Appendix 4-C

Open Channel Theory

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4.C.1 Open Channel Flow Theory

Design analysis of both natural and artificial channels proceeds according to the basic principles of open channel flow (see Chow, 1970; Henderson, 1966). The basic principles of fluid mechanics -- continuity, momentum, and energy -- can be applied to open channel flow with the additional complication that the position of the free surface is usually one of the unknown variables. The determination of this unknown is one of the principle problems of open channel flow analysis and it depends on quantification of the flow resistance. Natural channels display a much wider range of roughness values than artificial channels.

4.C.2 Concepts

4.C.2.1. Specific Energy

Specific energy, E, is defined as the energy head relative to the channel bottom. If the channel is not too steep (slope less than 10 percent) and the streamlines are nearly straight and parallel (so that the hydrostatic assumption holds), the specific energy, E, becomes the sum of the depth and velocity head:

$$E = y + \alpha\,(V^2/2g) \qquad (4.C.1)$$

Where: y = depth, feet
α = velocity distribution coefficient (see Equation 4.C.2)
V = mean velocity, fps
g = gravitational acceleration, 32.2 ft./s^2

The velocity distribution coefficient is taken to have a value of one for turbulent flow in prismatic channels but may be significantly different than one in natural channels (see Equation 4.C.2).

4.C.2.2 Velocity Distribution Coefficient

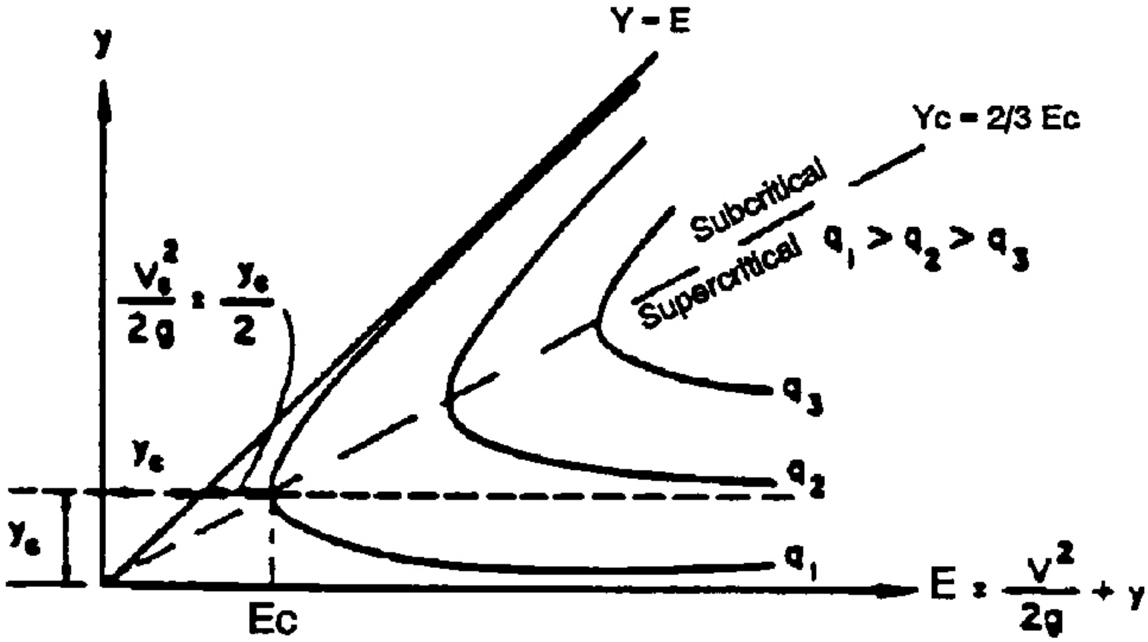
Due to the presence of a free surface and also due to friction along the channel boundary, the velocities in a channel are not uniformly distributed in the channel section. As a result of nonuniform distribution of velocities in a channel section, the velocity head of an open channel is usually greater than the average velocity head computed as $(Q/A_t)^2/2g$. A weighted average value of the velocity head is obtained by multiplying the average velocity head, above, by a velocity distribution coefficient, α, defined as:

