MATHEMATICS Companion Document

# Looking Across the Grades 6th Grade through Geometry

## GRADE LEVEL CONTENT EXPECTATIONS



v.12.05

### Looking Across the Grades 6<sup>th</sup>-Geometry

This "cross-grade" format shows how the 6<sup>th</sup>- 8<sup>th</sup> Grade Level Content Expectations (GLCE) provides the prerequisite skills and concepts for Geometry. The document is divided into tables, each focusing on a common concept. The Geometry expectations are listed in the order that they appear in Course Credit Requirements. The corresponding GLCE are from the Geometry and Measurement strands. While the topics across the columns are related, there is not necessarily a one-to-one correspondence of the expectations.

This document is intended to help middle school and Geometry teachers make the connections between what is expected in middle school to what will be taught in Geometry. This may be especially useful for those teaching Geometry in 8<sup>th</sup> grade to identify prerequisites students may be lacking who are going directly from 7<sup>th</sup> grade to Geometry.

A similar document is available aligning the 6<sup>th</sup>- 8<sup>th</sup> GLCE with the Algebra I course.

#### Looking Across the Grades 6th-Geometry

	Basic Euclidean and Coordinate Geometry			
Sixth Grade	Seventh Grade	Eighth Grade	Geometry	
Understand and apply basic			G1.1 Lines and Angles; Basic Euclidean and Coordinate Geometry	
<ul> <li>G.GS.06.01 Understand and apply basic</li> <li>properties</li> <li>G.GS.06.01 Understand and apply basic properties of lines, angles, and triangles, including: <ul> <li>triangle inequality</li> <li>relationships of vertical angles, complementary angles, and supplementary angles</li> <li>congruence of corresponding and alternate interior angles when parallel lines are cut by a transversal, and that such congruencies imply parallel lines</li> <li>locate interior and exterior angles of any triangle, and use the property that an exterior angle of a triangle is equal to the sum of the remote (opposite) interior angles</li> <li>know that the sum of the exterior angles of a convex polygon is 360°.</li> </ul> </li> <li>Construct geometric shapes</li> <li>G.SR.06.05 Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors; justify informally.</li> </ul>	<ul> <li>Draw and construct geometric objects</li> <li>G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles, and parallelograms with specified dimensions.</li> <li>G.SR.07.02 Use a compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.</li> </ul>		<ul> <li>Euclidean and Coordinate Geometry</li> <li>G1.1.1 Solve multi-step problems and construct proofs involving vertical angles linear pairs of angles, supplementary angles, complementary angles, and right angles.</li> <li>G1.1.2 Solve multi-step problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.</li> <li>G1.1.3 Perform and justify constructions including midpoint of a line segment and bisector of an angle, using a straightedge and compass.</li> <li>G1.1.4 Given a line and a point, construct a line through the point that is parallel to the original line using a straightedge and compass; given a line and a point, construct a line through the point that is perpendicular to the original line; justify the steps of the constructions.</li> <li>G1.1.5 Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.</li> <li>G1.1.6 Recognize Euclidean Geometry as an axiom system; know the key axioms and understand the meaning of and distinguish between undefined terms, axioms, definitions, and theorems.</li> </ul>	

	Triangles ar	nd their Properties	
Sixth Grade	Seventh Grade	Eighth Grade	Geometry
	Understand and use the Pythagorean Theorem	Pythagorean Theorem	G1.2 Triangles and Their Properties G1.2.1 Prove that the angle sum of a
		<ul> <li>G.GS.08.01 Understand at least one proof of the Pythagorean Theorem; use the Pythagorean Theorem and its converse to solve applied problems including perimeter, area, and volume problems.</li> <li>G.LO.08.02 Find the distance between two points on the coordinate plane using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.</li> </ul>	triangle is 180° and that an exterior angle of a triangle is the sum of the two remote interior angles.
			<b>G1.2.2</b> Construct and justify arguments and solve multi-step problems involving angle measure, side length, perimeter,
			and area of all types of triangles.
			<b>G1.2.3</b> Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem and its converse to solve multi- step problems.
			<b>G1.2.4</b> Prove and use the relationships among the side lengths and the angles of $30^{\circ}$ - $60^{\circ}$ - $90^{\circ}$ triangles and $45^{\circ}$ - $45^{\circ}$ - $90^{\circ}$ triangles.
			<b>G1.2.5</b> Solve multi-step problems and construct proofs about the properties of medians, altitudes, and perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle; using a straightedge and compass, construct these lines.

