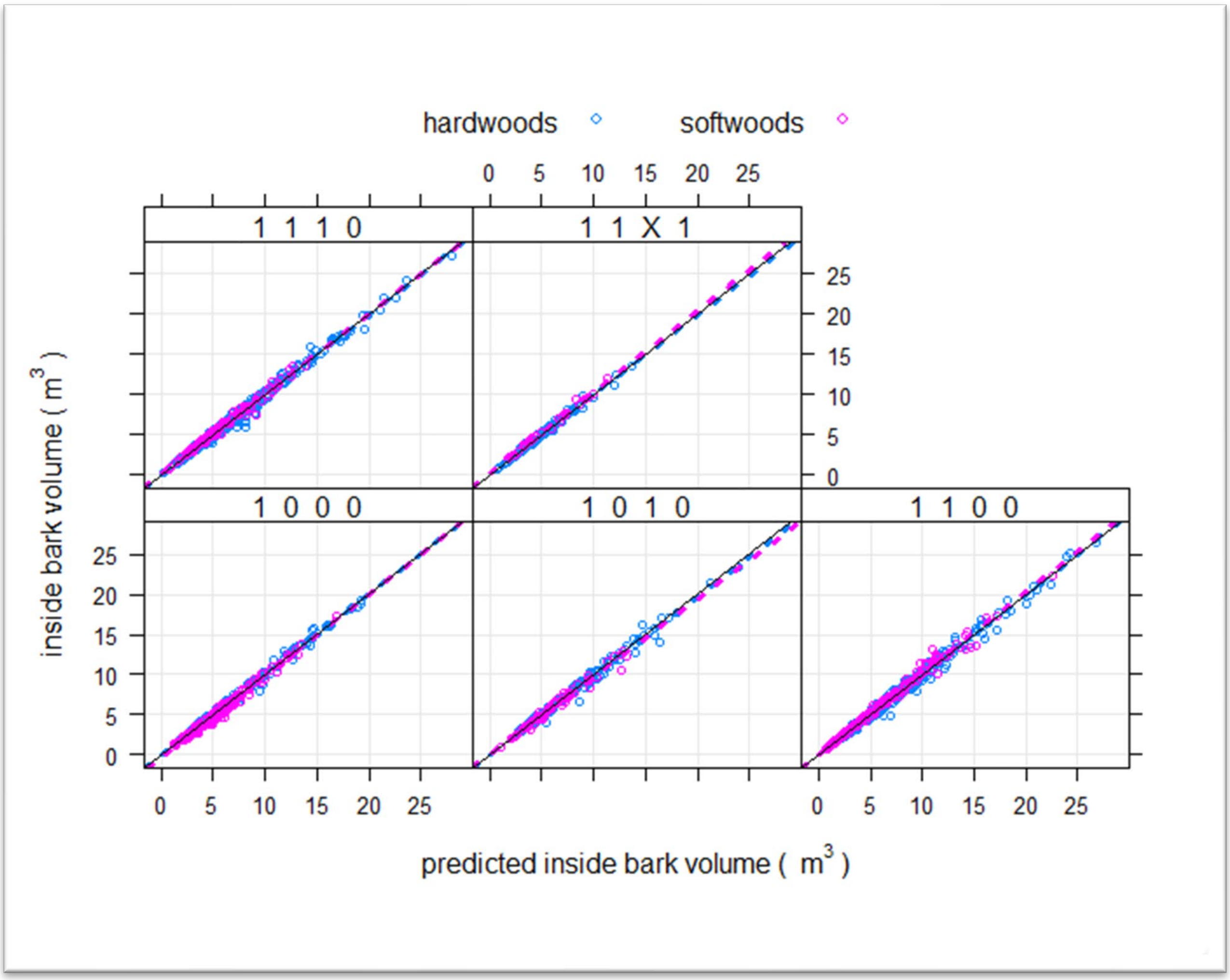
C.3.6.1. Saw volume inside bark

Sawtimber volume inside bark predicted from the outside bark equation was highly accurate across all trees of all MFTs (see figure below), with an $R^2 > 0.995$ and a MAPE of 0.035. So, it is expected that translating from outside to inside bark volume will introduce very little error into saw timber volume estimation from the taper model. Error statistics by MFT and species are shown in Table B.1.



C.3.6.2. Pulp volume inside bark

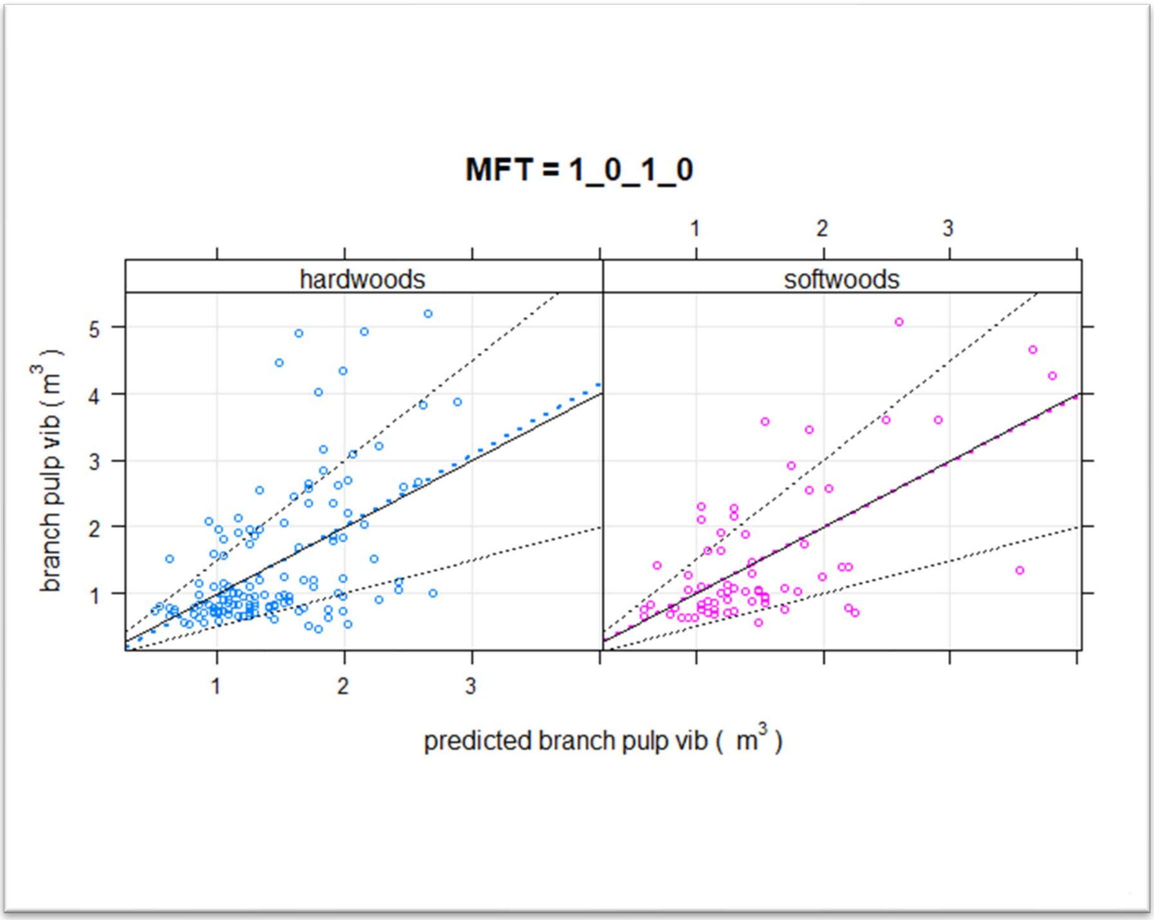
Pulp volume inside bark predicted from the outside bark equation was highly accurate across all trees of all MFTs (see figure below), with an $R^2 > 0.990$ and a MAPE of 0.047. So, it is expected that translating from outside to inside bark volume will introduce very little error into pulp volume estimation from the taper model. Error statistics by MFT and species are shown in Table B.2.



D. Branch volume

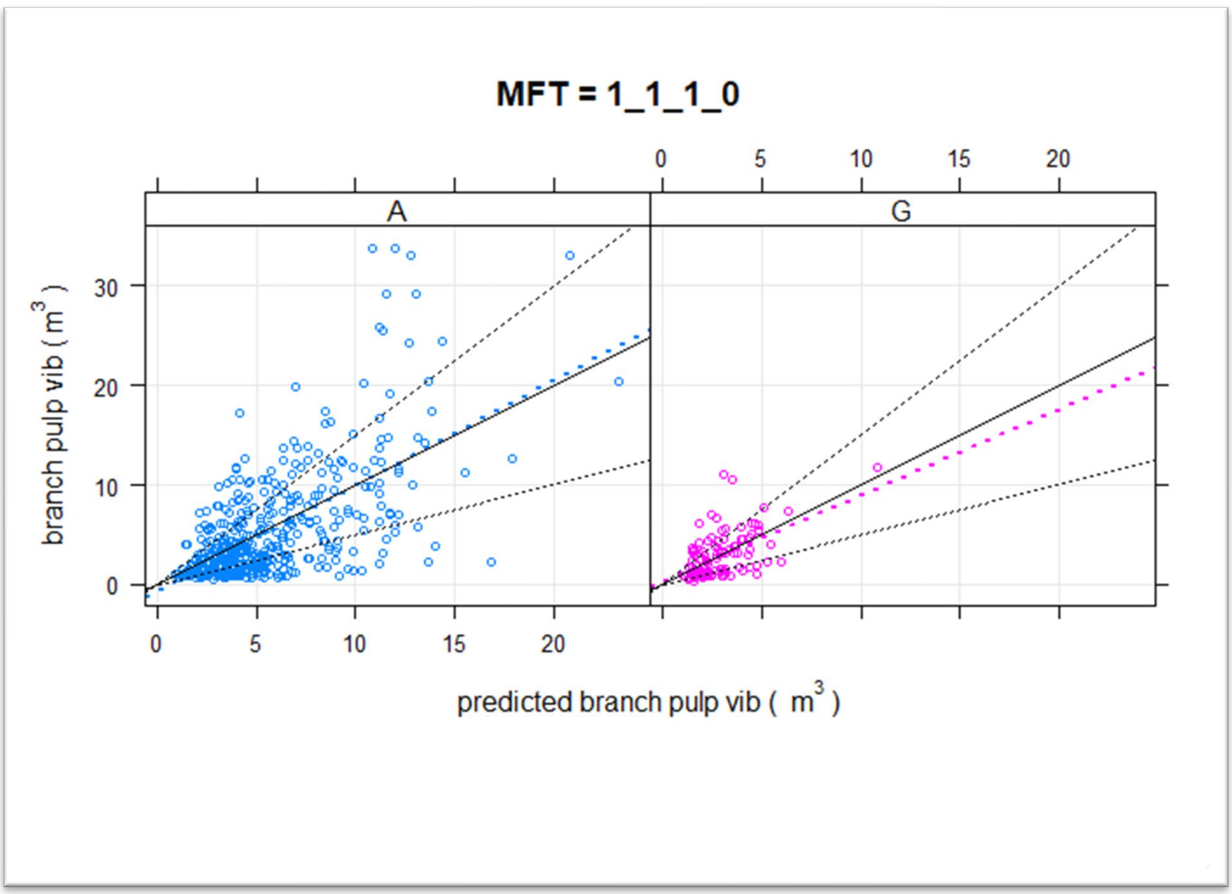
D.1. Branch pulp wood volume for trees with a MFT = 1_0_1_0.

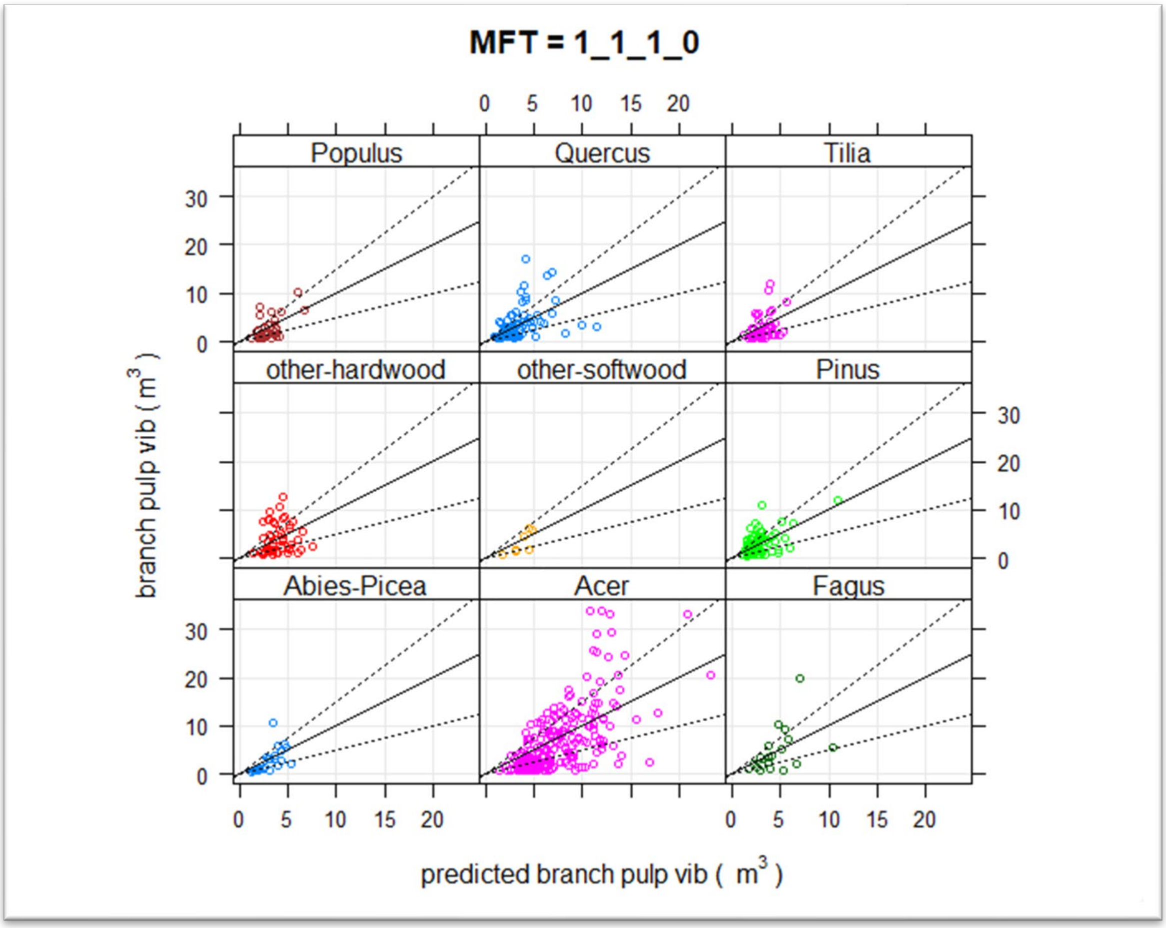
The models for predicting branch wood volume for pole-sized trees with pulp in branches (MFT = 1_0_1_0) had a generally high degree of imprecision, likely because the only significant predictor used was DBH. The overall $R^2 = 0.324$, so about 2/3 of the variation remained unexplained by the model. The MAPE = 0.515 indicating branch volume prediction was unreliable from tree to tree, though most trees were within ± 50% of the true value (see figure below). The model performed better for softwoods (Eq. 15a) than hardwoods (Eq. 15b) (see Table C). One positive aspect is that the model showed no significant bias across the range of predicted values (see figure below), so, while tree to tree predicted volume is highly imprecise, this should not lead to generally biased estimates of pulp volume in branches for pole-sized trees over large sample populations (this was confirmed in section E, below). MacFarlane (2013) showed that recording the height to the base of the first merchantable branch (a.k.a., height to base of merchantable crown) would allow for improved branch volume prediction, if the DNR wants to increase precision of branch volume estimation, beyond what is available from these models.