$$\alpha = \sum_{i=1}^{n}(K_i^3/A_i^2)/(K_t^3/A_t^2) \qquad (4.C.2)$$

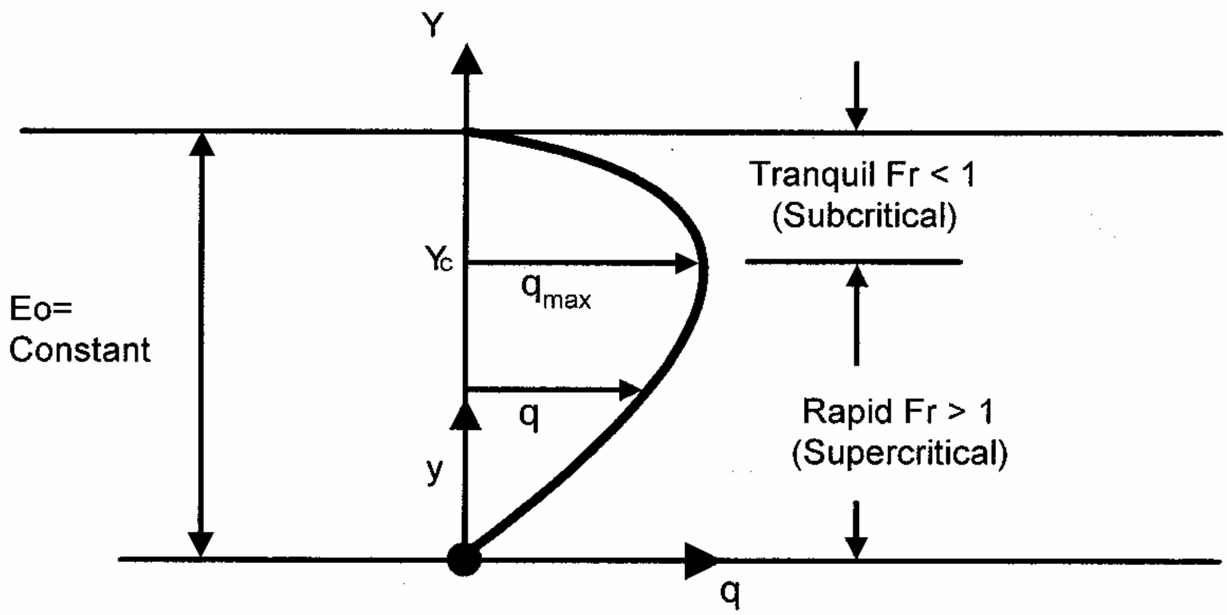
Where: K_i = conveyance in subsection (see Equation 4.C.15)
K_t = total conveyance in section (see Equation 4.C.15)
A_i = cross-sectional area of subsection, sf
A_t = total cross-sectional area of section, sf
n = number of subsections

4.C.2.3 Total Energy Head

The total energy head is the specific energy head plus the elevation of the channel bottom with respect to some datum. The locus of the energy head from one cross section to the next defines the energy grade line. See Figure 4-C-1, Specific Energy and Discharge Diagram for Rectangular Channels, for a plot of the specific energy diagram.



(a) Specific Energy Diagram



(b) Discharge Diagram

Figure 4-C-1 Specific Energy and Discharge Diagram for Rectangular Channels

(Adopted from *Highways in the River Environment*, 1990.)

4.C.2.4 Steady and Unsteady Flow

A steady flow is one in which the discharge passing a given cross section is constant with respect to time. The maintenance of steady flow in any reach requires that the rates of inflow and outflow be constant and equal. When the discharge varies with time, the flow is unsteady.

4.C.2.5 Uniform Flow and Nonuniform Flow

A nonuniform flow is one in which the velocity and depth vary in the direction of motion, while in uniform flow they remain constant. Uniform flow can only occur in a prismatic channel, which is a channel of constant cross section, roughness, and slope in the flow direction. Nonuniform flow can occur either in a prismatic channel or in a natural channel with variable properties.

