ESSENTIAL ELEMENTS FOR GRADES 11-12: MATHEMATICS

Note: These Essential Elements for Grades 11-12 with Michigan Range of Complexity are only assessed at the state level in Grade 11. They also serve as the foundation for instruction in grade 12. There is not a separate document for grade 12.

**Claim #1: Students demonstrate increasingly complex understanding of number sense.

Number and Quantity

Target Essential Element	Michigan Range of Complexity		
	High Range	**Medium Range**	**Low Range**
Michigan Grades 11-12 Standard for Mathematics: N-RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, "We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5."*			
EE.N-RN.1: Determine the value of a quantity that is squared or cubed.	**EE.N-RN.H.1**: The student can extend or describe a number pattern involving doubling, including determining the value of a quantity that is squared to solve a problem.	**EE.N-RN.M.1**: The student can select appropriate numbers and/or quantities to solve problems. Quantities limited to no more than 50.	**EE.N-RN.L.1**: The student can select appropriate numbers and/or quantities to solve problems. Quantities limited to no more than 5.

****Claim #2: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.**

Geometry

Target Essential Element	Michigan Range of Complexity		
	High Range	Medium Range	Low Range
Michigan Grades 11-12 Standard for Mathematics: G-CO.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **Michigan Grades 11-12 Standard for Mathematics: G-CO.7**: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **Michigan Grades 11-12 Standard for Mathematics: G-CO.8**: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.			
EE.G-CO.6-8: Identify corresponding congruent and similar parts of shapes.	**EE.G-CO.H.6-8**: The student can identify corresponding congruent angles in two similar triangles.	**EE.G-CO.M.6-8**: The student can identify corresponding sides in similar shapes when presented in context.	**EE.G-CO.L.6-8**: The student can determine which of two similar shapes or objects is bigger or smaller.

****Claim #3: Students demonstrate increasingly complex understanding of measurement, data and analytic procedures.**

Statistics and Probability

Target Essential Element	Michigan Range of Complexity		
	High Range	**Medium Range**	**Low Range**
Michigan Grades 11-12 Standard for Mathematics: S-ID.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).			
EE.S-ID.3: Interpret general trends on a graph or chart.	**EE.S-ID.H.3**: The student can solve problems, describe trends, or make predictions based on data in tables, charts, or graphs.	**EE.S-ID.M.3**: The student can use a graph or scatter plot to determine a trend using informal language (e.g., increasing, decreasing).	**EE.S-ID.L.3**: The student can sort given data into two groups.
Michigan Grades 11-12 Standard for Mathematics: S-IC.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **Michigan Grades 11-12 Standard for Mathematics: S-IC.2**: Decide if a specified model is consistent with results from a given data generating process, e.g., using simulation. For example, *"A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?"*			
EE.S-IC.1-2: Determine the likelihood of an event occurring when the outcomes are equally likely to occur.	**EE.S-IC.H.1-2**: When presented with a scenario, the student can correctly predict which outcome of a real-world event is more likely to occur.	**EE.S-IC.M.1-2**: The student can determine two possible outcomes of an event using real-world examples.	**EE.S-IC.L.1-2**: The student can identify one possible outcome of a real-world event (e.g., weather, games, etc.).

****Claim #4: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.**

Algebra

Target Essential Element	Michigan Range of Complexity		
	High Range	**Medium Range**	**Low Range**
Michigan Grades 11-12 Standard for Mathematics: A-SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments*.			
EE.A-SSE.4: Determine the successive term in a geometric sequence given the common ratio.	**EE.A-SSE.H.4**: The student can describe or extend a simple geometric sequence.	**EE.A-SSE.M.4**: The student can recognize a ratio relationship of 2:1 or 1:2 (e.g., 2 circles to 1 square).	**EE.A-SSE.L.4**: The student can recognize double the quantity of an item with a total quantity up to 10.

Functions

Target Essential Element	Michigan Range of Complexity		
	High Range	**Medium Range**	**Low Range**
Michigan Grades 11-12 Standard for Mathematics: F-IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x.* The graph of *f i*s the graph of the equation *y = f(x).* **Michigan Grades 11-12 Standard for Mathematics: F-IF.2**: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Michigan Grades 11-12 Standard for Mathematics: F-IF.3**: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by *f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.*			
EE.F-IF.1-3: Use the concept of function to solve problems.	**EE.F.IF.H.1-3**: The student can use a simple function table to solve a real-world problem.	**EE.F-IF.M.1-3**: The student can identify what quantity of data is needed to answer a question, solve a problem, or complete a graph or table involving a fixed pattern.	**EE.F-IF.L.1-3**: The student can identify which of 2 choices is needed to answer a question or solve a problem.

Target Essential Element	Michigan Range of Complexity		
	High Range	**Medium Range**	**Low Range**
Michigan Grades 11-12 Standard for Mathematics: F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **Michigan Grades 11-12 Standard for Mathematics: F-IF.5**: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function. **Michigan Grades 11-12 Standard for Mathematics: F-IF.6**: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.			
EE.F-IF.4-6: Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.	**Michigan Range of Complexity:** Not measured at state level, range of complexity determined at classroom level.		
Michigan Grades 11-12 Standard for Mathematics: F-BF.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.			
EE.F-BF.2: Determine an arithmetic sequence with whole numbers when provided a recursive rule.	**EE.F-BF.H.2**: The student can extend or describe repeating number patterns or patterns found in daily life such as calendars and schedules.	**EE.F-BF.M.2**: The student can order the days of the week or months of the year in the correct sequence, up to 7 consecutive days of the week and 12 consecutive months of the year.	**EE.F.BF.L.2**: The student can use ordinal terms to identify position in a pattern or sequence (e.g., 1st, 2nd, 3rd, first, last).

Target Essential Element	Michigan Range of Complexity		
	High Range	Medium Range	Low Range
Michigan Grades 11-12 Standard for Mathematics: F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. A. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. B. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. C. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **Michigan Grades 11-12 Standard for Mathematics: F-LE.2**: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **Michigan Grades 11-12 Standard for Mathematics: F-LE.3**: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.			
EE.F-LE.1-3: Model a simple linear function such as *y = mx* to show that these functions increase by equal amounts over equal intervals.	**Michigan Range of Complexity:** Not measured at state level, range of complexity determined at classroom level.		

Target Essential Elements as developed by: Dynamic Learning Maps Consortium (2013). Dynamic Learning Maps Essential Elements for Mathematics. Lawrence, KS: University of Kansas.