D.2. Branch pulp wood volume for trees with MFT = 1_1_1_0 or X_1_1_0.

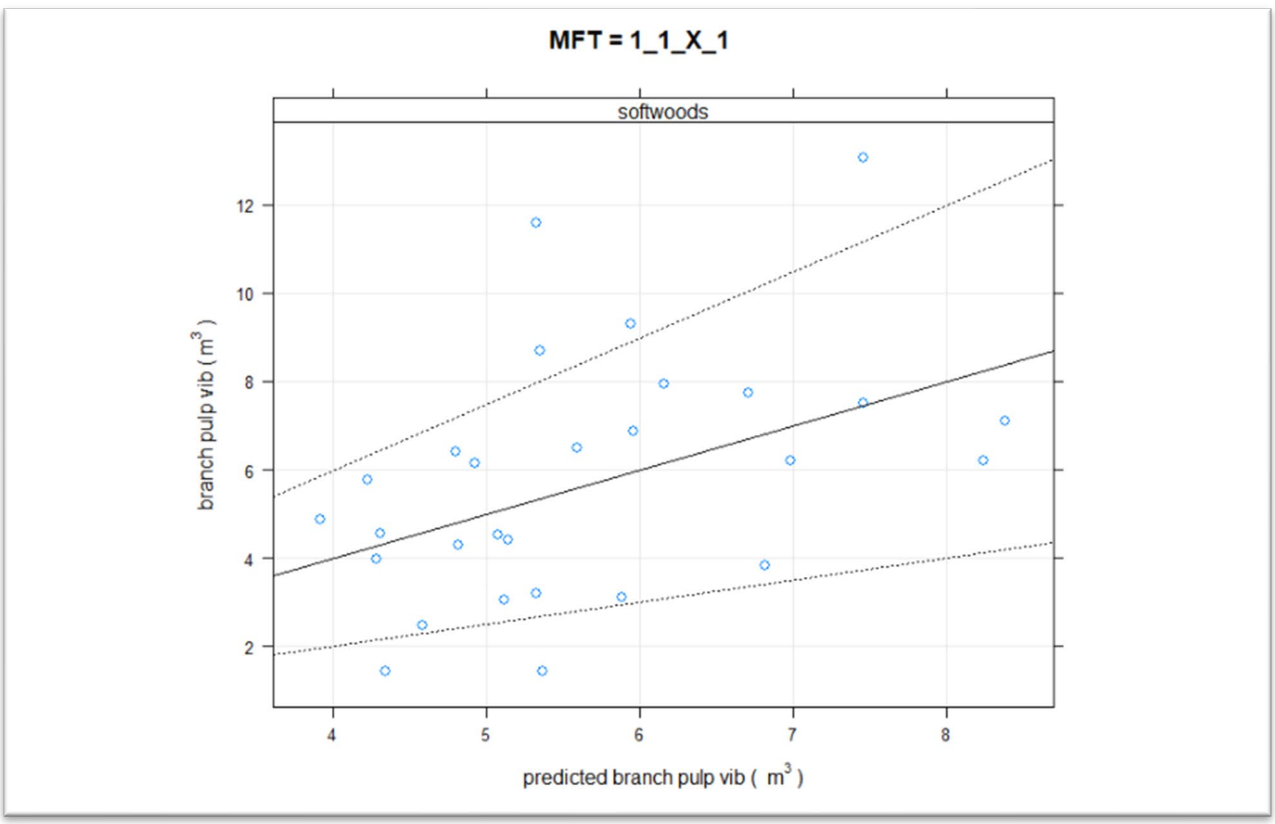
The models for predicting branch wood volume for saw-sized trees with pulp in branches had a high degree of imprecision, because the only significant predictor used was DBH. The overall R^2 = 0.419, so about 60% of the variation remained unexplained by the model. The MAPE = 1.011, indicating branch volume prediction was unreliable from tree to tree, with a significant number of trees outside of ± 50% of the true value (see upper figure below). The model performed better for softwoods than hardwoods (see Table C, below) and grouping similar species together improved the overall prediction of the model, in most cases (see Table A.8, see lower figure below). The model showed very little bias across the range of predicted values (see upper figure below), so, while tree to tree predicted volume is highly imprecise, this should not lead to generally biased estimates of pulp volume in branches for pole-sized trees over large sample populations (see also section E, below). MacFarlane (2013) showed that recording the height to the base of the first merchantable branch (a.k.a., height to base of merchantable crown) would allow for improved branch volume prediction, if the DNR wants to increase precision of branch volume estimation, beyond what is available from these models.



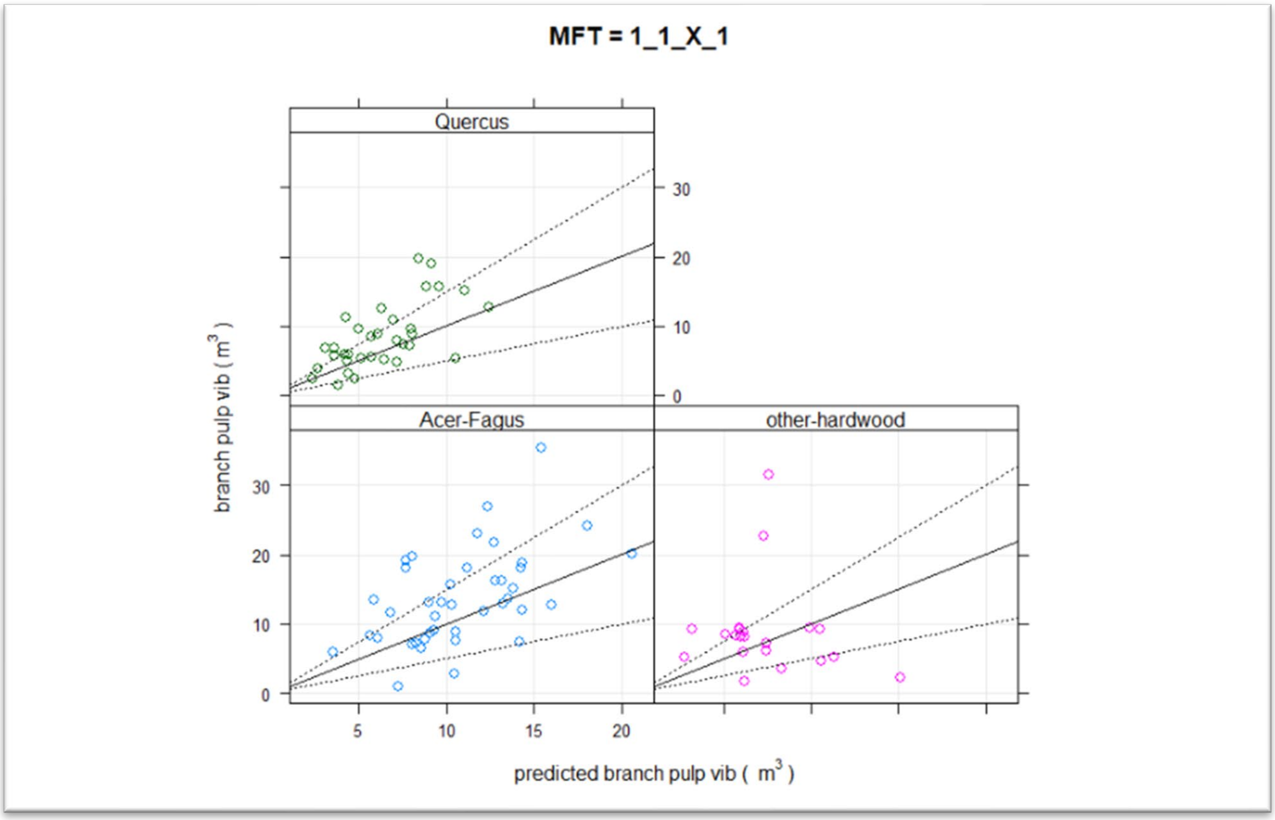

MFT = 1_1_1_0
Populus
Quercus
Tilia
other-hardwood
other-softwood
Pinus
Abies-Picea
Acer
Fagus
branch pulp vib (m^3)
predicted branch pulp vib (m^3)

D.3. Branch pulp wood volume for saw-sized trees with saw-sized branches; MFT = 1_1_X_1 or 0_1_X_1.

The model for softwoods (Eq. 17a), gave generally poor predictions of the branch pulp volume (see figure below and Table C), with R2 = 0.195 and a MAPE = 0.468, though without any statistically significant bias (dotted line, fig below)). This high degree of imprecision reflects both the limited scope of this data and large range of branch pulp logs which can be found in softwoods with branch saw logs, which are generally trees with major forks. The value estimated are essentially random within a reasonable range (within a mean range of about ± 47%), though for a few trees the value were almost double or half of the measured value (figure below, outer dotted lines at 50% and 150%). Overall, this model is not very reliable at the individual tree level, but should give reasonable values for populations of trees within the size range of the trees. Analysis suggested that these models could only be improved by collecting additional data of trees of this type to better characterize the amount of pulp logs in large branches in softwoods. Given the relative rarity of trees of this type in the validation study (MacFarlane and Weiskittel 2016), the error in these models should not contribute a large amount to the error to a timber volume inventory from trees of this type (Section E below).

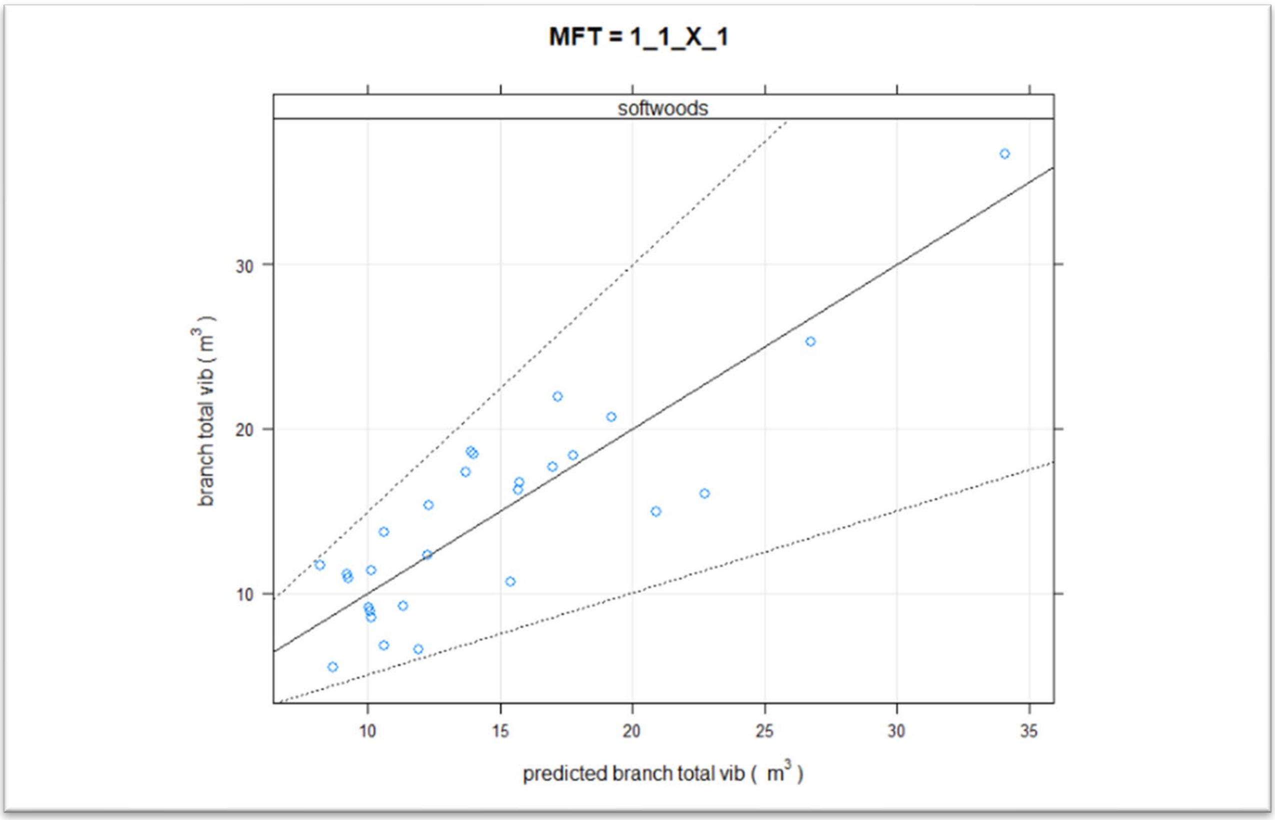


The hardwood model for predicting the pulp portion of total branch wood volume (Eq. 18a) gave generally poor predictions of the branch pulp volume (see figure below and Table C), with R2 = 0.16 and a MAPE = 0.537 (Table C and figure below). The model worked best for Quercus spp. and worst for the "Other Hardwoods" group (Table A.9). The issue for model predictions was that there was a very high degree of variation from tree to tree. In general, the reliability of the model for an individual tree was quite low, though the volume could be estimated with ±50% of the true value for most of the sample trees (see figure below). The best way to improve this model would be to add some reference to the relative size of the crown or branch log counts, in addition to the DBH (see MacFarlane 2013).