4.C.2.6 Gradually Varied and Rapidly Varied

A nonuniform flow in which the depth and velocity change gradually enough in the flow direction that vertical accelerations can be neglected is referred to as a gradually varied flow; otherwise, it is considered to be rapidly varied.

4.C.2.7 Froude Number

The Froude number, Fr, is an important dimensionless parameter in open channel flow. It represents the ratio of inertial forces to gravitational forces and is defined by:

$$\mathbf{Fr = V/[(gd/\alpha)^{.5}]} \qquad \textbf{(4.C.3)}$$

Where: α = velocity distribution coefficient
V = mean velocity = Q/A, fps
g = acceleration of gravity, 32.2 feet/s^2
d = hydraulic depth = A/T, feet
A = cross-sectional area of flow, sf
T = channel top width at the water surface, feet
Q = total discharge, cfs

This expression for Froude number applies to any normal depth channel. For rectangular channels, the hydraulic depth is equal to the flow depth.

4.C.2.8 Critical Flow

Critical flow occurs when the specific energy is a minimum. The variation of specific energy with depth at a constant discharge shows a minimum in the specific energy at a depth called critical depth at which *the Froude number has a value of one*. Critical depth is also the depth of maximum discharge when the specific energy is held constant. These relationships are illustrated in Figure 4-C-1, Specific Energy and Discharge Diagram for Rectangular Channels. During critical flow, the velocity head is equal to half the hydraulic depth. The general expression for flow at critical depth is:

$$\alpha Q^2/g = A^3/T \qquad \textbf{(4.C.4)}$$

Where: α = velocity distribution coefficient
Q = total discharge, cfs
g = gravitational acceleration, 32.2 feet/s^2
A = cross-sectional area of flow, sf
T = channel top width at the water surface, feet

When flow is at critical depth, Equation 4.C.4 must be satisfied, no matter what the shape of the channel.

4.C.2.9 Subcritical Flow

Depths greater than critical occur in subcritical flow and the *Froude number is less than one*. In this state of flow, small water surface disturbances can travel both upstream and downstream, and the control is always located downstream.

4.C.2.10 Supercritical Flow

Depths less than critical depth occur in supercritical flow and the *Froude number is greater than one*. Small water surface disturbances are always swept downstream in supercritical flow, and the location of the flow control is always upstream.

4.C.2.11 Hydraulic Jump

A hydraulic jump occurs as an abrupt transition from supercritical to subcritical flow in the flow direction. There are significant changes in depth and velocity in the jump, and energy is dissipated. For this reason, the hydraulic jump is often employed under controlled situations and facilities to dissipate energy and control erosion at highway drainage structures.

A hydraulic jump will not occur until the ratio of the flow depth y_1 in the approach channel to the flow depth y_2 in the downstream channel reaches a specific value that depends on the channel geometry. The depth before the jump is called the initial depth, y_1, and the depth after the jump is the conjugate depth, y_2. When a hydraulic jump is used as an energy dissipater, controls to create sufficient tailwater depth are often necessary to control the location of the jump and to ensure that a jump will occur during the desired range of discharges. Sills can be used to control a hydraulic jump if the tailwater depth is less than the conjugate depth. If the tailwater depth is higher than the conjugate depth, a drop in the channel floor must be used in order to ensure a jump.

4.C.2.12 Velocity Distribution

In stream channels, the transverse variation of velocity in any cross section is a function of subsection geometry and roughness and may vary considerably from one stage and discharge to another. It is important to know this variation for purposes of designing erosion control measures and locating relief openings in highway fills for example. The best method of establishing transverse velocity variations is by current meter measurements. If this is not possible, the normal depth method can be used by dividing the cross section into subsections of relatively uniform roughness and geometry. It is assumed that the energy grade line slope is the same across the cross section so that the total conveyance K_t of the cross section is the sum of the subsection conveyances. The total discharge is then $K_t S^{1/2}$ and the discharge in each subsection is proportional to its conveyance. The velocity in each subsection is obtained from the continuity equation, V = Q/A.

