MOTION MODULE

Updated and revised in 2017 for the Michigan Department of Transportation (MDOT)

This manual was originally composed as part of the Transportation and Civil Engineering (TRAC) Program created by the American Association of State Highway and Transportation Officials (AASHTO). For more information on the original manual, see the complete final report, NCHRP 20-52.

The manual was updated and revised in 2017 by the Center for Technology & Training (CTT) at Michigan Technological University for the Michigan Department of Transportation (MDOT).
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Executive Summary

This module contains six activities and covers the educational topics of momentum and impulse that are normally taught in a high school physics course or an intermediate school physical science course. Transportation topics include automobile collision analysis and roadside crash barrier design. Specific relationships to the National Science Education Standards (NSES) pertaining to momentum, impulse, energy, Newton’s laws of motion, and scientific inquiry skills are outlined in the individual activities. Each activity contains the following sections:

1. **Instructor’s Reference.** This section is intended for both instructor and volunteer use and contains an activity summary and preparation information for the activity.
2. **Instructor’s Answer Key & Discussion Ideas.** This section serves as an instructor companion to the Research Manual and Research Notes and contains all answers to the questions given to students. It also contains suggested points of discussion that relate to the activity.
3. **Research Manual.** This section is intended for student use and contains all background, setup, and procedure information and instructions for completing the activity.
4. **Research Notes.** This section is intended for student use while working on the activity and lists the same questions found in the Instructor’s Answer Key & Discussion Ideas section.

This manual also contains complete activity overview tables, instructor introduction to the module, and a copy of the National Education Standards. Below are summaries and potential volunteer topics for each activity.

**Activity 1: Bump N Run**

Activity 1 demonstrates the Law of Conservation of Momentum for middle school students using a series of collisions between maglev cars of different mass. Students will be introduced to the concept of momentum and will learn to represent velocity and momentum by drawing vectors. They will then be asked to apply the concepts they use in the activity to real-world scenarios.

**Volunteer Topics**

*Automotive engineers, insurance investigators, or police officers* could provide insight into how momentum is used in their fields to evaluate vehicular accidents and how they use these tools to help lessen the likelihood and severity of future incidents.

**Activity 2: Calculated Collisions**

Activity 2 demonstrates the Law of Conservation of Momentum for high school students using a series of collisions between maglev cars of different mass. Students will be introduced to the concept of momentum and will calculate the velocity and momentum of the maglev cars before and after the collisions. They will then use their calculations to determine the average momentum.
Volunteer Topics

Automotive engineers, insurance investigators, or police officers could provide insight into how momentum is used in their fields to evaluate vehicular accidents and how they use these tools to help lessen the likelihood and severity of future incidents.

Activity 3: Collision Analysis

Activity 3 consists of three story problems which require students to use the Law of Conservation of Momentum to solve for specific variables. They will then use their findings to answer questions about the collisions.

Volunteer Topics

Automotive engineers, insurance investigators, or police officers could provide insight into how momentum is used in their fields to evaluate vehicular accidents and how they use these tools to help lessen the likelihood and severity of future incidents.

Activity 4: Impulse

Activity 4 introduces the concept of impulse and involves two demonstrations. In the first demonstration, students will toss water balloons back and forth to visualize the concept of impulse, and to see how differences in mass affect impulse. In the second demonstration, students will try to break an egg by tossing it at a sheet.

Volunteer Topics

Transportation or safety engineers could discuss how impulse plays into the work they do. They could discuss the benefits of traffic safety devices and how they decide what is best to use for various site conditions.

Activity 5: Egg Drop

Activity 5 is a hands-on activity in which students are given a limited number of materials to build a structure that will protect an egg during falls from increasing heights. The students will apply their knowledge of momentum and impulse to their designs and will also consider how the same concepts are applied in real-world automobile designs.

Volunteer Topics

Transportation engineers, safety engineers, automotive engineers, or industrial (packaging) engineers could relate the theory, design, and experimentation processes of the egg drop activity to the work they do on a regular basis.
Activity 6: Major Impacts

Activity 6 is a hands-on activity in which students design and build a crash barrier. The barriers are then tested on an inclined track by a vehicle impacting the barrier. Students will learn about guardrails and other traffic attenuation devices in this activity and will apply the knowledge they’ve learned about momentum to their designs.

Volunteer Topics

*Transportation or safety engineers* could discuss the benefits of many available traffic safety devices and how they decide what is best to use for various site conditions.
Instructor’s Introduction

Engineering is not simply about solving problems. It is about solving problems in the most efficient and elegant manner possible, while not creating new problems along the way. In order to come up with the most efficient solution, some amount of prior knowledge is usually needed. Frequently, this knowledge is mathematical or experiential.

For centuries, scientists, mathematicians, and engineers have studied the physical world and recorded their observations. They have derived mathematical formulas that describe the way materials and systems behave. They have also conducted experiments and drawn conclusions from their results. This body of knowledge that has accumulated over time is what engineers study and apply to solve problems every day. This process is what differentiates engineering from tinkering.

Tinkering is what we do when we try to solve problems by relying on trial-and-error. Tinkering can be fun, but it is usually not the most efficient way of solving a problem. Although solutions to engineering problems can sometimes be found by tinkering, these solutions tend to be neither efficient nor optimal.

Engineering can be fun too. There is a great deal of satisfaction to be gained from approaching a problem theoretically. Typically, an engineer will try to find a set of equations that describe the problem mathematically. These equations will give the engineer clues about how to solve the problem at hand. Using these clues, engineers can arrive at the optimal solution much more quickly than they could have if they had relied on tinkering alone.