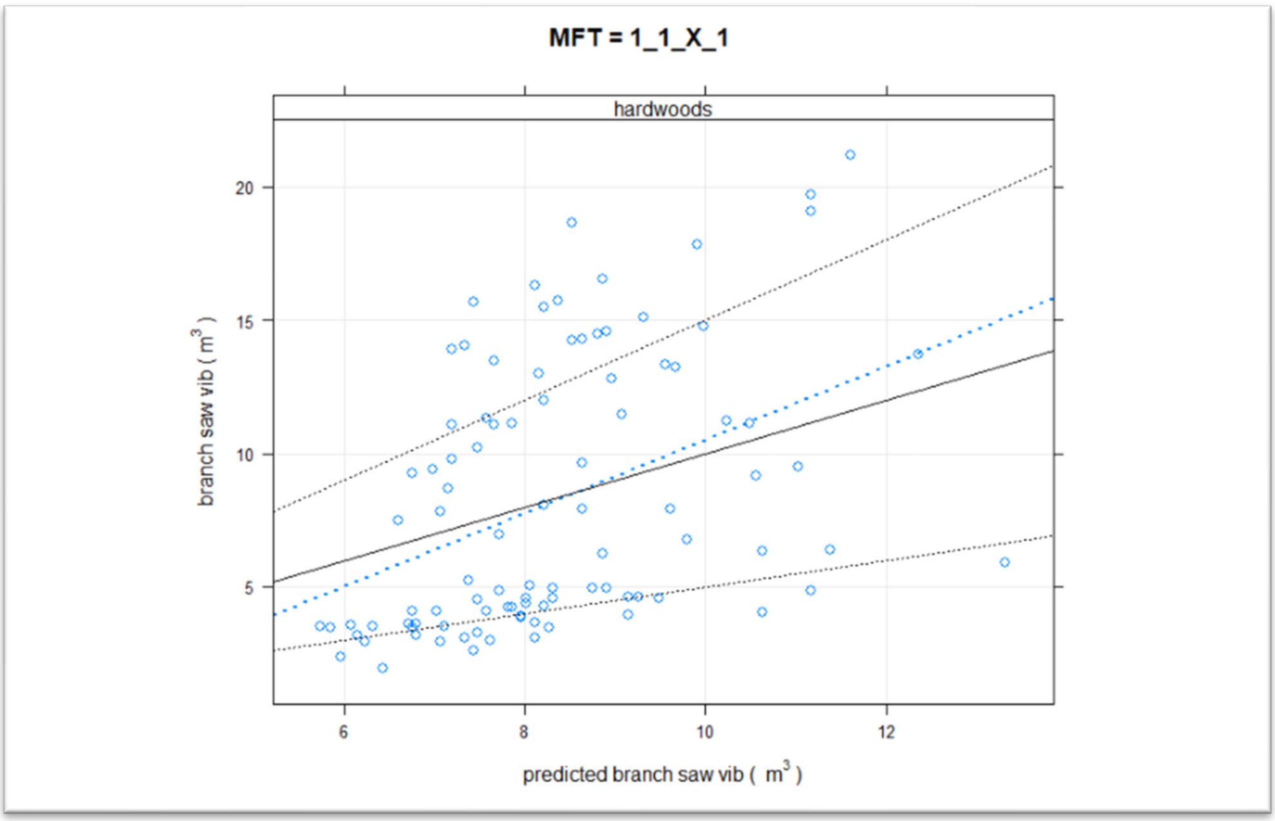


D4. Branch saw wood volume for saw-sized trees with saw-sized branches; MFT = 1_1_X_1 or 0_1_X_1.

The model (Eq. 17b) for predicting branch saw volume for softwoods, fit the limited data well with a R2 = 0.697 and a MAPE = 0.404 (see Table C). The model appeared unbiased across the range of predicted values (color dotted line figure below; outer lines are at 50% and 150%), though the sample size was not very large, especially for trees with larger branch saw volume.

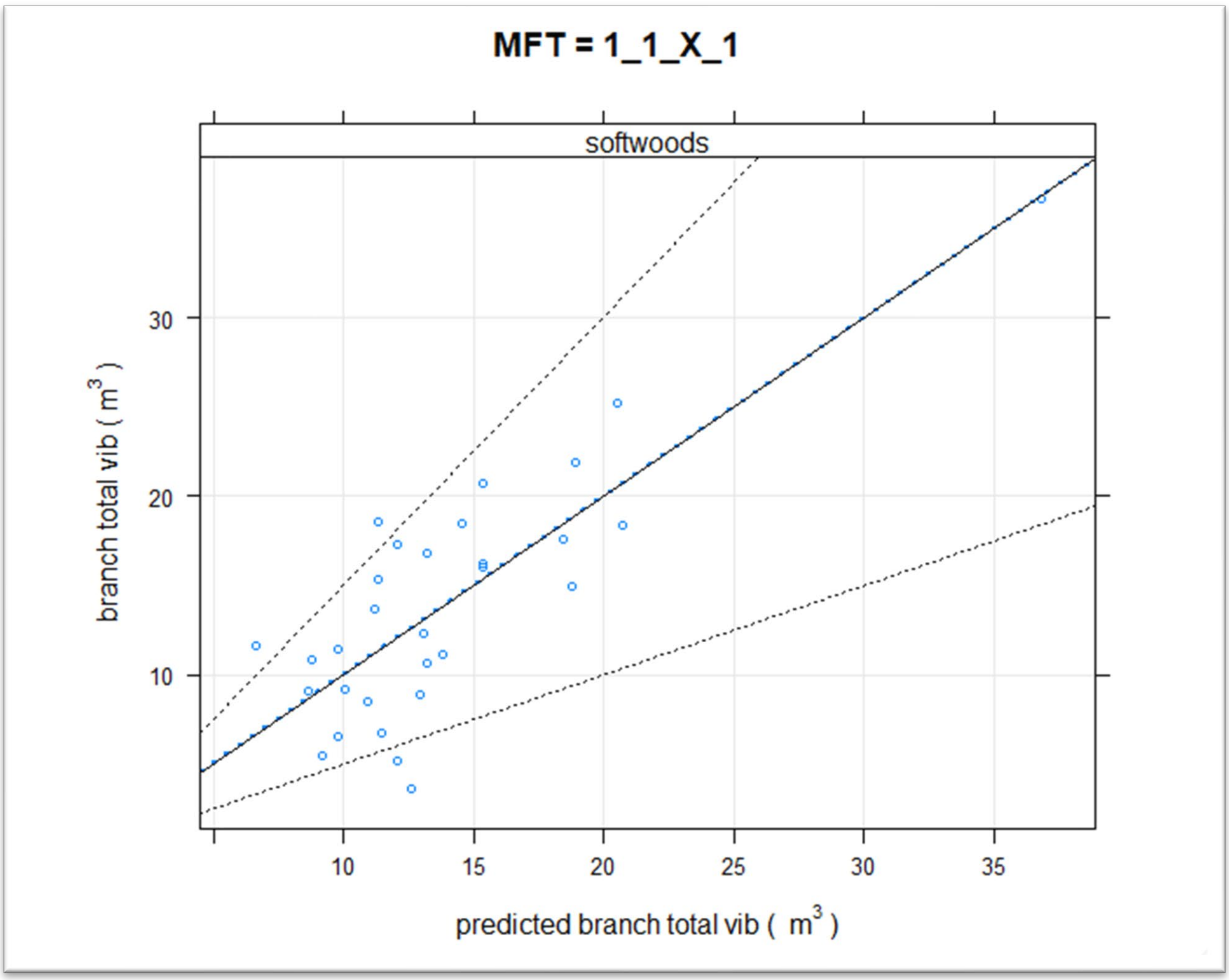


For hardwoods, the model for predicting the saw timber portion of total branch volume (Eq. 18b) was generally a poor predictor (Table C), with a large amount of scatter (outer lines at 50% and 150%; dotted lines in the figure below). This reflected the enormous variability in the number and size of saw logs in branches and large forks for saw-sized hardwoods with saw-sized branches, within and between species. Though the model was not statistically biased over all trees, the observed versus predicted plots (below) show clusters of trees with relatively low predicted branch saw volume and other clusters of trees with relatively high predicted volumes (clusters of trees along the 50 and 150% lines below), relative to the mean trend (blue dotted line, below). However, analyses of the data showed that these subgroups of trees within the hardwood 1_1_X_1 group could not be better differentiated without introducing more predictor variables into the model system, such as measurement of the relative height to the base of the merchantable crown or other metrics, such as counts of the number of branch logs (see MacFarlane 2013).

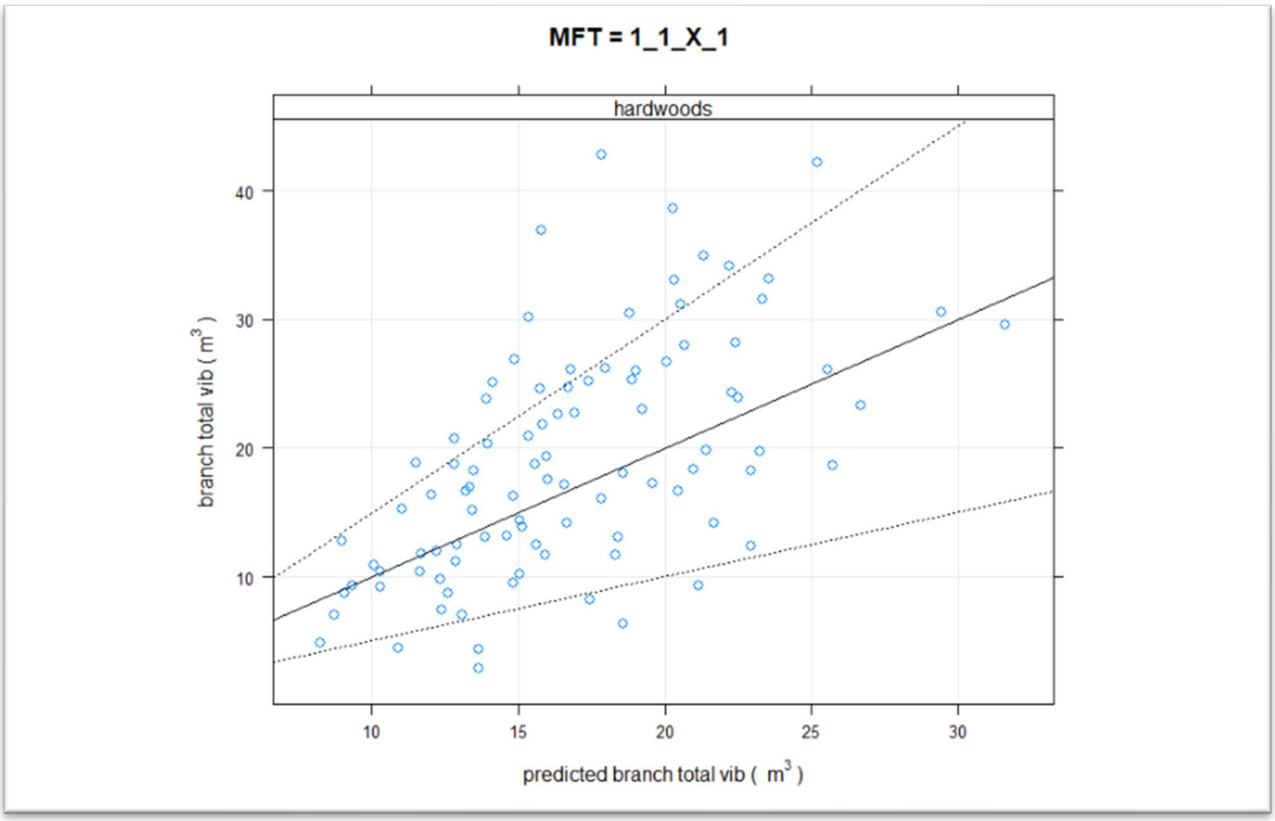


D.3. Branch total wood volume for saw-sized trees with saw-sized branches; MFT = 1_1_X_1 or 0_1_X_1.

For softwoods of this MFT, total branch wood volume predicted was generally good and unbiased R2 = 0.747 and a MAPE = 0.226 (see Table C), with most trees within ± 50% of the true value (see dotted lines in figure below at 50% and 150%). This total comes from the sum of two independent models (eqs. 17a + 17b) and most of the total branch volume in trees of this type is in branch saw logs.



For hardwoods, the total branch volume model had a much lower precision than the softwood model, for trees of this MFT (R2 = 0.284 and a MAPE = 0.374; see Table C). However, the model was statistically unbiased, with the most of trees within ± 50% of the true value (see dotted lines in figure below at 50% and 150%). The error in total branch volume prediction is the sum of the combined error from hardwood branch pulp (eq. 18a) and branch saw models (eq. 18b) for trees of this MFT, which ended up being lower than the error in both models. This means that the model system did better at predicting the total branch volume in trees with this MFT than it did predicting the individual components of that volume.



E. Comparisons to volume estimation under the new and preceding DNR model systems.

Total merchantable wood volume per tree was computed using the CSS-M modeling system, described in this report, and compared to values computed for the same trees computed by TSale.

TSale has models associated with a 'Mark and Tally' system (denoted here as TLY), where each tree that is marked to cut is also measured. The cruiser collects the species, DBH, number of saw and large pulp sticks per tree. TSale then calculates the volume of sawtimber in thousands of board feet (BF), denoted as MBF, and the volume of pulp in cords for each tree measured. Here, the estimates were generated by giving TSale the tallies from the destructive sampling to compute the cords and MBF for each tree.

TSale also has models that predict volume from the Cumulative VBAR Tally Method, aka Cumulative Tally (denoted here as CRL). On a cruise plot, the cruiser records the total number of 'in' trees, the total number of sticks (saw and pulp) in those trees, the total number of those trees that are saw trees and the total number of saw sticks within the saw trees, by species. TSale then calculates the volume of sawtimber in MBF per acre and the volume of pulp in cords per acre for each species on each plot. As above, the pulp and saw stick counts of each tree from destructive sampling were fed into TSALE, with the plot #s and tally counts known for the plots that contained the destructive sample trees, which were sampled using a Basal Area Factor (BAF) of 10 square feet per acre. Values per acre from the CRL files were converted into per-tree estimates for this analysis, with knowledge of the number of trees per acre represented by that tree, given the BAF.