4.C.2.13 Standard Step Method

The following discussion explains the theory behind the standard step method, beginning with the energy equation. The energy equation is:

$$h_1 + \alpha_1(V_1^2/2g) = h_2 + \alpha_2(V_2^2/2g) + h_L \quad \textbf{(4.C.5)}$$

Where: h_1 and h_2 are the upstream and downstream stages, respectively, feet
α = velocity distribution coefficient
V = mean velocity, fps
h_L = head loss due to local cross sectional changes (minor loss) as well as boundary resistance, feet

The stage h is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y, i.e., h = z+y. The energy equation is solved between successive stream reaches with nearly uniform roughness, friction slope, and cross-sectional properties.

The total head loss is calculated from:

$$h_L = K_m [(\alpha_1 V_1^2/2g) - (\alpha_2 V_2^2/2g)] + S_{f(avg)} L \quad \textbf{(4.C.6)}$$

Where:
K_m = expansion or contraction loss coefficient
$S_{f(avg)}$ = the mean slope of the energy grade line evaluated from Manning's equation and a selected averaging technique, feet/foot
L = discharge-weighted or conveyance-weighted reach length, feet

The standard step method solves these equations from cross section to cross section.

Guidance on the selection of expansion and contraction loss coefficients for bridges can be found in the HEC-RAS Hydraulic Reference Manual (USACE). Guidance on the design of transitions for culverts and the associated expansion and contraction loss coefficients can be found in HEC-14 (FHWA).

The default values of the minor loss coefficient K_m are 0.1 for contractions and 0.3 for expansions in HEC-RAS. The range of these coefficients, from ideal transitions to abrupt changes, are 0.1 to 0.5 for contractions and 0.3 to 1.0 for expansions, respectively.

HEC-RAS calculates a discharge-weighted reach length, L, as:

$$\mathbf{L = [(L_{lob}Q_{lob(avg)} + L_{ch}Q_{ch(avg)} + L_{rob}Q_{rob(avg)})/(Q_{lob(avg)} + Q_{ch(avg)} + Q_{rob(avg)})]} \qquad \textbf{(4.C.7)}$$

Where:

L_{lob}, L_{ch}, L_{rob} = flow distance between cross-sections in the left overbank, main channel and right overbank, respectively, feet

$Q_{lob(avg)}$, $Q_{ch(avg)}$, $Q_{rob(avg)}$ = arithmetic average of flows between cross section for the left overbank, main channel and right overbank, respectively, cfs

HEC-RAS allows the user the following options for determining the friction slope, $S_{f(avg)}$:

- Average conveyance equation

$$\mathbf{S_{f(avg)} = [(Q_u + Q_d)/(K_u + K_d)]^2} \qquad \textbf{(4.C.8)}$$

- Average friction slope equation

$$\mathbf{S_{f(avg)} = (S_{fu} + S_{fd})/2} \qquad \textbf{(4.C.9)}$$

- Geometric mean friction slope equation

$$\mathbf{S_{f(avg)} = (S_{fu}S_{fd})^{1/2}} \qquad \textbf{(4.C.10)}$$

- Harmonic mean friction slope equation

$$\mathbf{S_{f(avg)} = (2S_{fu}S_{fd})/(S_{fu} + S_{fd})} \qquad \textbf{(4.C.11)}$$

Where:

Q_u, Q_d = discharge at the upstream and downstream cross sections, respectively, cfs
K_u, K_d = conveyance at the upstream and downstream cross sections, respectively
S_{fu}, S_{fd} = friction slope at the upstream and downstream cross sections, respectively, feet/foot

The default option is the average conveyance equation in HEC-RAS.

4.C.2.14 Standard Step Computation Worksheet

A sample worksheet is taken from *"Hydrologic Engineering Methods For Water Resources Development - Volume 6, Water Surface Profiles,"* The Hydrologic Engineering Center, USACE, Davis, California. The worksheet is used in accordance with the concepts presented in Section 4.4.1.5. Because of the large amount of calculation required, most designers will choose to use the HEC-RAS software.