As part of the TRAC & RIDES Program it is key to understand where funding comes from and how decisions are made in the world transportation planning/engineering. Transportation plays a huge role in our everyday lives, and Metropolitan Planning Organizations/Transportation Planning Organizations (or MPOs/TPOs) are a critical component of a city’s transportation system. MPOs help plan the future of transportation in a region, and chances are, there is a MPO in your city making decisions that affect all of us and how we get around. MPOs are made up of local elected officials, elected by the people of a city or region, who decide how to spend taxpayer money on transportation projects. MPOs plan all types of transportation, from roads and highways to public transit and bike lanes. Public involvement is very important to decision makers, and your voice matters! Learn more about your local MPO, and find out how you can get involved in planning the transportation system of the future. As you implement any of the TRAC & RIDES modules we suggest you investigate the MPO/TPO in your area and encourage your students to do the same. It will open up a whole new area where students can explore career opportunities in transportation planning and engineering.

Two websites to begin your student’s research:

- Association of Metropolitan Planning Organizations [www.ampo.org](http://www.ampo.org)
- National Association of Regional Councils [www.narc.org](http://www.narc.org)

In this module, students will use the concepts of momentum and impulse to solve problems. They will perform several demonstrations using maglev cars, water balloons, and eggs to visualize those concepts. In Activities 5 and 6, they will have the opportunity to apply what they have learned by designing and building safety structures to lessen impact.
National Education Standards

National Science Education Standards: Physical Science

Grades 5-8

Science as Inquiry

- Use appropriate tools and techniques to gather, analyze, and interpret data.
- Use technology and mathematics to improve investigations and communications.
- Think critically and logically to make the relationships between evidence and explanations.
- Formulate and revise scientific explanations and models using logic and evidence.

Science Content

Motion and Forces

- The motion of an object can be described by its position, direction of motion and speed. That motion can be measured.
- If more than one force acts on an object along a straight line, then the forces will reinforce or cancel one another, depending on their direction and magnitude. Unbalanced forces will cause changes in the speed or direction of an object’s motion.

Grades 9-12

Science as Inquiry

- Design and conduct scientific investigations.
- Use technology and mathematics to improve investigations and communications.
- Formulate and revise scientific explanations and models using logic and evidence.
- Recognize and analyze alternative explanations and models.

Science Content

Motion and Forces

- Objects change their motion only when a net force is applied. Laws of motion are used to calculate precisely the effects of forces on the motion of objects.

Conservation of Energy and Increase in Disorder

- Everything tends to become less organized and less orderly over time. Therefore, in all energy transfers, the overall effect is that the energy is spread out uniformly.

For the full documentation of TRAC and the National Education Standards, see the TRAC/Michigan Education Standards page on the MDOT website: http://www.michigan.gov/mdot/0,4616,7-151-9623_38029_38059_41397-184233--,00.html.
Activity 1: Bump-N-Run

Activity Summary

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<tr>
<td>National Science Education Standards</td>
<td>Appropriate tools Technology and mathematics Think critically Motion and forces Conservation of Energy</td>
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Introduction

This lab provides a conceptual understanding of The Law of Conservation of Momentum. It is suitable for use in a middle school physical science course. Questions are included at the end of the activity for additional practice and application. Due to the length of this activity, the pre-lab is recommended to be setup and completed as a class demonstration.

Note: Activities 1 and 2 are very similar, although Activity 2: Calculated Collisions involves more advanced concepts than Activity 1: Bump-N-Run. Both activities introduce the Law of Conservation of Momentum using maglev car collisions, but Activity 2 asks students to perform more in-depth calculations. Activity 1 can either be used as an introduction to Activity 2, or Activity 1 can be skipped in favor of Activity 2 for a more advanced class.

Objective

Students will be able to:

- Visualize the Law of Conservation of Momentum using a series of collisions between maglev cars of similar and different mass.
- Explain what they observed in terms of mass and velocity.
- Apply their knowledge of mass and velocity to predict the results of real-world collisions.

Activity Expansion Ideas

The collisions that occur on the maglev track are elastic collisions, meaning both momentum and kinetic energy are conserved during the collision. When the maglev cars collide, they will bounce off each other. A real-world car collision tends to be somewhat inelastic, meaning some energy is converted to sound, heat, and the deformation of the cars; therefore, the total momentum before and after the collision is not the same. In a real car collision, the cars will deform under impact, meaning the collision will tend to
be more inelastic in nature. A brief prelab illustrates the difference between elastic and inelastic collisions. This can be expanded upon through the following interactive web-based activities.

From PBS Learning Media (Collisions on an Air Track):  
https://wnmu.pbslearningmedia.org/resource/lsp07.sci.phys.maf.airtrack/collisions-on-an-air-track/

From PHET Interactive Simulations at the University of Colorado Boulder (Collision of Particles):  
https://phet.colorado.edu/en/simulation/legacy/collision-lab

From Physics Classroom (Collision of Carts):  
http://www.physicsclassroom.com/Physics-Interactives/Momentum-and-Collisions/Collision-Carts
Activity 1: Bump-N-Run

Part 1 & Part 2 Sample Answer:

For purposes of being concise, here is a sample for the answers in Part 1 and Part 2 rather than providing all of the diagrams. Students should draw the arrows in the direction the mass is moving before and after collision with the length of the arrows representing the general speed (short arrow for slow, long arrow for fast).

Questions

1) When both cars were in motion, what happened to the relative velocity of the car on the right as the car’s mass was increased?

   The velocity should have decreased as the mass increased. Considering that force is equal to mass times a change in velocity (acceleration), mass and acceleration have an inverse relationship when force is held constant. Because the elastic band is stretched the same amount each time, the force is held constant. As the mass is increased then, the change in velocity should decrease.

2) Knowing how the relative velocity changed, how do you think the momentum changed as the car’s mass was increased?