Destructive sample trees (described above) were used to compute cubic-foot volumes for each validation tree. All volumes were standardized to the TSALE system outputs using 79 cubic feet per cord and 185 cubic feet per MBF, or vice versa to cubic TSALE volumes into cubic feet.

E.1. Overall differences and differences between trees of different size

In terms of total tree, cubic foot merchantable volume estimation, the median overall error was negligible for the new model across all size classes (Table E.1). TSALE-model cubic foot volumes, converted from cords and board foot volumes, showed a general overestimation of measured cubic foot volume for both the TLY and CRL methods, with errors that were an order of magnitude worse than CSS-M model predictions.

Table E.1. Median relative error ([predicted – measured]/[measured]; (%)) of cubic foot tree volume from the CSS-M system and two current methods used by the Michigan DNR (TLY and CRL), displayed by tree size class (2.5" wide classes, with the mid-point shown, e.g., 7.5" class ranges from 6.25 to 8.75").

Total Merchantable Wood Estimation Error (cubic feet)

dclass (in)	n	CSS-M	TLY	CRL
5	86	-1.87%	3.60%	4.16%
7.5	317	1.91%	6.18%	-2.47%
10	420	1.36%	4.37%	1.75%
12.5	478	-1.13%	20.27%	18.99%
15	335	-0.52%	19.28%	25.06%
17.5	175	0.33%	5.42%	29.35%
20	96	-2.26%	-10.31%	27.13%
22.5	36	-7.23%	-3.52%	27.34%
25	11	5.77%	59.78%	40.28%
27.5	4	-1.87%	78.08%	132.42%
30	1	41.10%	-91.69%	-78.91%
Grand Total	1959	0.03%	8.38%	13.13%

Overall, these results suggest that a change to the CCS-M model system will cause an overall reduction in the cubic foot volumes predicted for trees under the new modeling system, holding all other elements constant. A previous analysis of the CRL and TLY methods of timber volume inventory by **MacFarlane (2013) indicated that the biggest error in the TLY and CRL systems was measurement error, not modeling error, with the CRL system also including significant tallying errors during point sampling for the CRL method** (See MacFarlane 2013, Table 35). The new system assumes accurate tree measurements, particularly H4 and Hs, which will allow it to produce relatively high accuracy (low relative error) volumes estimates.

E.2. Differences across merchantable form types

Assigning trees to MFTs is part of the CSS-M system and it is likely that increased accuracy in volume estimation is partly a reflection of the value of assigning trees to MFTs. Comparing the differences between the tree modeling approaches, it is clear that the CSS-M system is much better at characterizing the volume of trees (compare Figs. E1, E2 and E3 and see Table E.2.). CSS-M provides highly precise and unbiased estimates of tree merchantable volume, though with a lower precision for trees with MFT = 0_1_X_X (Fig. E1). The TLY volume models showed good volume estimates only for pulp trees without branches (MFT = 1_0_0_0), with imprecise, but generally unbiased estimates of MFT = 1_1_1_0, and imprecise and biased estimates for trees with other MFTs (Figure E2). The CRL volume models also only showed good volume estimates only for trees with MFT = 1_0_0_0. CRL showed higher precision, but greater bias, relative to the TLY method (Figs. E2 & E3). The CRL method also showed a tendency to markedly underestimate volume for a subset of trees (see slightly bimodal distributions in Fig. E3).

Table E.2. Median relative error ([predicted – measured]/[measured]; (%)) of tree cubic foot volume from the CSS-M system and two current methods used by the Michigan DNR (TLY and CRL), displayed by merchantable form type (MFT).

Total Merchantable Wood Estimation Error (cubic feet)

MFT	n	CSS-M	TLY	CRL
0_1_X_X	57	**3.24%**	44.12%	32.03%
1_0_0_0	401	**0.75%**	1.31%	0.87%
1_0_1_0	178	**4.34%**	18.84%	-3.10%
1_1_0_0	612	**-1.17%**	18.07%	17.58%
1_1_1_0	579	0.58%	**0.23%**	17.34%
1_1_X_1	132	**-1.63%**	22.24%	50.45%
Grand Total	1959	**0.03%**	8.38%	13.13%

Figure E1.

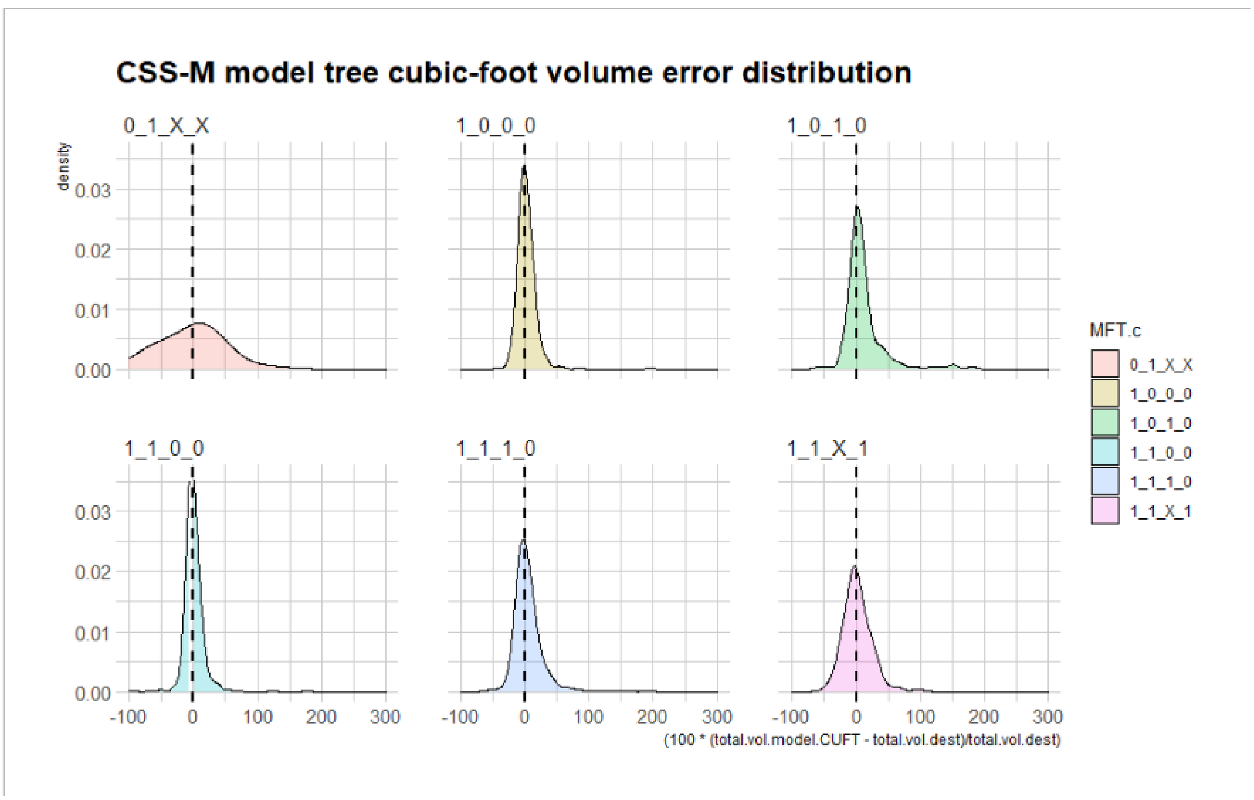


Figure E2.

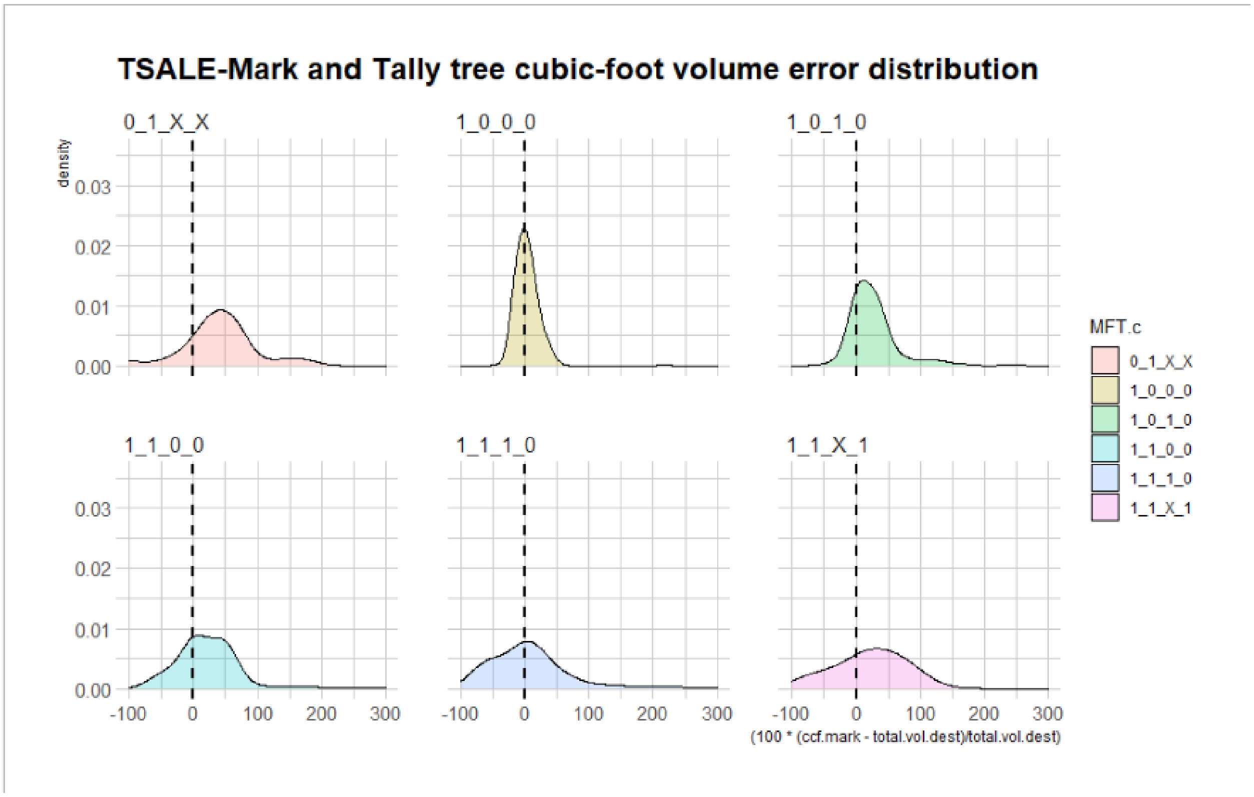
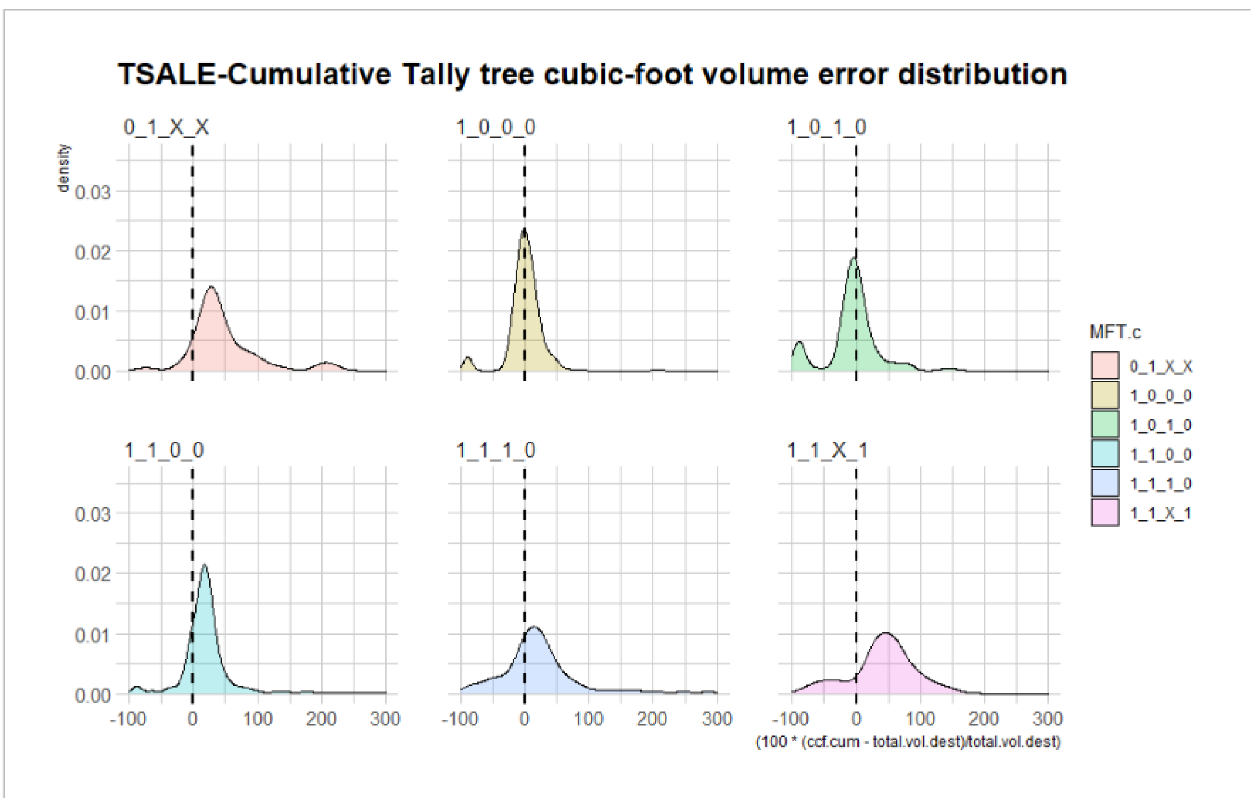


Figure E3.



Looking at the median error of tree total cubic foot volume estimates (Table E.2), the CSS-M model was generally superior across form types and competing methods examined (CRL and TLY). This pattern held when looking only at the most common MFTs (Table E.2.1).

Table E.2.1. Median relative cubic foot volume error for a subset of Table E.2.2 showing only the three most common tree MFTs (as shown by MacFarlane and Weiskittel 2016).