A convenient form for use in calculating water surface profiles is shown in Figure 4-C-2, Standard Step Computation Worksheet. In summary, Columns 2 and 4 through 12 are devoted to solving Manning's equation to obtain the energy loss due to friction, Columns 13 and 14 contain calculations for the velocity distribution across the section, Columns 15 through 17 contain the average kinetic energy, Column 18 contains calculations for "other losses" (expansion and contraction losses due to interchanges between kinetic and potential energies as the water flows), and Column 19 contains the computed change in water surface elevation. Conservation of energy is accounted for by proceeding from section to section down the computation form.

Column 1 - CROSS SECTION NO. is the cross section identification number. Feet upstream from the mouth are recommended.

Column 2 - ASSUMED is the water surface elevation which must agree with the resulting computed water surface elevation within $\pm$ 0.5 inch, or some allowable tolerance, for trial calculations to be successful.

Column 3 - COMPUTED is the rating curve value for the first section, but thereafter is the value calculated by adding WS to the computed water surface elevation for the previous cross-section.

Column 4 - A is the cross section area. If the section is complex and has been subdivided into several parts (e.g., left overbank, channel, and right overbank) use one line of the form for each sub-section and sum to get A_t, the total area of cross section.

Column 5 - R is the hydraulic radius. Use the same procedure as for Column 4 if section is complex, but do not sum subsection values.

Column 6 - $R^{2/3}$ is 2/3 power of hydraulic radius.

Column 7 - n is Manning's roughness coefficient.

Column 8 - K is conveyance and is defined as ($C\ AR^{2/3}/n$) where C is 1.49 for English units (C is 1.0 for Metric units). If the cross section is complex, sum subsection K values to get K_t.

Column 9 - $K_{t(avg)}$ is average conveyance for the reach, and is calculated by $0.5(K_{td} + K_{tu})$ where subscripts D and U refer to downstream and upstream ends of the reach, respectively.

Column 10 - $S_{f(avg)}$ is the average slope through the reach determined by $(Q/K_{t(avg)})^2$.

Column 11 - L is the discharge-weighted or conveyance-weighted reach length.

<u>Column 12</u> - h_f is energy loss due to friction through the reach and is calculated by $h_f = (Q/K_{t(avg)})^2L = S_{f(avg)}L$.

<u>Column 13</u> - (K^3/A^2) is part of the expression relating distributed flow velocity to an average value. If the section is complex, calculate one of these values for each subsection and sum all subsection values to get a total. If one subsection is used, Column 13 is not needed and Column 14 equals one.

<u>Column 14</u> - α is the velocity distribution coefficient and is calculated by $(K^3/A^2)/(K_t^3/A_t^2)$ where the numerator is the sum of values in Column 13 and the denominator is calculated from K_t and A_t.

<u>Column 15</u> - V is the average velocity and is calculated by Q/A_t.

<u>Column 16</u> - $\alpha V^2/2g$ is the average velocity head corrected for flow distribution.

<u>Column 17</u> - $\Delta(\alpha V^2/ 2g)$ is the difference between velocity heads at the downstream and upstream sections. A positive value indicates velocity is increasing; therefore, use a contraction coefficient for "other losses." A negative value indicates the expansion coefficient should be used in calculating "other losses."

<u>Column 18</u> - h_o is "other losses," and is calculated by multiplying either the expansion or contraction coefficient, K_m, times the absolute value of Column 17.

<u>Column 19</u> - ΔWS is the change in water surface elevation from the previous cross section. It is the algebraic sum of Columns 12, 17, and 18.