   The momentum would have remained constant. The elastic band applies the same force each time. Because momentum is the product of mass and velocity, as mass increases, velocity has to decrease if momentum is held constant.

3) In a real-world scenario, when a small sedan collides with a large truck traveling the same speed but in the opposite direction, which direction will the sedan likely travel after the collision? Describe your answer in terms of the same or opposite direction the sedan was traveling before the collision.

   The sedan will likely travel in the opposite direction.
4) In a real-world scenario, when a semi-truck collides with a small SUV traveling the same speed but in the opposite direction, in which direction will the semi-truck likely travel after the collision? Describe your answer in terms of the same or opposite direction the semi-truck was traveling before the collision.

The semi-truck will likely travel in the same direction after the crash.

Discussion
To understand momentum in this activity, we don’t consider every variable present. In reality, there are many factors that affect the outcome of a collision, such as friction, wind resistance, and the type and severity of the collision. Most collisions are neither truly elastic nor inelastic, but instead have elements of both. Even if we don’t notice these factors, they can have a great impact on the final results. Consider a scenario with high winds, icy roads, or other factors not represented in this activity and discuss how the results would differ.
Activity 1: Bump-N-Run

Introduction
In this activity, you will use maglev cars to demonstrate the Law of Conservation of Momentum. You will record the results of a series of collisions between maglev cars of different mass to visualize real-world collisions between vehicles of different sizes.

Objective
In these experiments, you will observe the following:

- How momentum is transferred between two vehicles in a collision.
- How vehicles with similar and dissimilar mass react in a collision.

Background
Momentum is defined as mass multiplied by velocity, and it is one of the core concepts to understanding basic physics. The Law of Conservation of Momentum can be stated many ways. One of those ways is as follows: during a collision, the sum of the momentum of all the vehicles before a collision equals the sum of the momentum of all vehicles after a collision. Mathematically this can be stated as:

\[
\text{Total momentum before collision } (p_i) = \text{Total momentum after collision } (p_f)
\]

\[
m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf}
\]

In this equation, the first vehicle is represented by the letter \(a\), and its mass and velocity before the collision are represented as \(m_a v_{ai}\) while its mass and velocity after the collision are represented as \(m_a v_{af}\). The second vehicle is represented by the letter \(b\).

If \(m_a = m_b\), then the equation simplifies to:

\[
m(v_{ai} + v_{bi}) = m(v_{af} + v_{bf})
\]

Where:

- \(m\) = mass
- \(v\) = velocity
- \(i\) = initial value before collision
- \(f\) = final value after collision

Momentum is a vector. A vector is defined as a quantity with a magnitude and a direction and is represented by an arrow. Magnitude is shown by the length of the arrow, while direction is indicated by where the arrow points. When entering data into this equation, consider the vehicles moving from left to right as moving in a positive direction. Vehicles traveling from left to right have a positive velocity in the formula, while vehicles traveling from right to left have a negative velocity. This concept is demonstrated in Figure 1-1.
**Figure 1-1: Magnitude and Direction of Vectors**

**Note:** When drawing a vector, the arrow gives the direction of the vector. When using magnitudes mathematically, it is important to use the sign convention stated previously to establish direction. Therefore, for the velocities shown in Figure 1-1 we would say that they are 2 m/s and -4 m/s when describing them or using them mathematically.

**Example 1**

Two cars of equal mass (1000 kg) collide as shown in Figure 1-2. Car A is moving at 2 m/s (with a positive velocity because it is traveling left to right) and Car B is moving at -4 m/s (with a negative velocity because it is traveling right to left). After the collision, Car A is moving at a velocity of -1 m/s. What is Car B’s velocity after the collision?

![Vehicle Vectors before Collision](image)

**Solution**

Using the equation for the Law of Conservation of Momentum, we can find the velocity of Car B after the collision:

\[ m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf} \]

\[ (1000 \text{ kg} \times 2 \text{ m/s}) + (1000 \text{ kg} \times (-4 \text{ m/s})) = (1000 \text{ kg} \times (-1 \text{ m/s})) + (1000 \text{ kg} \times v_{bf}) \]

\[ 2000 \frac{\text{kg} \times \text{m}}{\text{s}} + (-4000 \frac{\text{kg} \times \text{m}}{\text{s}}) = (-1000 \frac{\text{kg} \times \text{m}}{\text{s}}) + (1000 \text{ kg} \times v_{bf}) \]

\[ -2000 \frac{\text{kg} \times \text{m}}{\text{s}} = (-1000 \frac{\text{kg} \times \text{m}}{\text{s}}) + (1000 \text{ kg} \times v_{bf}) \]

\[ v_{bf} = -1 \frac{\text{m}}{\text{s}} \]
From our solution, we can determine that Car B was moving at a velocity of -1 m/s after the collision, as shown in Figure 1-3.

Figure 1-3: Vehicle Vectors after Collision

When two vehicles of the same mass collide, the transfer of momentum can be observed by looking at the velocity and direction of the cars. It is not so obvious when two vehicles of different mass collide. The momentum of each vehicle may change, but the total momentum must remain the same (ignoring losses from external forces).

There are two types of collisions, elastic and inelastic. Before a collision occurs, one or both of the objects is in motion and, therefore, has kinetic energy. When a vehicle collision occurs, some of that kinetic energy is converted to other forms of energy, such as sound, heat, and deformation of the vehicles. This is why the Law of Conservation of Energy is not fully observable in a real-world collision. If the transformation of kinetic energy to other forms of energy is significant, the vehicles will not bounce off each other and will instead stick together. This type of collision is called inelastic. In a collision with little or no loss in kinetic energy, the vehicles will bounce off each other, which is an elastic collision.