Total Merchantable Wood Estimation Error (cubic feet)

MFT	n	CSS-M	TLY	CRL
1_0_0_0	401	**0.75%**	1.31%	0.87%
1_1_0_0	612	**-1.17%**	18.07%	17.58%
1_1_1_0	579	0.58%	**0.23%**	17.34%
Grand Total	1592	**-0.05%**	7.36%	13.29%

E.3. Species differences in volume estimation between the methods

Where samples sizes were large for species (n > 50 in Table E.3), the overall trends in error estimation (Grand Totals at bottoms of Tables E.1, E.2, E.3) across the model systems were generally preserved at the species levels. Looking at total tree cubic foot volume, the CSS-M was better in terms having a consistently low median prediction error (less than ±10%) across all species (Table E.3.1). Whereas the TLY and CRL methods significantly over or under -estimated total volumes by a median relative error greater than ±10% for many species: The TLY method 9 or 27 species were greater than ±10% median error and for CRL, 18 of 27 species greater than ±10% median error, giving it a lower bias than TLY estimates, but generally a lower precision. There were a few cases when TLY or CRL showed lower median relative volume errors (bolded in Table E.3).

It is important to note that species-level patterns in error are more subject to sampling error. Every MFT was not equally represented within each species group, so that there is likely a form type bias buried within the species numbers, especially where the sample sizes are small (Table E.3). MacFarlane and Weiskittel (2015) showed that MFTs were more reliable than species as predictors of tree volume, so the MFT-level comparisons are probably more meaningful. Most importantly, some species were only drawn from a few stands (Table 1).

Finally, in all the error comparisons in this section, median relative errors were used, so as not to inflate volume error comparisons with unusual trees or due to small sample sizes. Since the volume error distributions showed in Figs. E1, E2 and E3, show a much broader range in volume estimation errors for TLY and CRL, the differences between the CSS-M and TSale presented here are conservative.

Table E.3. Median relative error ([predicted – measured]/[measured]; (%)) of cubic foot tree volume from the CSS-M system and two current methods used by the Michigan DNR (TLY and CRL), displayed by tree species or Genus group.

Total Merchantable Wood Estimation Error (% of cubic feet)

Genus.spp	N	CSS-M	TLY	CRL
Abies balsamea	78	**-0.76%**	3.56%	**0.23%**
Acer rubrum	161	**0.22%**	6.18%	13.55%
Acer saccharinum	2	10.90%	**-3.03%**	18.84%
Acer saccharum	305	**-2.35%**	-8.28%	5.82%
Amelanchier spp.	1	**6.33%**	-32.18%	31.91%
Betula alleghaniensis	16	**6.66%**	8.82%	18.78%
Betula papyrifera	29	**0.74%**	-7.59%	5.92%
Fagus grandifolia	62	**-0.51%**	-4.70%	5.81%
Fraxinus americana	24	6.50%	**0.56%**	12.21%
Fraxinus nigra	30	-2.14%	**-0.21%**	17.71%
Ostrya virginiana	2	**-7.75%**	-15.17%	-15.51%
Picea glauca	40	**2.49%**	4.73%	21.27%
Picea mariana	36	**-2.20%**	6.64%	8.24%
Pinus banksiana	242	**6.48%**	11.86%	11.18%
Pinus resinosa	281	**-1.21%**	47.76%	18.12%
Pinus strobus	30	**8.25%**	20.71%	-45.99%
Populus balsamifera	21	**1.79%**	2.06%	16.23%
Populus grandidentata	38	**-0.88%**	8.38%	3.67%
Populus spp.	74	**-0.89%**	-8.38%	-7.05%
Populus tremuloides	37	-2.63%	**-0.51%**	1.14%
Prunus serotina	37	-2.36%	**1.02%**	7.37%
Quercus alba	44	**2.74%**	13.62%	25.03%
Quercus ellipsoidalis	71	**4.57%**	13.43%	19.21%
Quercus rubra	96	**-1.24%**	12.14%	18.31%
Quercus velutina	67	**-0.19%**	17.21%	24.43%
Thuja occidentalis	25	9.60%	**8.91%**	30.12%
Tilia americana	109	-2.75%	**-1.19%**	17.77%
Grand Total	1958	**0.03%**	8.38%	13.13%

Appendix: coefficients and model statistics

Table A.1. Coefficients for the CSS taper model: 4" top dob - Michigan variant

tree.group	Gen.spp	MFT	c	e	r	p	q
hardwoods	*Acer rubrum*	1_0_0_0	0.9122555	106.0824	58.47741	2.5227173	1.0884408
hardwoods	*Acer rubrum*	1_0_1_0	0.7113925	106.0824	58.47741	2.305335	1.1669794
hardwoods	*Acer rubrum*	1_1_0_0	0.7114521	106.0824	58.47741	3.1922247	0.9723102
hardwoods	*Acer rubrum*	1_1_1_0	0.680764	106.0824	58.47741	0.9867537	1.4611448
hardwoods	*Acer rubrum*	1_1_X_1	0.4194959	106.0824	58.47741	0.5658934	1.5935555
hardwoods	*Acer saccharum*	1_0_0_0	1.52994	0	48.39271	3.629251	0.946668
hardwoods	*Acer saccharum*	1_0_1_0	0.3603127	0	17.81215	1.525871	1.305894
hardwoods	*Acer saccharum*	1_1_0_0	0.8840663	0	32.91748	4.585492	0.8756624
hardwoods	*Acer saccharum*	1_1_1_0	0.5112	1327.9146	47.0444	5.4737	1.2021
hardwoods	*Acer saccharum*	1_1_X_1	0.90716	0	52.70521	1	1.60353
hardwoods	*Betula alleghaniensis*	1_0_0_0	0.7109831	332.7966	37.77914	1	0.9351129
hardwoods	*Betula alleghaniensis*	1_0_1_0	0.7109831	332.7966	36.67505	1	0.8654905
hardwoods	*Betula alleghaniensis*	1_1_0_0	0.7109831	332.7966	37.4325	1	0.9132546
hardwoods	*Betula alleghaniensis*	1_1_1_0	0.7109831	332.7966	38.46253	1	0.9781809
hardwoods	*Betula alleghaniensis*	1_1_X_1	0.7109831	332.7966	43.31132	1	1.2839543
hardwoods	*Betula papyrifera*	1_0_0_0	0.7109831	332.7966	37.77914	1	0.9351129
hardwoods	*Betula papyrifera*	1_0_1_0	0.7109831	332.7966	36.67505	1	0.8654905
hardwoods	*Betula papyrifera*	1_1_0_0	0.7109831	332.7966	37.4325	1	0.9132546
hardwoods	*Betula papyrifera*	1_1_1_0	0.7109831	332.7966	38.46253	1	0.9781809
hardwoods	*Betula papyrifera*	1_1_X_1	0.7109831	332.7966	43.31132	1	1.2839543
hardwoods	*Fagus grandifolia*	1_0_0_0	1.736842	0	67.80945	2.493319	1.106597
hardwoods	*Fagus grandifolia*	1_0_1_0	1.680814	0	67.94876	3.8773146	1.068123
hardwoods	*Fagus grandifolia*	1_1_0_0	1.736842	0	67.80945	2.493319	1.106597
hardwoods	*Fagus grandifolia*	1_1_1_0	1.736842	0	67.80945	2.493319	1.106597
hardwoods	*Fagus grandifolia*	1_1_X_1	1.831034	0	67.94876	2.3106204	1.165035
hardwoods	*Fraxinus americana*	1_0_0_0	1.0690268	0	49.24582	2.426381	1.0307403

tree.group	Gen.spp	MFT	c	e	r	p	q
hardwoods	*Fraxinus americana*	1_0_1_0	1.1107947	0	49.24582	2.870625	1.1439017
hardwoods	*Fraxinus americana*	1_1_0_0	0.9902608	0	49.24582	1.588582	0.8174218
hardwoods	*Fraxinus americana*	1_1_1_0	1.0552849	0	49.24582	2.280182	0.9935849
hardwoods	*Fraxinus americana*	1_1_X_1	1.1159978	0	49.24582	2.925971	1.1579864
hardwoods	*Fraxinus nigra*	1_0_0_0	1.0690268	0	49.24582	2.426381	1.0307403
hardwoods	*Fraxinus nigra*	1_0_1_0	1.1107947	0	49.24582	2.870625	1.1439017
hardwoods	*Fraxinus nigra*	1_1_0_0	0.9902608	0	49.24582	1.588582	0.8174218
hardwoods	*Fraxinus nigra*	1_1_1_0	1.0552849	0	49.24582	2.280182	0.9935849
hardwoods	*Fraxinus nigra*	1_1_X_1	1.1159978	0	49.24582	2.925971	1.1579864
hardwoods	*Populus balsamifera*	1_0_0_0	0.6395536	0	40.03038	3.376637	0.8611608
hardwoods	*Populus balsamifera*	1_0_1_0	0.6249338	0	52.69335	2.997845	1.082256
hardwoods	*Populus balsamifera*	1_1_0_0	0.7077233	0	40.03038	3.376637	0.8398876
hardwoods	*Populus balsamifera*	1_1_1_0	0.6881715	0	52.69335	2.997845	1.023778
hardwoods	*Populus balsamifera*	1_1_X_1	0.4820342	0	52.69335	2.997845	1.214367
hardwoods	*Populus grandidentata*	1_0_0_0	0.6395536	0	40.03038	3.376637	0.8611608
hardwoods	*Populus grandidentata*	1_0_1_0	0.6249338	0	52.69335	2.997845	1.082256
hardwoods	*Populus grandidentata*	1_1_0_0	0.7077233	0	40.03038	3.376637	0.8398876
hardwoods	*Populus grandidentata*	1_1_1_0	0.6881715	0	52.69335	2.997845	1.023778
hardwoods	*Populus grandidentata*	1_1_X_1	0.4820342	0	52.69335	2.997845	1.214367
hardwoods	*Other Populus spp.*	1_0_0_0	0.6395536	0	40.03038	3.376637	0.8611608
hardwoods	*Other Populus spp.*	1_0_1_0	0.6249338	0	52.69335	2.997845	1.082256
hardwoods	*Other Populus spp.*	1_1_0_0	0.7077233	0	40.03038	3.376637	0.8398876
hardwoods	*Other Populus spp.*	1_1_1_0	0.6881715	0	52.69335	2.997845	1.023778
hardwoods	*Other Populus spp.*	1_1_X_1	0.4820342	0	52.69335	2.997845	1.214367
hardwoods	*Populus tremuloides*	1_0_0_0	0.6395536	0	40.03038	3.376637	0.8611608
hardwoods	*Populus tremuloides*	1_0_1_0	0.6249338	0	52.69335	2.997845	1.082256
hardwoods	*Populus tremuloides*	1_1_0_0	0.7077233	0	40.03038	3.376637	0.8398876
hardwoods	*Populus tremuloides*	1_1_1_0	0.6881715	0	52.69335	2.997845	1.023778

tree.group	Gen.spp	MFT	c	e	r	p	q
hardwoods	*Populus tremuloides*	1_1_X_1	0.4820342	0	52.69335	2.997845	1.214367
hardwoods	*Prunus serotina*	1_0_0_0	1.000919	0	84.55735	3.231215	0.9701911
hardwoods	*Prunus serotina*	1_0_1_0	1.000919	0	84.55735	3.039403	0.9999448
hardwoods	*Prunus serotina*	1_1_0_0	1.000919	0	84.55735	4.386846	0.7912697
hardwoods	*Prunus serotina*	1_1_1_0	1.000919	0	84.55735	1.266855	1.2742416
hardwoods	*Prunus serotina*	1_1_X_1	1.000919	0	84.55735	2.807133	1.0358795
hardwoods	*Quercus alba*	1_0_0_0	1.012791	202.7475	47.7482	2.249527	1.001046
hardwoods	*Quercus alba*	1_0_1_0	1.012791	202.7475	47.7482	2.53819	1.149834
hardwoods	*Quercus alba*	1_1_0_0	1.012791	202.7475	47.7482	2.334655	1.045056
hardwoods	*Quercus alba*	1_1_1_0	1.012791	202.7475	47.7482	2.671394	1.218693
hardwoods	*Quercus alba*	1_1_X_1	1.012791	202.7475	47.7482	3.221256	1.502371
hardwoods	*Quercus ellipsoidalis*	1_0_0_0	1.012791	202.7475	47.7482	2.249527	1.001046
hardwoods	*Quercus ellipsoidalis*	1_0_1_0	1.012791	202.7475	47.7482	2.53819	1.149834
hardwoods	*Quercus ellipsoidalis*	1_1_0_0	1.012791	202.7475	47.7482	2.334655	1.045056
hardwoods	*Quercus ellipsoidalis*	1_1_1_0	1.012791	202.7475	47.7482	2.671394	1.218693
hardwoods	*Quercus ellipsoidalis*	1_1_X_1	1.012791	202.7475	47.7482	3.221256	1.502371
hardwoods	*Quercus rubra*	1_0_0_0	1.012791	202.7475	47.7482	2.249527	1.001046
hardwoods	*Quercus rubra*	1_0_1_0	1.012791	202.7475	47.7482	2.53819	1.149834
hardwoods	*Quercus rubra*	1_1_0_0	1.012791	202.7475	47.7482	2.334655	1.045056
hardwoods	*Quercus rubra*	1_1_1_0	1.012791	202.7475	47.7482	2.671394	1.218693
hardwoods	*Quercus rubra*	1_1_X_1	1.012791	202.7475	47.7482	3.221256	1.502371
hardwoods	*Quercus velutina*	1_0_0_0	1.012791	202.7475	47.7482	2.249527	1.001046
hardwoods	*Quercus velutina*	1_0_1_0	1.012791	202.7475	47.7482	2.53819	1.149834
hardwoods	*Quercus velutina*	1_1_0_0	1.012791	202.7475	47.7482	2.334655	1.045056
hardwoods	*Quercus velutina*	1_1_1_0	1.012791	202.7475	47.7482	2.671394	1.218693
hardwoods	*Quercus velutina*	1_1_X_1	1.012791	202.7475	47.7482	3.221256	1.502371
hardwoods	*Tilia americana*	1_0_0_0	1.640403	-324.6962	84.65083	7.00625	0.9839392
hardwoods	*Tilia americana*	1_0_1_0	1.640403	-324.6962	84.65083	6.615482	1.1070703