Properties of Geometric Figures			
Sixth Grade	Seventh Grade	Eighth Grade	Geometry
		Solve problems about geometric figures	G1.5 Other Polygons and Their Properties
		<ul> <li>G.SR.08.04 Find area and perimeter of complex figures by sub-dividing them into basic shapes (quadrilaterals, triangles, circles).</li> <li>G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles.</li> <li>Solve problems about geometric figures</li> <li>G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems.</li> </ul>	<b>G1.5.1</b> Know and use subdivision or circumscription methods to find areas of polygons.
			<b>G1.5.2</b> Know, justify, and use formulas for the perimeter and area of a regular n-gon and formulas to find interior and exterior angles of a regular n-gon and their sums.
			G1.6 Circles and Their Properties
			<b>G1.6.1</b> Solve multi-step problems involving circumference and area of circles.
			<b>G1.6.2</b> Solve problems and justify arguments about chords and lines tangent to circles.
			<b>G1.6.3</b> Solve problems and justify arguments about central angles, inscribed angles and triangles in circles.
			<b>G1.6.4</b> Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.

Volume and Surface Area			
Sixth Grade	Seventh Grade	Eighth Grade	Geometry
		Understand concepts of volume	G1.8 Three-Dimensional Figures
		and surface area, and apply formulas G.SR.08.06 Know the volume formulas for generalized cylinders ((area of base) x height), generalized cones and pyramids (1/3 (area of base) x height), and spheres ( 4/3 n (radius) 3 ) and apply them to solve problems.	<b>G1.8.1</b> Solve multi-step problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.
			<b>G1.8.2</b> Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.
			G2.1 Relationships Between Area and Volume Formulas
		<b>G.SR.08.07</b> Understand the concept of surface area, and find the surface area of prisms, cones, spheres, pyramids, and cylinders.	<b>G2.1.1</b> Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.
			<b>G2.1.2</b> Know and demonstrate the relationships between the area formulas of various quadrilaterals.
Find volume and surface area			<b>G2.1.3</b> Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).
<b>M.PS.06.02</b> Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets).		Visualize solids	G2.2 Relationships Between Two- Dimensional and Three-Dimensional Representations
<b>M.TE.06.03</b> Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides, using formulas.		<b>G.SR.08.08</b> Sketch a variety of two- dimensional representations of three- dimensional solids including orthogonal views (top, front, and side), picture	<b>G2.2.1</b> Identify or sketch a possible 3- dimensional figure, given 2-dimensional views; create a 2-dimensional representation of a 3-dimensional figure.
Convert within measurement	views (projective or isometric), and nets; use such two-dimensional representations to help solve problems.	nets; use such two-dimensional representations to help solve	<b>G2.2.2</b> Identify or sketch cross-sections of 3-dimensional figures; identify or sketch solids formed by revolving 2-dimensional figures around lines.
systems			L2.3 Measurement Units, Calculations,
<b>M.UN.06.01</b> Convert between basic units of measurement within a single measurement system, e.g., square inches to square feet.			and Scales L2.3.1 Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.