In the real world, there are many additional factors, such as friction and air resistance, that we will not consider in this activity. The maglev track reduces friction enough to assume it is non-existent, which allows us to examine the relationship between mass and velocity directly. Assuming friction is negligible, use the formula for average velocity to calculate the velocities of the cars.

\[
\text{Average velocity } \left( \frac{\text{m}}{\text{s}} \right) = \frac{\text{Total distance traveled (m)}}{\text{Total travel time (s)}}
\]

It is also important to note that we will be holding force constant in this activity by using an elastic band to launch the cars. It is assumed that during the first phase of the collision, when the cars collide, they

---

Does contact area affect friction?

Leonardo da Vinci was one of the first scholars to study friction systematically. In 1699, nearly 180 years after da Vinci’s death, the French architect and engineer Guillaume Amontons built upon da Vinci’s findings and discovered that frictional force is independent of the area of contact. This is why if you have two objects with the same mass, but one is half as long and twice as tall as the other, they will still experience the same amount of friction when dragged along a surface.

Coulomb’s Law of Friction is used to determine the frictional force of sliding objects.

\[
F_f = \mu_k N
\]

Where:
- \(F_f\) = frictional force
- \(\mu_k\) = coefficient of kinetic friction between two surfaces

Two Blocks of Equal Mass Experience the Same Frictional Force
are traveling with equal momentum. This activity focuses on what happens in the instant after the cars collide, before the second phase of the collision when the cars come to rest.

### Materials

<table>
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<tr>
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</tr>
<tr>
<td>Maglev car mass M</td>
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</tr>
<tr>
<td>Maglev car mass 2M</td>
<td>1</td>
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<tr>
<td>Maglev car mass 3M</td>
<td>2</td>
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<tr>
<td>Maglev car mass 4M</td>
<td>1</td>
</tr>
<tr>
<td>Maglev car mass 5M</td>
<td>1</td>
</tr>
<tr>
<td>Launcher block</td>
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</tr>
<tr>
<td>Button-hole elastic</td>
<td>2</td>
</tr>
<tr>
<td>Velcro</td>
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</tr>
<tr>
<td>Pencil</td>
<td>1 per student</td>
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### Setup

For this activity, you will work in teams of three or four to create and observe the collisions and record the results.

### Pre-Lab

This pre-lab demonstrates inelastic and elastic collisions, as shown in Figure 1-4. Place the track on a level surface. Now, select two maglev vehicles with Velcro on one end of each. The Velcro should be paired so that one car has the hook side and the other has the loop side. Place one of the maglev cars approximately halfway along the track.

Now place the other vehicle in the launcher with the Velcro end against the launcher (in our first test we want the cars to collide without the Velcro ends making contact). Launch the car and discuss what happened following the collision.

Now rotate the vehicles so that the Velcro sides are facing each other. (The Velcro should stick together). Place one vehicle halfway along the track, and the other in the launcher. Launch the car and discuss what happened following the collision.

![Figure 1-4A: After an inelastic collision](image1)

![Figure 1-4B: After an elastic collision](image2)
Procedure
In this activity, you will launch maglev cars toward each other and observe the resulting collisions. Before beginning, attach an elastic band to the screws on the ends of the track, as shown in Figure 1-5. For this portion of the activity, you will create a launcher to apply a consistent force to the maglev cars. For each collision, pull back the elastic band so that it pushes the launcher block, which will push the maglev car toward the center of the track.

Launching the cars with consistency takes practice; significant force is needed for the car to reach the end of the track, yet the car must travel slow enough to make accurate measurements. Experiment with launching a car by pulling back the launcher several distances. When you’ve found a distance that provides adequate velocity, mark the launcher car, as shown in Figure 1-5. (The launcher blocks at both ends of the track should have lines marked at the same location.) When running tests, pull the launcher back until the line is at the end of the track. This will create a consistent force applied to the car, which will result in consistent velocity.

Figure 1-5: Launcher Setup on Maglev Track

When you launch the cars, pull each launcher away from the center of the track. Pull both launchers back equal amounts so the force applied by the elastic bands is equal. The goal is NOT to see how much impact the vehicles can withstand. Be reasonable about the length that the elastic band is stretched. You should pull both cars back the same amount in all the scenarios so you can compare the results. The more consistent you can be with the elastic band, the easier it will be to compare the effects of different mass.

This activity will require teamwork. Two students should launch the cars and one or two should record the results. After Part 1, switch roles so the recorder(s) from Part 1 do the launching in Part 2.

Your observations will be recorded using the figures in the Research Notes section. Figure 1-6 illustrates how the data should be recorded. Both the direction and the relative velocity of the car should be recorded using vectors. The length of each vector should represent the relative velocity of each car. For example, if one car appears to move twice as fast as the other, then its vector representation should be twice as long. Although it is difficult, try and make the arrows consistent throughout all the scenarios.
When the lab is finished, you should be able to look at all of the vectors and distinguish the fastest car from the slowest.

**Tip:** It may help to first try launching a car of each mass to get a general idea of how fast each car moves down the track.

![Figure 1-6: Showing Conditions Before and After Impact](image)

**Figure 1-6: Showing Conditions Before and After Impact**

**Part 1: Two cars moving toward each other**

1. **Collision 1: M vs M, moving toward each other**
   a. Place two cars of mass M on the maglev track.
   b. Pull each car back, away from the center, to launch them toward each other.
   c. Record the results of the collision in the *Research Notes*.

2. **Collision 2: M vs 2M, moving toward each other**
   a. Place one car of mass M and one car of mass 2M on the maglev track.
   b. Pull each car back, away from the center, to launch them toward each other.
   c. Record the results of the collision in the *Research Notes*.

3. **Collision 3: M vs 3M, moving toward each other**
   a. Place one car of mass M and one car of mass 3M on the maglev track.
   b. Pull each car back, away from the center, to launch them toward each other.
   c. Record the results of the collision in the *Research Notes*.