tree.group	Gen.spp	MFT	c	e	r	p	q
hardwoods	*Tilia americana*	1_1_0_0	1.640403	-324.6962	84.65083	7.596583	0.7979443
hardwoods	*Tilia americana*	1_1_1_0	1.640403	-324.6962	84.65083	7.322946	0.8841552
hardwoods	*Tilia americana*	1_1_X_1	1.640403	-324.6962	84.65083	6.559547	1.12469
hardwoods	*other*	1_0_0_0	0.8238644	100.9851	42.14267	2.491757	0.9217195
hardwoods	*other*	1_0_1_0	0.20108	190.33858	20.71768	2.93832	1.07767
hardwoods	*other*	1_1_0_0	0.72661	166.46637	41.17879	3.40187	0.94989
hardwoods	*other*	1_1_1_0	0.6780743	896.8386	52.5376	3.17384	1.157206
hardwoods	*other*	1_1_X_1	1.255625	0	66.50766	3.20466	1.388952
hardwoods	*other*	Other	1.032836	157.2767	50.42207	1.753931	1.011706
softwoods	*Abies balsamea*	1_0_0_0	0.8740271	252.2322	37.18211	0.5717145	1.1051322
softwoods	*Abies balsamea*	1_0_1_0	0.9048766	252.2322	37.18211	0.5717145	1.3002848
softwoods	*Abies balsamea*	1_1_0_0	1.0170224	252.2322	37.18211	0.5717145	1.0408293
softwoods	*Abies balsamea*	1_1_1_0	0.9855994	252.2322	37.18211	0.5717145	1.5048309
softwoods	*Picea glauca*	1_0_0_0	0.7742605	133.2581	27.25562	1.9512295	1.382734
softwoods	*Picea glauca*	1_0_1_0	0.6305204	133.2581	27.25562	5.4626832	1.930251
softwoods	*Picea glauca*	1_1_0_0	1.2563706	0	48.53335	3.744156	1.2845841
softwoods	*Picea glauca*	1_1_1_0	1.3252595	0	48.53335	2.877387	1.8577427
softwoods	*Picea glauca*	1_1_X_1	0.1870705	0	48.53335	11.921118	0.8435077
softwoods	*Picea mariana*	1_0_0_0	0.7742605	133.2581	27.25562	1.9512295	1.382734
softwoods	*Picea mariana*	1_0_1_0	0.6305204	133.2581	27.25562	5.4626832	1.930251
softwoods	*Picea mariana*	1_1_0_0	1.2563706	0	48.53335	3.744156	1.2845841
softwoods	*Picea mariana*	1_1_1_0	1.3252595	0	48.53335	2.877387	1.8577427
softwoods	*Pinus banksiana*	1_0_0_0	0.3941175	133.2814	34.18755	1.730156	1.088171
softwoods	*Pinus banksiana*	1_0_1_0	0.3703646	133.2814	34.18755	2.706335	1.227106
softwoods	*Pinus banksiana*	1_1_0_0	0.3819297	133.2814	34.18755	1.541147	1.015374
softwoods	*Pinus banksiana*	1_1_1_0	0.3768865	133.2814	34.18755	2.847314	1.274376
softwoods	*Pinus banksiana*	1_1_X_1	0.2731668	133.2814	34.18755	3.797821	1.189366
softwoods	*Pinus resinosa*	1_0_0_0	0.4096433	206.6203	41.69924	3.111635	0.8609625

tree.group	Gen.spp	MFT	c	e	r	p	q
softwoods	*Pinus resinosa*	1_0_1_0	0.3417525	206.6203	41.69924	3.946611	1.2614389
softwoods	*Pinus resinosa*	1_1_0_0	0.4059883	206.6203	41.69924	6.312203	0.9462035
softwoods	*Pinus resinosa*	1_1_1_0	0.3596222	206.6203	41.69924	2.268741	1.126587
softwoods	*Pinus resinosa*	1_1_X_1	0.2955761	206.6203	41.69924	2.44908	1.4921677
softwoods	*Pinus strobus*	1_0_0_0	0.47968	91.48812	29.49214	1.51646	1.09187
softwoods	*Pinus strobus*	1_0_1_0	0.48112	0	22.756719	2.573201	1.296256
softwoods	*Pinus strobus*	1_1_0_0	0.93098	0	39.30387	1.30493	1.34117
softwoods	*Pinus strobus*	1_1_1_0	0.44643	0	37.41702	2.62248	1.3024
softwoods	*Pinus strobus*	1_1_X_1	0.384663	0	41.88435	2.86532	1.4147907
softwoods	other	1_0_0_0	0.7223867	0	25.6979	1.603014	1.046712
softwoods	other	1_0_1_0	0.5537865	0	25.6979	3.947643	1.42756
softwoods	other	1_1_0_0	1.013667	0	41.65965	2.23366	1.173357
softwoods	other	1_1_1_0	0.44643	0	37.41702	2.62248	1.3024
softwoods	other	1_1_X_1	0.52673	183.3235	37.93698	3.164949	1.178214
softwoods	other	Other	0.2949	616.979	36.6591	2.2394	1.1185
all	other	0_1_X_X	1.54401	0	88.28184	1	0.5137

Table A.1.1. Fit statistics for models in Table A.1

tree.group	Gen.spp	MFT	R^2	MAPE
hardwoods	*Acer rubrum*	1_0_0_0	0.921824	0.059312
hardwoods	*Acer rubrum*	1_0_1_0	0.896606	0.066467
hardwoods	*Acer rubrum*	1_1_0_0	0.938419	0.061099
hardwoods	*Acer rubrum*	1_1_1_0	0.908579	0.097316
hardwoods	*Acer rubrum*	1_1_X_1	0.924109	0.102252
hardwoods	*Acer saccharum*	1_0_0_0	0.971355	0.053867
hardwoods	*Acer saccharum*	1_0_1_0	0.964158	0.060514
hardwoods	*Acer saccharum*	1_1_0_0	0.936688	0.063062
hardwoods	*Acer saccharum*	1_1_1_0	0.912391	0.091828
hardwoods	*Acer saccharum*	1_1_X_1	0.931212	0.100605
hardwoods	*Betula alleghaniensis*	1_0_0_0	0.944554	0.051544
hardwoods	*Betula alleghaniensis*	1_0_1_0	0.898973	0.069844
hardwoods	*Betula alleghaniensis*	1_1_0_0	0.908676	0.084411
hardwoods	*Betula alleghaniensis*	1_1_1_0	0.915425	0.079733
hardwoods	*Betula alleghaniensis*	1_1_X_1	0.918139	0.125123
hardwoods	*Betula papyrifera*	1_0_0_0	0.944554	0.051544
hardwoods	*Betula papyrifera*	1_0_1_0	0.898973	0.069844
hardwoods	*Betula papyrifera*	1_1_0_0	0.908676	0.084411
hardwoods	*Betula papyrifera*	1_1_1_0	0.915425	0.079733
hardwoods	*Betula papyrifera*	1_1_X_1	0.918139	0.125123
hardwoods	*Fagus grandifolia*	1_0_0_0	0.989718	0.038791
hardwoods	*Fagus grandifolia*	1_0_1_0	0.986297	0.054976
hardwoods	*Fagus grandifolia*	1_1_0_0	0.913770	0.080680
hardwoods	*Fagus grandifolia*	1_1_1_0	0.929784	0.086723
hardwoods	*Fagus grandifolia*	1_1_X_1	0.967956	0.082439
hardwoods	*Fraxinus americana*	1_0_0_0	0.968280	0.052359
hardwoods	*Fraxinus americana*	1_0_1_0	0.954463	0.051009
hardwoods	*Fraxinus americana*	1_1_0_0	0.991737	0.049214
hardwoods	*Fraxinus americana*	1_1_1_0	0.932749	0.081495
hardwoods	*Fraxinus americana*	1_1_X_1	0.964825	0.075672
hardwoods	*Fraxinus nigra*	1_0_0_0	0.968280	0.052359
hardwoods	*Fraxinus nigra*	1_0_1_0	0.954463	0.051009
hardwoods	*Fraxinus nigra*	1_1_0_0	0.991737	0.049214
hardwoods	*Fraxinus nigra*	1_1_1_0	0.932749	0.081495
hardwoods	*Fraxinus nigra*	1_1_X_1	0.964825	0.075672
hardwoods	*Populus balsamifera*	1_0_0_0	0.960622	0.042908
hardwoods	*Populus balsamifera*	1_0_1_0	0.937650	0.056567
hardwoods	*Populus balsamifera*	1_1_0_0	0.971430	0.059530

tree.group	Gen.spp	MFT	R^2	MAPE
hardwoods	*Populus balsamifera*	1_1_1_0	0.928073	0.075909
hardwoods	*Populus balsamifera*	1_1_X_1	0.964834	0.072723
hardwoods	*Populus grandidentata*	1_0_0_0	0.960622	0.042908
hardwoods	*Populus grandidentata*	1_0_1_0	0.937650	0.056567
hardwoods	*Populus grandidentata*	1_1_0_0	0.971430	0.059530
hardwoods	*Populus grandidentata*	1_1_1_0	0.928073	0.075909
hardwoods	*Populus grandidentata*	1_1_X_1	0.964834	0.072723
hardwoods	*Other Populus spp.*	1_0_0_0	0.960622	0.042908
hardwoods	*Other Populus spp.*	1_0_1_0	0.937650	0.056567
hardwoods	*Other Populus spp.*	1_1_0_0	0.971430	0.059530
hardwoods	*Other Populus spp.*	1_1_1_0	0.928073	0.075909
hardwoods	*Other Populus spp.*	1_1_X_1	0.964834	0.072723
hardwoods	*Populus tremuloides*	1_0_0_0	0.960622	0.042908
hardwoods	*Populus tremuloides*	1_0_1_0	0.937650	0.056567
hardwoods	*Populus tremuloides*	1_1_0_0	0.971430	0.059530
hardwoods	*Populus tremuloides*	1_1_1_0	0.928073	0.075909
hardwoods	*Populus tremuloides*	1_1_X_1	0.964834	0.072723
hardwoods	*Prunus serotina*	1_0_0_0	0.953154	0.062088
hardwoods	*Prunus serotina*	1_0_1_0	0.882044	0.114965
hardwoods	*Prunus serotina*	1_1_0_0	0.905609	0.058502
hardwoods	*Prunus serotina*	1_1_1_0	0.890618	0.084389
hardwoods	*Prunus serotina*	1_1_X_1	0.954677	0.064081
hardwoods	*Quercus alba*	1_0_0_0	0.989970	0.042527
hardwoods	*Quercus alba*	1_0_1_0	0.971272	0.063594
hardwoods	*Quercus alba*	1_1_0_0	0.938849	0.056424
hardwoods	*Quercus alba*	1_1_1_0	0.924047	0.080511
hardwoods	*Quercus alba*	1_1_X_1	0.939851	0.103552
hardwoods	*Quercus ellipsoidalis*	1_0_0_0	0.989970	0.042527
hardwoods	*Quercus ellipsoidalis*	1_0_1_0	0.971272	0.063594
hardwoods	*Quercus ellipsoidalis*	1_1_0_0	0.938849	0.056424
hardwoods	*Quercus ellipsoidalis*	1_1_1_0	0.924047	0.080511
hardwoods	*Quercus ellipsoidalis*	1_1_X_1	0.939851	0.103552
hardwoods	*Quercus rubra*	1_0_0_0	0.989970	0.042527
hardwoods	*Quercus rubra*	1_0_1_0	0.971272	0.063594
hardwoods	*Quercus rubra*	1_1_0_0	0.938849	0.056424
hardwoods	*Quercus rubra*	1_1_1_0	0.924047	0.080511
hardwoods	*Quercus rubra*	1_1_X_1	0.939851	0.103552
hardwoods	*Quercus velutina*	1_0_0_0	0.989970	0.042527
hardwoods	*Quercus velutina*	1_0_1_0	0.971272	0.063594
hardwoods	*Quercus velutina*	1_1_0_0	0.938849	0.056424

tree.group	Gen.spp	MFT	R^2	MAPE
hardwoods	*Quercus velutina*	1_1_1_0	0.924047	0.080511
hardwoods	*Quercus velutina*	1_1_X_1	0.939851	0.1035520
hardwoods	*Tilia americana*	1_0_0_0	0.979090	0.0483600
hardwoods	*Tilia americana*	1_0_1_0	0.985342	0.0506440
hardwoods	*Tilia americana*	1_1_0_0	0.935715	0.0551210
hardwoods	*Tilia americana*	1_1_1_0	0.935689	0.0748430
hardwoods	*Tilia americana*	1_1_X_1	0.942875	0.0824660
hardwoods	*Other*	1_0_0_0	0.9000248	0.05033184
hardwoods	*Other*	1_0_1_0	0.8995392	0.06414775
hardwoods	*Other*	1_1_0_0	0.9213138	0.06123766
hardwoods	*Other*	1_1_1_0	0.9124944	0.08981790
hardwoods	*Other*	1_1_X_1	0.9228505	0.10008930
hardwoods	*Other*	Other	0.9376222	0.05621315
softwoods	*Abies balsamea*	1_0_0_0	0.9289514	0.03692099
softwoods	*Abies balsamea*	1_0_1_0	0.8902664	0.06695446
softwoods	*Abies balsamea*	1_1_0_0	0.9585111	0.05732138
softwoods	*Abies balsamea*	1_1_1_0	0.9700554	0.05905447
softwoods	*Picea glauca*	1_0_0_0	0.9088897	0.04642958
softwoods	*Picea glauca*	1_0_1_0	0.8562384	0.07722053
softwoods	*Picea glauca*	1_1_0_0	0.9596712	0.05136538
softwoods	*Picea glauca*	1_1_1_0	0.9465789	0.06764225
softwoods	*Picea glauca*	1_1_X_1	0.8260340	0.09304350
softwoods	*Picea mariana*	1_0_0_0	0.9088897	0.04642958
softwoods	*Picea mariana*	1_0_1_0	0.8562384	0.07722053
softwoods	*Picea mariana*	1_1_0_0	0.9596712	0.05136538
softwoods	*Picea mariana*	1_1_1_0	0.9465789	0.06764225
softwoods	*Picea mariana*	1_1_X_1	0.8260340	0.09304350
softwoods	*Pinus banksiana*	1_0_0_0	0.9499303	0.03899202
softwoods	*Pinus banksiana*	1_0_1_0	0.9091816	0.05401895
softwoods	*Pinus banksiana*	1_1_0_0	0.9400417	0.05196173
softwoods	*Pinus banksiana*	1_1_1_0	0.9169482	0.07340622
softwoods	*Pinus banksiana*	1_1_X_1	0.9213958	0.07386332
softwoods	*Pinus resinosa*	1_0_0_0	0.9775470	0.02729231
softwoods	*Pinus resinosa*	1_0_1_0	0.8759303	0.05978781
softwoods	*Pinus resinosa*	1_1_0_0	0.9669119	0.05225324
softwoods	*Pinus resinosa*	1_1_1_0	0.9506784	0.07777416
softwoods	*Pinus resinosa*	1_1_X_1	0.9001605	0.09816469
softwoods	*Pinus strobus*	1_0_0_0	0.9726341	0.02698561
softwoods	*Pinus strobus*	1_0_1_0	0.8658701	0.06985634
softwoods	*Pinus strobus*	1_1_0_0	0.9453034	0.06020546

tree.group	Gen.spp	MFT	R^2	MAPE
softwoods	*Pinus strobus*	1_1_1_0	0.7020768	0.16804429
softwoods	*Pinus strobus*	1_1_X_1	0.9187470	0.08905200
softwoods	Other	1_0_1_0	0.9231940	0.04039900
softwoods	Other	1_0_1_0	0.8789990	0.06546700
softwoods	Other	1_1_0_0	0.9473433	0.05927744
softwoods	Other	1_1_1_0	0.9349917	0.09133313
softwoods	Other	1_1_X_1	0.8961432	0.09570614
softwoods	Other	Other	0.9566957	0.04177918
all	Other	0_1_X_X	0.7947859	0.08788533