Congruence and Similarity			
Sixth Grade	Seventh Grade	Eighth Grade	Geometry
Understand the concept of congruence and basic transformations	Understand the concept of similar polygons, and solve related problems		<b>G2.3 Congruence and Similarity</b> <b>G2.3.1</b> Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria,
<ul> <li>G.GS.06.02 Understand that for polygons, congruence means corresponding sides and angles have equal measures.</li> <li>G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.</li> <li>G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection.</li> </ul>	G.TR.07.03 Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor. G.TR.07.04 Solve problems about similar figures and scale drawings. G.TR.07.05 Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments. G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r <sup>2</sup> .	Understand and apply concepts of transformation and symmetry G.TR.08.09 Understand the definition of a dilation from a point in the plane, and relate it to the definition of similar polygons. G.TR.08.10 Understand and use reflective and rotational symmetries of two-dimensional shapes and relate them to transformations to solve problems.	<ul> <li>using the 355, 3A5, A3A, and AAS criteria, and for right triangles, the hypotenuse-leg criterion.</li> <li>G2.3.2 Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.</li> <li>G2.3.3 Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.</li> <li>G2.3.4 Use theorems about similar triangles to solve problems with and without use of coordinates.</li> <li>G2.3.5 Know and apply the theorem stating that the effect of a scale factor of k relating one two dimensional figure to another or one three dimensional figure to another or one three dimensional figure to another, on the length, area, and volume of the figures is to multiply each by k, k<sup>2</sup>, and k<sup>3</sup>, respectively.</li> <li>G3.1.1 Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.</li> <li>G3.1.2 Given two figures that are images of each other under an isometry, find the isometry and describe it completely.</li> <li>G3.1.3 Find the image of a figure under the composition of two or more isometries, and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.</li> <li>G3.2.1 Know the definition of dilation, and find the image of a figure under a given isometries.</li> </ul>

The following expectations appear in the Geometry course/credit descriptions but have no direct link to the middle school expectations.

#### L1.1 Number Systems and Number Sense

**L1.1.6** Explain the importance of the irrational numbers  $\sqrt{2}$  and  $\sqrt{3}$  in basic right triangle trigonometry; and the importance of  $\pi$  because of its role in circle relationships.

#### L1.2 Representations and Relationships

**L1.2.3** Use vectors to represent quantities that have magnitude and direction; interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.

#### L3.1 Mathematical Reasoning

**L3.1.1** Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

**L3.1.2** Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.

**L3.1.3** Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics; identify and give examples of each.

#### L3.2 Language and Laws of Logic

L3.2.1 Know and use the terms of basic logic.

**L3.2.2** Use the connectives "NOT," "AND," "OR," and "IF...,THEN," in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.

**L3.2.3** Use the quantifiers "THERE EXISTS" and "ALL" in mathematical and everyday settings and know how to logically negate statements involving them.

**L3.2.4** Write the converse, inverse, and contrapositive of an "If..., then..." statement; use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original while the inverse and converse are not.

#### L3.3 Proof

L3.3.1 Know the basic structure for the proof of an "If..., then..." statement (assuming the hypothesis and ending with the conclusion) and know that proving the contrapositive is equivalent.
L3.3.2 Construct proofs by contradiction; use counterexamples, when appropriate, to disprove a statement.

**L3.3.3** Explain the difference between a necessary and a sufficient condition within the statement of a theorem; determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.

#### **G1.3 Triangles and Trigonometry**

**G1.3.1** Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides; solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles. **G1.3.2** Know and use the Law of Sines and the Law of Cosines and use them to solve problems; find the area of a triangle with sides a and b and included angle using the formula Area = (1/2) a b sin. **G1.3.3** Determine the exact values of sine, cosine, and tangent for 0°, 30°, 45°, 60°, and their integer multiples, and apply in various contexts.

#### **G1.4 Quadrilaterals and Their Properties**

**G1.4.1** Solve multi-step problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.

**G1.4.2** Solve multi-step problems and construct proofs involving quadrilaterals using Euclidean methods or coordinate geometry. **G1.4.3** Describe and justify hierarchical relationships among quadrilaterals.

**G1.4.4** Prove theorems about the interior and exterior angle sums of a quadrilateral.



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