4. **Collision 4: M vs 4M, moving toward each other**
   a. Place one car of mass M and one car of mass 4M on the maglev track.
   b. Pull each car back, away from the center, to launch them toward each other.
   c. Record the results of the collision in the *Research Notes*.

5. **Collision 5: M vs 5M, moving toward each other**
   a. Place one car of mass M and one car of mass 5M on the maglev track.
   b. Pull each car back, away from the center, to launch them toward each other.
   c. Record the results of the collision in the *Research Notes*.
Part 2: One stationary and one moving car

1. Collision 6: Stationary M vs moving 3M
   a. Place one car of mass M in the center of the maglev track.
   b. Pull back and then release the launcher to send the 3M car toward the center of the track.
   c. Record the results of the collision in the Research Notes.

2. Collision 7: Stationary 2M vs moving 3M
   a. Place one car of mass 2M in the center of the maglev track.
   b. Pull back and then release the launcher to send the 3M car toward the center of the track.
   c. Record the results of the collision in the Research Notes.

3. Collision 8: Stationary 3M vs moving 3M
   a. Place one car of mass 3M in the center of the maglev track.
   b. Pull back and then release the launcher to send the 3M car toward the center of the track.
   c. Record the results of the collision in the Research Notes.

4. Collision 9: Stationary 4M vs moving 3M
   a. Place one car of mass 4M in the center of the maglev track.
   b. Pull back and then release the launcher to send the 3M car toward the center of the track.
   c. Record the results of the collision in the Research Notes.

5. Collision 10: Stationary 5M vs moving 3M
   a. Place one car of mass 5M in the center of the maglev track.
   b. Pull back and then release the launcher to send the 3M car toward the center of the track.
   c. Record the results of the collision in the Research Notes.

Proceed to the collision figures and questions in the Research Notes section.
Activity 1: Bump-N-Run

Collision Figures

Example

Part 1: Two cars moving toward each other
For each collision, draw your vectors in the appropriate collision figure to show each car’s relative velocity before and after the collision.
Part 2: One stationary and one moving car

Because only the 3M car will have a velocity before the collision, the vector before the collision only needs to be drawn once. Draw the “before” figure below, and then use the other figures to draw the cars’ velocities after each collision.
Questions

1) When both cars were in motion, what happened to the relative velocity of the car on the right as the car’s mass was increased?

2) Knowing how the relative velocity changed, how do you think the momentum changed as the car’s mass was increased?
3) In a real-world scenario, when a small sedan collides with a large truck traveling the same speed but in the opposite direction, which direction will the sedan likely travel after the collision? Describe your answer in terms of the same or opposite direction the sedan was traveling before the collision.

4) In a real-world scenario, when a semi-truck collides with a small SUV traveling the same speed but in the opposite direction, in which direction will the semi-truck likely travel after the collision? Describe your answer in terms of the same or opposite direction the semi-truck was traveling before the collision.

Discussion Notes
Activity 2: Calculated Collisions

<table>
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<th>Activity Summary</th>
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<td>Technology</td>
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<td>National Science Education Standards</td>
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</table>

Introduction
This lab provides a conceptual and mathematical understanding of The Law of Conservation of Momentum. It demonstrates how a traffic engineer would conduct an analysis of a vehicle collision. The analysis involves a minimal amount of algebra and some basic high school lab equipment.

Note: Activities 1 and 2 are very similar, although Activity 2: Calculated Collisions involves more advanced concepts than Activity 1: Bump-N-Run. Both activities introduce the Law of Conservation of Momentum using maglev car collisions, but Activity 2 asks students to perform more in-depth calculations. Activity 1 can either be used as an introduction to Activity 2, or Activity 1 can be skipped in favor of Activity 2 for a more advanced class.

Objective
Students will be able to:
- Visualize and verify mathematically the Law of Conservation of Momentum.
- Calculate the momentum before and after each collision.
- Consider how outside forces affected their results.

Activity Expansion Ideas

Time-lapse Photography
This activity may be expanded with a discussion on measurement errors and ways to mitigate this type of error when designing an experiment. Following this discussion, different means of measuring the velocities can be employed to remove any error associated with students’ differing reaction times and perceptions of the position of the moving cars. For example, time-lapse photography could be used to record the position of the cars at a known time interval and the average velocity could be calculated using the recorded change in position over the known time interval. These photos could be overlaid in
order to visually see the distance traveled. Similarly, students could discuss errors associated with the launchers, air resistance, friction, or other sources and propose alternatives in the experimental design to mitigate these.

Using Time-lapse Photography to Accurately Measure Velocity

Innovative Launcher

Another expansion activity is to design a better, more accurate launcher. Students could be encouraged to come up with a better way to provide a consistent force to propel the cars or an accurate way to measure the applied force. These improvements could be tested by the students to determine how well they perform. For example, test the launchers by launching a maglev car with mass M in a series of trials—with three runs for each launcher—and measure the velocity over a set distance. The trial with the three closest velocity measurements has the most precise launcher.
Activity 2: Calculated Collisions

Calculation Tables

Part 1: Initial Momentum

Calculate the average total momentum BEFORE the collision using the data collected in Part 1 by completing the table.

Note: The M represents the mass of the 1M car. Therefore, the relative mass of the 3M car is three times the mass of the 1M car and should be accounted for when calculating the momentum.

For example:
Car Velocity = 1.2 m/s; Car Mass = 3M kg;
Car Momentum = [(3M kg)*(1.2 m/s)] = 3.6M kg*m/s

The following calculations are sample data for your reference.