Table A.2. Coefficients for predicting H4 from Hp

tree.group	MFT	b	m
hardwoods	1_0_0_0	4.389908	0.986139
hardwoods	1_0_1_0	5.294911	0.976627
hardwoods	1_1_0_0	5.351743	0.976045
hardwoods	1_1_1_0	5.50345	0.974462
hardwoods	1_1_X_1	5.319288	0.976378
hardwoods	other	5.17186	0.977931
softwoods	1_0_0_0	4.190229	0.959946
softwoods	1_0_1_0	5.046335	0.958154
softwoods	1_1_0_0	5.621835	0.956779
softwoods	1_1_1_0	5.340153	0.957708
softwoods	1_1_X_1	6.788218	0.954941
softwoods	other	5.397354	0.957506

Table A.3. Coefficients for predicting DFH from DBH and H4

tree.group	Gen.spp	MFT	a	b	w
hardwoods	*Acer rubrum*	1_0_0_0	-0.925557137	0.774121393	0.036836999
hardwoods	*Acer rubrum*	1_0_1_0	-0.850032362	0.66770745	0.056412917
hardwoods	*Acer rubrum*	1_1_0_0	-0.688314934	0.704236248	0.044318722
hardwoods	*Acer rubrum*	1_1_1_0	-1.079504154	0.745602979	0.047054889
hardwoods	*Acer rubrum*	1_1_X_1	-1.116761641	0.653587135	0.066946636
hardwoods	*Acer saccharum*	1_0_0_0	-0.637721649	0.82878058	0.017356823
hardwoods	*Acer saccharum*	1_0_1_0	-0.958171294	0.753616627	0.041936756
hardwoods	*Acer saccharum*	1_1_0_0	-0.847939385	0.839347915	0.021218749
hardwoods	*Acer saccharum*	1_1_1_0	-0.942510802	0.766733235	0.038838826
hardwoods	*Acer saccharum*	1_1_X_1	-1.156805425	0.781389085	0.041940334
hardwoods	*Betula alleghaniensis*	1_0_0_0	-0.9258683	0.7525782	0.0412215

tree.group	Gen.spp	MFT	a	b	w
hardwoods	*Betula alleghaniensis*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula alleghaniensis*	1_1_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula alleghaniensis*	1_1_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula alleghaniensis*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula papyrifera*	1_0_0_0	-0.904124234	0.744872744	0.042175507
hardwoods	*Betula papyrifera*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula papyrifera*	1_1_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula papyrifera*	1_1_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Betula papyrifera*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Fagus grandifolia*	1_0_0_0	-0.90051805	0.755182526	0.039962735
hardwoods	*Fagus grandifolia*	1_0_1_0	-0.667960698	0.707836313	0.042978296
hardwoods	*Fagus grandifolia*	1_1_0_0	-1.088077331	0.801390378	0.035871225
hardwoods	*Fagus grandifolia*	1_1_1_0	-1.007013698	0.849598699	0.023675757
hardwoods	*Fagus grandifolia*	1_1_X_1	-0.940985284	0.856704584	0.020322343
hardwoods	*Fraxinus americana*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Fraxinus americana*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Fraxinus americana*	1_1_0_0	-0.646300001	0.738542283	0.036082043
hardwoods	*Fraxinus americana*	1_1_1_0	-1.038695456	0.777669564	0.039313703
hardwoods	*Fraxinus americana*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Fraxinus nigra*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Fraxinus nigra*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Fraxinus nigra*	1_1_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Fraxinus nigra*	1_1_1_0	-1.027889612	0.764406008	0.041725092
hardwoods	*Fraxinus nigra*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus balsamifera*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus balsamifera*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus balsamifera*	1_1_0_0	-0.901312284	0.762381575	0.038509607
hardwoods	*Populus balsamifera*	1_1_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus grandidentata*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus grandidentata*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus grandidentata*	1_1_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus grandidentata*	1_1_1_0	-0.921932033	0.819071843	0.027479259
hardwoods	*Populus grandidentata*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Other Populus spp.*	1_0_0_0	-0.932802534	0.788082303	0.034146816
hardwoods	*Other Populus spp.*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Other Populus spp.*	1_1_0_0	-1.049115303	0.80525566	0.033968799
hardwoods	*Other Populus spp.*	1_1_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus tremuloides*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Populus tremuloides*	1_0_1_0	-0.899646425	0.790451544	0.03272511
hardwoods	*Populus tremuloides*	1_1_0_0	-1.0305798	0.790673667	0.036414017

tree.group	Gen.spp	MFT	a	b	w
hardwoods	*Populus tremuloides*	1_1_1_0	-1.160475633	0.791008271	0.040077212
hardwoods	*Populus tremuloides*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Prunus serotina*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Prunus serotina*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Prunus serotina*	1_1_0_0	-1.089232623	0.794676501	0.037280826
hardwoods	*Prunus serotina*	1_1_1_0	-1.148843744	0.782805391	0.041418185
hardwoods	*Prunus serotina*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus alba*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus alba*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus alba*	1_1_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus alba*	1_1_1_0	-1.063804291	0.665082768	0.063098556
hardwoods	*Quercus alba*	1_1_X_1	-1.50092921	0.866044787	0.03445283
hardwoods	*Quercus ellipsoidalis*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus ellipsoidalis*	1_0_1_0	-0.794597892	0.645459914	0.05938719
hardwoods	*Quercus ellipsoidalis*	1_1_0_0	-1.033306646	0.788143557	0.037019357
hardwoods	*Quercus ellipsoidalis*	1_1_1_0	-1.079686524	0.787642247	0.038433214
hardwoods	*Quercus ellipsoidalis*	1_1_X_1	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus rubra*	1_0_0_0	-0.845897455	0.695527176	0.050610273
hardwoods	*Quercus rubra*	1_0_1_0	-0.796362302	0.702078652	0.04784861
hardwoods	*Quercus rubra*	1_1_0_0	-0.908184558	0.766181962	0.037927191
hardwoods	*Quercus rubra*	1_1_1_0	-1.300843081	0.763818344	0.049654517
hardwoods	*Quercus rubra*	1_1_X_1	-0.907947385	0.813642152	0.028177829
hardwoods	*Quercus velutina*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Quercus velutina*	1_0_1_0	-0.83699247	0.60467566	0.068954454
hardwoods	*Quercus velutina*	1_1_0_0	-0.936833698	0.720139739	0.048176714
hardwoods	*Quercus velutina*	1_1_1_0	-1.173234297	0.828001579	0.032856637
hardwoods	*Quercus velutina*	1_1_X_1	-1.03228953	0.734281877	0.048037194
hardwoods	*Tilia americana*	1_0_0_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Tilia americana*	1_0_1_0	-0.9258683	0.7525782	0.0412215
hardwoods	*Tilia americana*	1_1_0_0	-0.789359797	0.751559076	0.037525781
hardwoods	*Tilia americana*	1_1_1_0	-0.98762306	0.766735434	0.040095611
hardwoods	*Tilia americana*	1_1_X_1	-0.835731617	0.852987726	0.018064129
softwoods	*Abies balsamea*	1_0_0_0	-0.839901037	0.745583738	0.040204931
softwoods	*Abies balsamea*	1_0_1_0	-0.9258683	0.7525782	0.0412215
softwoods	*Abies balsamea*	1_1_0_0	-0.672931815	0.751382849	0.03421532
softwoods	*Abies balsamea*	1_1_1_0	-0.932682208	0.729626944	0.046110633
softwoods	*Picea glauca*	1_0_0_0	-0.927608143	0.75070145	0.041651879
softwoods	*Picea glauca*	1_0_1_0	-0.9258683	0.7525782	0.0412215
softwoods	*Picea glauca*	1_1_0_0	-0.512127035	0.660142182	0.048304182
softwoods	*Picea glauca*	1_1_1_0	-0.9258683	0.7525782	0.0412215

tree.group	Gen.spp	MFT	a	b	w
softwoods	*Picea glauca*	1_1_X_1	-0.9258683	0.7525782	0.0412215
softwoods	*Picea mariana*	1_0_0_0	-0.9258683	0.7525782	0.0412215
softwoods	*Picea mariana*	1_0_1_0	-0.9258683	0.7525782	0.0412215
softwoods	*Picea mariana*	1_1_0_0	-0.884394305	0.651807162	0.060669409
softwoods	*Picea mariana*	1_1_1_0	-0.9258683	0.7525782	0.0412215
softwoods	*Pinus banksiana*	1_0_0_0	-0.854389931	0.65578467	0.059003478
softwoods	*Pinus banksiana*	1_0_1_0	-0.906033143	0.686849879	0.054108185
softwoods	*Pinus banksiana*	1_1_0_0	-0.894604555	0.671591675	0.056910973
softwoods	*Pinus banksiana*	1_1_1_0	-0.951081311	0.704510314	0.051771762
softwoods	*Pinus banksiana*	1_1_X_1	-0.9258683	0.7525782	0.0412215
softwoods	*Pinus resinosa*	1_0_0_0	-0.854286182	0.742359446	0.041258922
softwoods	*Pinus resinosa*	1_0_1_0	-0.884160018	0.696138851	0.051581499
softwoods	*Pinus resinosa*	1_1_0_0	-0.905655295	0.808454173	0.02919809
softwoods	*Pinus resinosa*	1_1_1_0	-0.752262671	0.686253201	0.049812904
softwoods	*Pinus resinosa*	1_1_X_1	-0.557751032	0.720385007	0.037256269
softwoods	*Pinus strobus*	1_0_0_0	-0.909132613	0.746388111	0.042009145
softwoods	*Pinus strobus*	1_0_1_0	-0.9258683	0.7525782	0.0412215
softwoods	*Pinus strobus*	1_1_0_0	-1.191581688	0.822463697	0.034510944
softwoods	*Pinus strobus*	1_1_1_0	-0.9258683	0.7525782	0.0412215
softwoods	*Pinus strobus*	1_1_X_1	-0.9258683	0.7525782	0.0412215
all	Other	Other	-0.9258683	0.7525782	0.0412215

Table A.3.1. Fit statistics for the DFH model in Table A.3.

tree.group	Gen.spp	MFT	R^2	MAPE
hardwoods	*Acer rubrum*	1_0_0_0	0.984757583	0.058052492
hardwoods	*Acer rubrum*	1_0_1_0	0.888133617	0.121392814
hardwoods	*Acer rubrum*	1_1_0_0	0.837817172	0.065162045
hardwoods	*Acer rubrum*	1_1_1_0	0.893132118	0.082498342
hardwoods	*Acer rubrum*	1_1_X_1	0.945507023	0.086398948
hardwoods	*Acer saccharum*	1_0_0_0	0.976521425	0.061008995
hardwoods	*Acer saccharum*	1_0_1_0	0.893789353	0.080732788
hardwoods	*Acer saccharum*	1_1_0_0	0.936310237	0.037667731
hardwoods	*Acer saccharum*	1_1_1_0	0.943145820	0.063930274
hardwoods	*Acer saccharum*	1_1_X_1	0.972575677	0.073557406
hardwoods	*Betula papyrifera*	1_0_0_0	0.989876354	0.053517691
hardwoods	*Fagus grandifolia*	1_0_0_0	0.996011121	0.050050157
hardwoods	*Fagus grandifolia*	1_0_1_0	0.907741722	0.089226233
hardwoods	*Fagus grandifolia*	1_1_0_0	0.928919164	0.043586674
hardwoods	*Fagus grandifolia*	1_1_1_0	0.967811891	0.056493249
hardwoods	*Fagus grandifolia*	1_1_X_1	0.987047346	0.051668389
hardwoods	*Fraxinus americana*	1_1_0_0	0.971017504	0.040545022
hardwoods	*Fraxinus americana*	1_1_1_0	0.969327991	0.042207895
hardwoods	*Fraxinus nigra*	1_1_1_0	0.962493313	0.041848674
hardwoods	*Other Populus spp.*	1_0_0_0	0.979442596	0.034260419
hardwoods	*Other Populus spp.*	1_1_0_0	0.971675836	0.029033900
hardwoods	*Populus balsamifera*	1_1_0_0	0.874811955	0.032837684
hardwoods	*Populus grandidentata*	1_1_1_0	0.967564367	0.044932331
hardwoods	*Populus tremuloides*	1_0_1_0	0.881758499	0.075853449
hardwoods	*Populus tremuloides*	1_1_0_0	0.936820140	0.040151796
hardwoods	*Populus tremuloides*	1_1_1_0	0.932764086	0.041088705
hardwoods	*Prunus serotina*	1_1_0_0	0.905640291	0.04539923
hardwoods	*Prunus serotina*	1_1_1_0	0.801335219	0.089381167
hardwoods	*Quercus alba*	1_1_1_0	0.910258190	0.067635098
hardwoods	*Quercus alba*	1_1_X_1	0.981488168	0.070126394
hardwoods	*Quercus ellipsoidalis*	1_0_1_0	0.914729125	0.120670747
hardwoods	*Quercus ellipsoidalis*	1_1_0_0	0.762397777	0.072338953
hardwoods	*Quercus ellipsoidalis*	1_1_1_0	0.739401824	0.121498424
hardwoods	*Quercus rubra*	1_0_0_0	0.989379247	0.053486858
hardwoods	*Quercus rubra*	1_0_1_0	0.941542719	0.096647936
hardwoods	*Quercus rubra*	1_1_0_0	0.825090407	0.044843232
hardwoods	*Quercus rubra*	1_1_1_0	0.931547765	0.075338650
hardwoods	*Quercus rubra*	1_1_X_1	0.957340026	0.091253744
hardwoods	*Quercus velutina*	1_0_1_0	0.971636647	0.078198529
hardwoods	*Quercus velutina*	1_1_0_0	0.824661603	0.065958759
hardwoods	*Quercus velutina*	1_1_1_0	0.914539068	0.071112487
hardwoods	*Quercus velutina*	1_1_X_1	0.862506055	0.136817925
hardwoods	*Tilia americana*	1_1_0_0	0.947195872	0.044667673
hardwoods	*Tilia americana*	1_1_1_0	0.971403935	0.043961546
hardwoods	*Tilia americana*	1_1_X_1	0.984360933	0.050285308