Cross Section No.	Water Surface Elevation Assumed	Water Surface Elevation Computed	Area	Hydraulic Radius R	$R^{2/3}$	n	K	$\overline{K}_t$	1000 $\overline{S}_f$	L	h_f	K^3/A^2	α	V	$\alpha V^2/2g$	$\Delta(\alpha V^2/2g)$	h_o	Δ Water Surface Elevation
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)

Figure 4-C-2 Standard Step Computation Worksheet

4.C.3 Flow Classification

The classification of open channel flow can be summarized as follows:

Steady Flow:

1. Uniform Flow
2. Nonuniform Flow
   a. Gradually Varied Flow
   b. Rapidly Varied Flow

Unsteady Flow:

1. Unsteady Uniform Flow (rare)
2. Unsteady Nonuniform Flow
   a. Gradually Varied Unsteady Flow
   b. Rapidly Varied Unsteady Flow

The steady uniform flow case and the steady nonuniform flow case are the most fundamental types of flow treated in highway engineering hydraulics. The following procedures address steady flow.

4.C.4 Equations

The following equations are those most commonly used to analyze open channel flow. The use of these equations in analyzing open channel hydraulics is discussed in Sections 4.C.1 and 4.C.2.

4.C.4.1 Continuity Equation

The continuity equation is the statement of conservation of mass in fluid mechanics. For the special case of one dimensional, steady flow of an incompressible fluid, it assumes the simple form:

$$\mathbf{Q = A_1V_1 = A_2V_2} \qquad \textbf{(4.C.12)}$$

Where: Q = discharge, cfs
A = cross-sectional area of flow, sf
V = mean cross-sectional velocity, fps (velocity perpendicular to the cross section)

Subscripts 1 and 2 refer to successive cross sections along the flow path.

4.C.4.2 Manning's Equation

For a given depth of flow in an open channel with a steady, uniform flow, the mean velocity, V, can be computed with Manning's equation:

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \qquad \textbf{(4.C.13)}$$

Where: V = velocity, fps
n = Manning's roughness coefficient
R = hydraulic radius = A/P, feet
P = wetted perimeter, feet
S = slope of the energy gradeline, feet/foot.
(Note: For steady uniform flow, S = channel slope, feet/foot)

The selection of Manning's n is generally based on observation; however, considerable experience is essential in selecting appropriate n values. The selection of Manning's n is discussed in Section 4.4.1.2. The range of n values for various types of channels and floodplains is given in Table 4-1.

The continuity equation can be combined with Manning's equation to obtain the steady, uniform flow discharge as:

$$Q = (1.49/n)AR^{2/3}S^{1/2} \qquad \textbf{(4.C.14)}$$

For a given channel geometry, slope, and roughness, and a specified value of discharge Q, a unique value of depth occurs in steady, uniform flow. It is called normal depth and is computed from Equation 4.C.13 by expressing the area and hydraulic radius in terms of depth. The resulting equation may require a trial and error solution. See Section 4.4.1.5 for a more detailed discussion of the computation of normal depth.

If the normal depth is greater than critical depth, the slope is classified as a mild slope, while on a steep slope, the normal depth is less than critical depth. Thus, uniform flow is subcritical on a mild slope and supercritical on a steep slope.

<u>Conveyance</u>

In channel analysis, it is often convenient to group the channel properties in a single term called the channel conveyance K:

$$K = (1.49/n)AR^{2/3} \qquad \textbf{(4.C.15)}$$

and then Equation 4.C.14 can be written as:

$$Q = KS^{1/2} \qquad \textbf{(4.C.16)}$$

The conveyance represents the carrying capacity of a stream cross section based upon its geometry and roughness characteristics alone and is independent of the streambed slope.

The concept of channel conveyance is useful when computing the distribution of overbank flood flows in the stream cross-section and the flow distribution through the opening in a proposed stream crossing. It is also used to determine the velocity distribution coefficient, α (see Equation 4.C.6).