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Trial Number</th>
<th>Car Mass (kg)</th>
<th>Car Travel Distance (m)</th>
<th>Car Travel Time (s)</th>
<th>Car Velocity (m/s)</th>
<th>Car Momentum (mass<em>velocity) (kg</em>m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3M</td>
<td>0.455</td>
<td>0.64</td>
<td>0.71</td>
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<td>0.72</td>
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<td></td>
<td></td>
<td><strong>Average Momentum for 3 Trials in Part 1</strong> 2.38M</td>
</tr>
</tbody>
</table>

* The mass of the cars may be measured and used in this experiment or the mass may be left as a variable to see the relative difference between cars of varying masses.
## Part 2: After the Collision

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Trial Number</th>
<th>Moving Car (3M)</th>
<th>Stationary Car (varies)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Car</td>
<td>Car Mass (kg)</td>
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</tbody>
</table>
Total Momentum after the Collision

Calculate the total momentum AFTER each collision using the data collected in Part 2 by completing the calculation table.

<table>
<thead>
<tr>
<th>Values Calculated from Experimental Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving Car (3M)</td>
</tr>
<tr>
<td>Car Velocity (distance/time) (m/s)</td>
</tr>
<tr>
<td>Experiment Number</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>Average Momentum for all 5 collisions in Part 2</td>
</tr>
</tbody>
</table>

Calculate the average total momentum BEFORE the collision using the data collected in Part 1 by completing the table.

Students should use the $p = mv$ formula to calculate the initial momentum of the two cars and then add them to get the total initial momentum (the momentum of one of the cars will be zero because it is stationary). Be sure to have the students define a reference for the direction of travel (motion in one direction should be assumed positive and the other negative).

Calculate the total momentum AFTER each collision using the data collected in Part 2 by completing the calculation table.

Students will have to calculate the movement of two vehicles. It is critical that students use the reference system established to differentiate directions of travel and indicate velocity as positive or negative. After they have calculated the momentum for each car, they should add the numbers to get the total momentum after the collision.

Questions

1) Part 1 established the momentum of the two cars before the collision (moving 3M car and stationary car). What is the percent difference between the average momentum calculated in Part 1 and the average momentum after the collision determined in Part 2?

Remind students to compare the shaded boxes in Part 1 and Part 2 because they are, in effect, comparing average momentums, which masks the variation between individual trials. The difference SHOULD be small; however, there will be a difference due to variables that are unaccounted for, such as errors in measurement, air resistance, friction, variation in applied force, etc.
2) What caused the differences between the momentum before and after the collision?

Friction, air resistance, measurement errors, and variations in the applied force all cause differences between initial and final momentum, as well as the fact that the collision is somewhat inelastic.

3) What conclusions can you draw about The Law of Conservation of Momentum based on the calculated before and after collision momentum?

Students should note that, in practice, the final momentum tends to be lower than the initial momentum. In most scientific equations, many factors contribute to the outcome of a closed system and are believed to have a minimal effect are often neglected to help simplify the calculations. In this experiment, we assumed frictionless surfaces when, in fact, there is some degree of friction along the sides of the track, which leads to a lower final momentum than initially present in the system. Air resistance on the moving cars has a similar effect.

4) What would you do to improve this experiment to better observe conservation of momentum?

Possibilities include: (1) improving the launcher so there is less variation in applied force between trials, (2) developing a more precise method to measure the velocity, and (3) eliminating variations in the mass of the maglev cars (material imperfections may cause a 5M car’s mass to be more or less than five times that of an M car).

Discussion

The equations that we used in this activity are simplified and do not consider every variable present. In reality there are a large amount of factors that affect the outcome of the experiment, such as friction, wind resistance, and the type of collision (elastic or inelastic). Most collisions are somewhere between being full elastic or inelastic. During a car crash, all of these factors come into play, even if we do not notice them. In general, however, we can simplify a scenario and use both the data we are given and our own deductive reasoning and experience to determine what happened in a collision.
Activity 2: Calculated Collisions

Introduction

In this activity, you will analyze collisions between maglev cars to demonstrate the Law of Conservation of Momentum. This activity demonstrates how a traffic engineer might analyze a real-world vehicle collision using both conceptual knowledge and mathematical calculations.

Objective

In these experiments, you will observe the following:

- How to calculate momentum before and after a collision.
- How momentum is transferred between two vehicles in a collision.

Background

Momentum is defined as mass multiplied by velocity, and it is one of the core concepts to understanding basic physics.

\[
\rho = mv
\]

Where:

- \( m \) = mass (kg)
- \( v \) = velocity (m/s)

An adult weighing 90 kg walking at 1.5 m/s has a momentum of 90 kg * 1.5 m/s = 135 kg*m/s. A child walking beside the adult, weighing 45 kg, would have a momentum of 67.5 kg*m/s. For the child and the adult to have the same momentum, the child would have to jog at 3 m/s (45 kg * 3 m/s = 135 kg*m/s). See Figure 2-1 for a visual representation of this.

![Figure 2-1: Understanding Momentum](image)
The Law of Conservation of Momentum can be stated many ways. One of those ways is to say that the sum of the momentum of all the vehicles before a collision equals the sum of the momentum of all vehicles after a collision. Mathematically this can be stated as:

\[ \text{Total momentum before collision} (p_i) = \text{Total momentum after collision} (p_f) \]

\[ m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf} \]

*In this equation, the first vehicle is represented by the letter a, and its mass and velocity before the collision are represented as \( m_a v_{ai} \) while its mass and velocity after the collision are represented as \( m_a v_{af} \). The second vehicle is represented by the letter b.*

If \( m_a = m_b \), then the equation simplifies to:

\[ m(v_{ai} + v_{bi}) = m(v_{af} + v_{bf}) \]

Where:

- \( m \) = mass
- \( v \) = velocity
- \( i \) = initial value before collision
- \( f \) = final value after collision

Momentum is a vector. A **vector** is defined as a quantity with a **magnitude** and a direction, and is represented by an arrow. Magnitude is shown by the length of the arrow, while direction is indicated by where the arrow points. When entering data into this equation, consider the vehicles moving from left to right as moving in a positive direction. Anything traveling from left to right has a positive velocity in the formula, while anything traveling from right to left has a negative velocity. This concept is demonstrated in Figure 2-2.