tree.group	Gen.spp	MFT	R^2	MAPE
softwoods	*Pinus banksiana*	1_0_0_0	0.991612742	0.054371087
softwoods	*Pinus banksiana*	1_0_1_0	0.956530007	0.103514513
softwoods	*Pinus banksiana*	1_1_0_0	0.815682911	0.059380311
softwoods	*Pinus banksiana*	1_1_1_0	0.801942945	0.090573026
softwoods	*Abies balsamea*	1_0_0_0	0.993949972	0.035928028
softwoods	*Abies balsamea*	1_1_0_0	0.780590361	0.053007407
softwoods	*Abies balsamea*	1_1_1_0	0.820240863	0.088688928
softwoods	*Pinus resinosa*	1_0_0_0	0.975747249	0.048220501
softwoods	*Pinus resinosa*	1_0_1_0	0.960118324	0.086741475
softwoods	*Pinus resinosa*	1_1_0_0	0.971035962	0.030655558
softwoods	*Pinus resinosa*	1_1_1_0	0.882128496	0.090011863
softwoods	*Pinus resinosa*	1_1_X_1	0.808344810	0.151448262
softwoods	*Pinus strobus*	1_0_0_0	0.994761961	0.043382835
softwoods	*Pinus strobus*	1_1_0_0	0.992113405	0.039853413
softwoods	*Thuja occidentalis*	1_1_0_0	0.828366894	0.056491091
softwoods	*Tsuga canadensis*	1_1_0_0	0.985348462	0.054829198
softwoods	*Picea glauca*	1_0_0_0	0.995671408	0.034547688
softwoods	*Picea glauca*	1_1_0_0	0.849752399	0.068926319
softwoods	*Picea mariana*	1_1_0_0	0.645536392	0.049491854
all	Other	other	0.9591506	0.056450700

Table A.4. Outside bark stem saw volume fit statistics from the DNR-Clark model.

MFT	R^2	MAPE
1_1_0_0	0.9786492	0.05551317
1_1_1_0	0.9764432	0.06268464
1_1_X_1	0.9621300	0.08681121
Gen.spp	R^2	MAPE
Abies balsamea	0.9966227	0.03674598
Acer rubrum	0.9766050	0.06458940
Acer saccharum	0.9739569	0.05911485
Betula alleghaniensis	0.8335537	0.09722617
Betula papyrifera	0.9693535	0.06514655
Fagus grandifolia	0.9623319	0.08313466
Fraxinus americana	0.9856108	0.04901616
Fraxinus nigra	0.9962271	0.05509242
Picea glauca	0.9866719	0.06628383
Picea mariana	0.9982752	0.07117491
Pinus banksiana	0.9963058	0.06339132
Pinus resinosa	0.9269114	0.06220374
Pinus strobus	0.9926167	0.07769462
Populus balsamifera	0.9854804	0.07898130
Populus grandidentata	0.9824469	0.05425856
Populus tremuloides	0.9816902	0.06617263

Prunus serotina	0.9671746	0.05612033
Quercus alba	0.9794992	0.07332724
Quercus ellipsoidalis	0.9827108	0.08071641
Quercus rubra	0.9669077	0.06914054
Quercus velutina	0.9103148	0.10493434
Tilia americana	0.9863873	0.04142137

Table A.5. Outside bark pulp volume fit statistics from the DNR-Clark model.

MFT	R^2	MAPE
1_0_0_0	0.9989177	0.05576525
1_0_1_0	0.9973762	0.09380777
1_1_0_0	0.9618299	0.06280366
1_1_1_0	0.9581123	0.09685371
1_1_X_1	0.9641327	0.09952030
Gen.spp	R^2	MAPE
Abies balsamea	0.9946576	0.04799655
Acer rubrum	0.9622587	0.08820554
Acer saccharum	0.9701631	0.09042236
Betula alleghaniensis	0.8545639	0.11204347
Betula papyrifera	0.9847051	0.0605791
Fagus grandifolia	0.9452893	0.08289203
Fraxinus americana	0.9818852	0.07423543
Fraxinus nigra	0.9848091	0.04825176
Picea glauca	0.9794483	0.07736551
Picea mariana	0.9903943	0.06818825
Pinus banksiana	0.9945095	0.06281261
Pinus resinosa	0.9552355	0.06530756
Pinus strobus	0.9443935	0.12779653
Populus balsamifera	0.8833457	0.06822076
Populus grandidentata	0.9795787	0.06749172
Populus tremuloides	0.9841154	0.08020847
Prunus serotina	0.9328071	0.10693340
Quercus alba	0.9706003	0.06737981
Quercus ellipsoidalis	0.9810017	0.07889336
Quercus rubra	0.9560482	0.12824867
Quercus velutina	0.9509565	0.08769833
Tilia americana	0.9836262	0.05519453

Table A.6. Fit statistics for taper model predictions of pulp volume above saw volume (volume between Hs and Hp).

MFT	R^2	MAPE
1_1_0_0	0.9452542	0.1787309
1_1_1_0	0.8603427	0.1814943
1_1_X_1	0.7438652	0.2451652
Gen.spp	R^2	MAPE
Abies balsamea	0.8853746	0.08771392
Acer rubrum	0.7571125	0.19016789
Acer saccharum	0.8669543	0.15943329
*Betula alleghaniensis**	0.2522406	0.24653626
Betula papyrifera	0.9280011	0.10460046
Fagus grandifolia	0.7201647	0.33764875
Fraxinus americana	0.8494013	0.16192280
Fraxinus nigra	0.8933954	0.15669784
Picea glauca	0.8883162	0.14291459
Picea mariana	0.8932658	0.09530834
Pinus banksiana	0.9064566	0.12393481
Pinus resinosa	0.9760919	0.24941632
*Pinus strobus**	0.3597141	0.30494713
Populus balsamifera	0.9787318	0.07819522
Populus grandidentata	0.9619665	0.13720890
Populus tremuloides	0.9828579	0.08117716
Prunus serotina	0.7803310	0.17551391
Quercus alba	0.8681646	0.15492410
Quercus ellipsoidalis	0.8601216	0.21085503
Quercus rubra	0.8695992	0.21712078
Quercus velutina	0.9060684	0.18352255
Tilia americana	0.8345779	0.14319653

*influenced by significant outliers and /or small sample sizes in validation data.

Table A.7. Fit statistics for cull model (eqs. 7c and 9c).

MFT	R^2	MAPE
1_0_0_0*	0.55028	0.920854
1_0_1_0	0.808706	0.538755
1_1_0_0	0.745688	1.646442
1_1_1_0	0.667358	0.932050
1_1_X_1	0.732975	0.390993
All	0.71872	0.802836

*statistics are based on a relatively small sample size compared to others in table.

Table A.8. Coefficients and fit statistics for computing branch pulp wood volume (vib) from DBH for trees with MFT = 1_1_1_0 or X_1_1_0.

tree.group	spp.group	Λ	R^2	MAPE
hardwood	Acer spp.	0.02765820	0.444872	1.250648
hardwood	Fagus spp.	0.02059980	0.246278	0.982760
hardwood	other spp.	0.01804579	-0.05634	1.037809
hardwood	Populus spp.	0.01472092	0.587131	0.958597
hardwood	Quercus spp.	0.01563093	0.245395	0.710211
hardwood	Tilia spp.	0.01272266	0.410470	0.929022
softwood	Abies-Picea	0.01644674	0.622177	0.788865
softwood	other spp.	0.01610288	0.642754	0.819791
softwood	Pinus spp.	0.01672588	0.537566	0.773132

Table A.9. Coefficients and fit statistics for computing branch pulp wood volume (vib) from DBH and H4 for hardwood trees with MFT = 1_1_X_1_0 or X_1_X_1.

tree.group	spp.group	λ	μ	R^2	MAPE
hardwood	Acer & Fagus spp.	2.315388	0.000470802	0.098339	0.520389
hardwood	Quercus spp.	2.540968	0.000288213	0.209530	0.377351
hardwood	other spp.	2.549056	0.000282963	-0.34431	0.808507

Table B. Coefficients for computing inside bark volume (vib) from outside bark volume (vob) of the main stems of trees.

tree.group	Gen.spp	K	z
hardwoods	*Acer rubrum*	0.8716072	1.006526
hardwoods	*Acer saccharum*	0.8717193	1.006526
hardwoods	*Betula alleghaniensis*	0.8350805	1.006526
hardwoods	*Betula papyrifera*	0.8729216	1.006526
hardwoods	*Fagus grandifolia*	0.9136276	1.006526
hardwoods	*Fraxinus americana*	0.8652457	1.006526
hardwoods	*Fraxinus nigra*	0.8161391	1.006526
hardwoods	*Other Populus species*	0.8471101	1.006526
hardwoods	*Populus balsamifera*	0.8407326	1.006526
hardwoods	*Populus grandidentata*	0.8535249	1.006526
hardwoods	*Populus tremuloides*	0.8632788	1.006526
hardwoods	*Prunus serotina*	0.8713451	1.006526
hardwoods	*Quercus alba*	0.8296280	1.006526
hardwoods	*Quercus ellipsoidalis*	0.8452838	1.006526
hardwoods	*Quercus rubra*	0.8556711	1.006526
hardwoods	*Quercus velutina*	0.8578214	1.006526
hardwoods	*Tilia americana*	0.8498516	1.006526
softwoods	*Abies balsamea*	0.8738557	1.006526
softwoods	*Picea glauca*	0.8971559	1.006526
softwoods	*Picea mariana*	0.8806053	1.006526
softwoods	*Pinus banksiana*	0.8556939	1.006526
softwoods	*Pinus resinosa*	0.8753540	1.006526
softwoods	*Pinus strobus*	0.8667190	1.006526
softwoods	*Thuja occidentalis*	0.8734730	1.006526
any	other	0.8618102	1.006526
small trees	all	0.7799500	1.003010

Table B.1. Fit statistics for predicting inside bark saw volume from outside bark saw volume.

MFT	R^2	MAPE
1_1_0_0	0.9966181	0.03380487
1_1_1_0	0.9957324	0.03731509
1_1_X_1	0.9940725	0.03060583
Gen.spp	R^2	**MAPE**
Abies balsamea	0.9934818	0.07074056
Acer rubrum	0.9929690	0.03537652
Acer saccharum	0.9966678	0.02594086
Betula alleghaniensis	0.9761027	0.05789689
Betula papyrifera	0.9993204	0.01893625
Fagus grandifolia	0.9990173	0.02114696
Fraxinus americana	0.9968011	0.03505866
Fraxinus nigra	0.9934174	0.04529215
Picea glauca	0.9990885	0.03959304
Picea mariana	0.9993639	0.05164271
Pinus banksiana	0.9989258	0.08581080
Pinus resinosa	0.9937688	0.02623053
Pinus strobus	0.9958109	0.02252225
Populus balsamifera	0.9972493	0.05079600
Populus grandidentata	0.9987936	0.04253006
Populus tremuloides	0.9810782	0.05992648
Prunus serotina	0.9927301	0.03696848
Quercus alba	0.9926777	0.07111953
Quercus ellipsoidalis	0.9981651	0.03845241
Quercus rubra	0.9896889	0.03906661
Quercus velutina	0.9843506	0.04893295
Tilia americana	0.9956084	0.02651046

Table B.2. Fit statistics for predicting inside bark pulp volume from outside bark pulp volume.

MFT	R^2	MAPE
1_0_0_0	0.989505	0.05508301
1_0_1_0	0.986148	0.04350994
1_1_0_0	0.991372	0.04878773
1_1_1_0	0.99122	0.04108554
1_1_X_1	0.990678	0.04486963
Gen.spp	R^2	MAPE
Abies balsamea	0.989991	0.02950078
Acer rubrum	0.990498	0.04362018
Acer saccharum	0.994666	0.03138313
Betula alleghaniensis	0.940507	0.1055473
Betula papyrifera	0.987527	0.03510305
Fagus grandifolia	0.995671	0.02230842
Fraxinus americana	0.988477	0.04299735
Fraxinus nigra	0.986190	0.04625635
Picea glauca	0.988157	0.04185238
Picea mariana	0.971605	0.05648842
Pinus banksiana	0.976877	0.07536381
Pinus resinosa	0.997594	0.05309998
Pinus strobus	0.980855	0.05304150
Populus balsamifera	0.981290	0.07285231
Populus grandidentata	0.988070	0.04330514
Populus tremuloides	0.986550	0.04798457
Prunus serotina	0.996689	0.03550426
Quercus alba	0.987694	0.03475740
Quercus ellipsoidalis	0.983814	0.05075169
Quercus rubra	0.986566	0.04222683
Quercus velutina	0.993654	0.04448724
Thuja occidentalis	0.969552	0.04858028
Tilia americana	0.991681	0.04532404

Table C. Fit statistics for branch volume prediction models.

volume.type	tree.group	MFT	R^2	MAPE
pulp	All	1_0_1_0	0.3237751	0.5151644
pulp	hardwoods	1_0_1_0	0.2781700	0.5336200
pulp	softwoods	1_0_1_0	0.4114200	0.4779600
pulp	All	1_1_1_0	0.4192773	1.0112700
pulp	hardwoods	1_1_1_0	0.4097028	1.0648601
pulp	softwoods	1_1_1_0	0.5653791	0.7799029
pulp	hardwoods	1_1_X_1	0.1579553	0.5372473
pulp	softwoods	1_1_X_1	0.1952240	0.4687440
saw	hardwoods	1_1_X_1	0.1612799	0.6675936
saw	softwoods	1_1_X_1	0.6897340	0.4048618
total	hardwoods	1_1_X_1	0.2846762	0.3743349
total	softwoods	1_1_X_1	0.7472214	0.2264368

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