4.C.4.3 Energy Equation

The energy equation expresses conservation of energy in open channel flow. The energy is expressed as energy per unit weight of fluid, which has dimensions of length, and is called energy head. The energy head is composed of potential energy head (elevation head), pressure head, and kinetic energy head (velocity head). These energy heads are scalar quantities which give the total energy head at any cross-section when added. Written between an upstream open channel cross section designated 1 and a downstream cross section designated 2, the energy equation is:

$$h_1 + \alpha_1(V_1^2/2g) = h_2 + \alpha_2(V^2_2/2g) + h_L \qquad \textbf{(4.C.17)}$$

Where: h_1 and h_2 are the upstream and downstream stages, respectively, feet
- α = velocity distribution coefficient
- V = mean velocity, fps
- h_L = head loss due to local cross-sectional changes (minor loss) as well as boundary resistance, feet

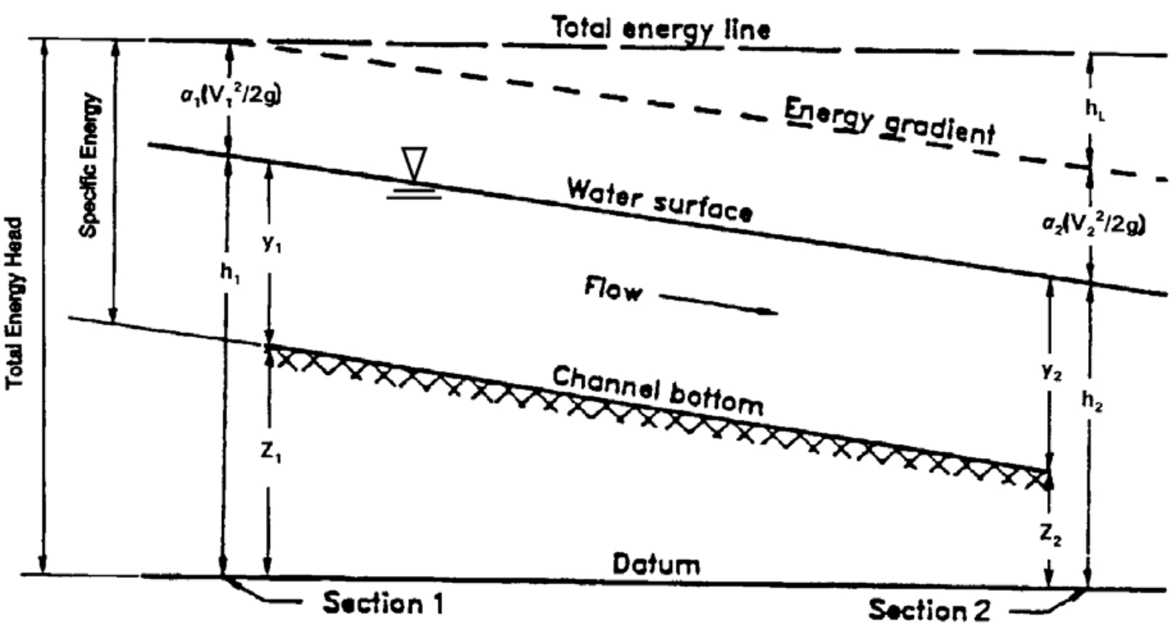


Figure 4-C-3 Terms in the Energy Equation

Source: FHWA, 1990

The stage h is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y, i.e., h = z + y. The terms in the energy equation are illustrated graphically in Figure 4-C-3. The energy equation states that the total energy head at an upstream cross section is equal to the energy head at a downstream section plus the intervening energy head loss. The energy equation can only be applied between two cross sections at which the streamlines are nearly straight and parallel so that vertical accelerations can be neglected (slopes approximately less than 10 percent).

4.C.4.4 Shear Stress Equation

Shear stress is the longitudinal stress on a channel boundary produced by flowing water. Shear stress is important as it is a primary cause of channel bank erosion.

The equation for shear stress is:

$$\tau_O = \gamma RS_f \qquad \textbf{(4.C.18)}$$

Where: τ_O = shear stress (lb./in.^{2} or lb./ft.^{2})
γ = unit weight of water (62.4 lb./ft.^{3})
R = hydraulic radius of channel (ft.)
S_f = slope of energy grade line (ft./ft.)

In wide channels, the hydraulic radius is approximately equivalent to water depth. For uniform flow, the channel slope equals the slope of the energy grade line.