![Figure 2-2: Magnitude and Direction in Vectors](image)

**Note:** When drawing a vector, the arrow gives the direction of the vector. When using magnitudes mathematically, it is important to use the sign convention stated previously to establish direction. Therefore, for the velocities shown in Figure 2-2, we would say that they are 2 m/s and -4 m/s when describing them or using them mathematically.
Example 1
Two cars of equal mass (1000 kg) collide as shown in Figure 2-3. Car A is moving at 2 m/s (with a positive velocity because it is traveling left to right) and Car B is moving at -4 m/s (with a negative velocity because it is traveling right to left). After the collision, Car A is moving at a velocity of -1 m/s. What is car B’s velocity after the collision?

![Figure 2-3: Vehicle Vectors before Collision](image)

Solution
Using the equation for the Law of Conservation of Momentum, we can find the velocity of Car B after the collision:

\[
m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf}
\]

\[
(1000 \text{ kg} \times 2 \text{ m/s}) + (1000 \text{ kg} \times (-4 \text{ m/s})) = (1000 \text{ kg} \times (-1 \text{ m/s})) + (1000 \text{ kg} \times v_{bf})
\]

\[
2000 \frac{\text{kg} \cdot \text{m}}{\text{s}} + (-4000 \frac{\text{kg} \cdot \text{m}}{\text{s}}) = (-1000 \frac{\text{kg} \cdot \text{m}}{\text{s}}) + (1000 \text{ kg} \times v_{bf})
\]

\[
-2000 \frac{\text{kg} \cdot \text{m}}{\text{s}} = (-1000 \frac{\text{kg} \cdot \text{m}}{\text{s}}) + (1000 \text{ kg} \times v_{bf})
\]

\[
v_{bf} = -1 \frac{\text{m}}{\text{s}}
\]

From our solution, we can determine that Car B was moving at a velocity of -1 m/s after the collision, as shown in Figure 2-4.

![Figure 2-4: Vehicle Vectors after Collision](image)
When two vehicles of the same mass collide, the transfer of momentum can be observed by looking at the velocity and direction of the cars. It is not so obvious when two vehicles of different mass collide. The momentum of each vehicle may change, but the total momentum of the system must remain the same (ignoring losses from external forces).

There are two types of collisions: elastic and inelastic. Before a collision occurs, one or both of the objects is in motion and, therefore, has kinetic energy. When a vehicle collision occurs, some of that kinetic energy is converted to other forms of energy, such as sound, heat, and deformation of the vehicles. This is why the Law of Conservation of Energy is not fully observable in a real-world collision. If the transformation of kinetic energy to other forms of energy is significant, the vehicles will not bounce off each other and will instead stick together. This type of collision is called inelastic. In a collision with little or no loss in kinetic energy, the vehicles will bounce off each other, which is an elastic collision.

In the real world, there are many additional factors (such as friction and air resistance) that we will not consider in this activity. The maglev track reduces friction enough to assume it is non-existent, which allows us to examine the relationship between mass and velocity directly. Assuming friction is negligible, use the formula for average velocity to calculate the velocities of the cars.

\[
\text{Average Velocity} \left( \frac{\text{m}}{\text{s}} \right) = \frac{\text{Total distance traveled}}{\text{Total travel time}}
\]

It is also important to note that we will be holding force constant in this activity by using an elastic band to launch the cars. It is assumed that during the first phase of the collision, when the cars collide, they are traveling with equal momentum. This activity focuses on what happens in the instant after the cars collide, before the second phase of the collision when the cars come to rest.
Materials

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maglev track</td>
<td>1</td>
</tr>
<tr>
<td>Maglev car mass M</td>
<td>2</td>
</tr>
<tr>
<td>Maglev car mass 2M</td>
<td>1</td>
</tr>
<tr>
<td>Maglev car mass 3M</td>
<td>2</td>
</tr>
<tr>
<td>Maglev car mass 4M</td>
<td>1</td>
</tr>
<tr>
<td>Maglev car mass 5M</td>
<td>1</td>
</tr>
<tr>
<td>Stopwatch</td>
<td>2</td>
</tr>
<tr>
<td>Meter stick</td>
<td>2</td>
</tr>
<tr>
<td>Launcher block</td>
<td>2</td>
</tr>
<tr>
<td>Pencil, 1 per student</td>
<td></td>
</tr>
</tbody>
</table>

Part 1: Initial Momentum

Setup

In this activity, you will launch maglev cars toward each other and observe the resulting collisions. Before beginning, attach an elastic band to the hooks on the ends of the track, as shown in Figure 2-5. For this portion of the activity, you will create a launcher to apply a consistent force to the maglev cars, see instructions on the following page. For each collision, pull back the elastic band so that it pushes the launcher block, which will push the maglev car toward the center of the track.

Launching the cars with consistency takes practice; significant force is needed for the car to reach the end of the track, yet the car must travel slow enough to make accurate measurements. Experiment with launching a car by pulling back the launcher to several different distances. When you’ve found a distance that provides adequate velocity, draw a line on the launcher car as shown in Figure 2-5. When running tests, pull the launcher back until the line is at the end of the track. This will create a consistent force applied to the car, which will result in consistent velocity.

Figure 2-5: Launcher Setup
**Procedure**

**Setting Measuring Distance**
Record the distance over which the car will be timed, as shown in Figure 2-6. This distance should begin after the car is launched and continue approximately to the center of the track to provide a good representation of the velocity just before impact. It may be useful to mark this distance with a piece of tape on the side of the track. You should consider the length of the car when determining your distance to be sure that you will be able to measure the time it takes for one location on the car to pass over both points. It might be helpful to place a dot on the car and record the time it takes for the dot to pass over your start and stop points.

![Figure 2-6: Part 1, Measuring Distance](image)

**Experiment**

1. Mark the distance over which the car will be timed. It may be useful to mark this distance with a piece of tape.
2. Record the distance in centimeters in the Initial Momentum calculation table. Then convert the distance to meters.
3. Place a car with mass 3M on the maglev track in front of the launcher.
4. Pull the launcher back from the center to launch the 3M car forward.
5. Measure the time over which the car travels between the points you chose in Step 1.
6. Record the mass, distance, and time in the table in your Research Notes section.
7. Calculate the velocity and momentum for each trial in the table in your Research Notes section.
8. Repeat Steps 2 through 6 until you have data for three trials. These will be used to determine an average momentum for the 3M car.

**Part 2: After the Collision**
We will continue to use the setup from Part 1; however, we will now adjust the experiment to see the effect of the momentum from the 3M car after it collides with stationary cars of varying mass. When performing the activity, one student should launch the car, two students should time the cars after the collision, and the fourth student should take notes. Students should rotate so that all team members get an opportunity to try each role.

**Setting Measuring Distance**
Record the distance over which each car will be timed, as shown in Figure 2-7. This distance should begin near the center of the track and continue away from the center, as the distance and time after the collision...
collision will be recorded. It may be useful to mark this distance with a piece of tape. You should consider the length of the car when determining your distances to be sure that you will be able to measure the time it takes for one location on the car to pass over both points. It might be helpful to place a dot on each car and record the time it takes for the dot to pass over your start and stop points. You may need to determine different distances for each experiment to accommodate the different lengths of each car.

![Figure 2-7: Part 2, Measuring Distance](image)

**Experiment 1: 3M vs M**
1. Place one car of mass M in the center of the maglev track.
2. Pull back and release the launcher to send the 3M car toward the center of the track.
3. Record the time it takes each car to move across the predetermined distance after the collision.
4. Record the mass, distance traveled, and time for both cars in your Research Notes.
5. Repeat this procedure three times to collect data from three trials.

**Experiment 2: 3M vs 2M**
1. Place one car of mass 2M in the center of the maglev track.
2. Pull back and release the launcher car to send the 3M car toward the center of the track.
3. Record the time it takes each car to move across the predetermined distance after the collision.
4. Record the mass, distance traveled, and time for both cars in your Research Notes.
5. Repeat this procedure three times to collect data from three trials.

**Experiment 3: 3M vs 3M**
1. Place one car of mass 3M in the center of the maglev track.
2. Pull back and release the launcher car to send the 3M car toward the center of the track.
3. Record the time it takes each car to move across the predetermined distance after the collision.
4. Record the mass, distance traveled, and time for both cars in your Research Notes.
5. Repeat this procedure three times to collect data from three trials.

**Experiment 4: 3M vs 4M**
1. Place one car of mass 4M in the center of the maglev track.
2. Pull back and release the launcher car to send the 3M car toward the center of the track.
3. Record the time it takes each car to move across the predetermined distance after the collision.
4. Record the mass, distance traveled, and time for both cars in your Research Notes.

5. Repeat this procedure three times to collect data from three trials.

**Experiment 5: 3M vs 5M**

1. Place one car of mass 5M in the center of the maglev track.
2. Pull back and release the launcher car to send the 3M car toward the center of the track.
3. Record the time it takes each car to move across the predetermined distance after the collision.
4. Record the mass, distance traveled, and time for both cars in your Research Notes.
5. Repeat this procedure three times to collect data from three trials.

Proceed to the calculations and questions in the Research Notes section.
Activity 2: Calculated Collisions

Equations

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ v = \frac{d}{t} ]</td>
<td>[ \rho = mv ]</td>
</tr>
</tbody>
</table>

Where:
- \( d = \) distance (m)
- \( t = \) time (s)
- \( m = \) mass (kg)
- \( v = \) velocity (m/s)

Calculation Tables

Part 1: Initial Momentum

Calculate the average total momentum BEFORE the collision using the data collected in Part 1 by completing the table.

<table>
<thead>
<tr>
<th>Data Collected During Experiment</th>
<th>Values Calculated From Experimental Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment Number</td>
<td>Trial Number</td>
</tr>
<tr>
<td>-------------------</td>
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<tr>
<td>1</td>
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<td>3</td>
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Average Momentum for 3 Trials in Part 1
### Part 2: After the Collision

#### Data Collected During Experiment

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Trial Number</th>
<th>Car</th>
<th>Car Mass (kg)</th>
<th>Distance Traveled (m)</th>
<th>Travel Time (s)</th>
<th>Car</th>
<th>Car Mass (kg)</th>
<th>Distance Traveled (m)</th>
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</thead>
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<td>3M</td>
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<td>3</td>
<td>3M</td>
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<td>3M</td>
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<td>3M</td>
<td></td>
<td></td>
<td>4M</td>
<td></td>
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<tr>
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<td>2</td>
<td>3M</td>
<td></td>
<td></td>
<td>4M</td>
<td></td>
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<td></td>
<td>3</td>
<td>3M</td>
<td></td>
<td></td>
<td>4M</td>
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<td></td>
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<td>3M</td>
<td></td>
<td></td>
<td>5M</td>
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<td></td>
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<td>3M</td>
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<td></td>
<td>5M</td>
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<td></td>
<td>3</td>
<td>3M</td>
<td></td>
<td></td>
<td>5M</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average</td>
</tr>
</tbody>
</table>
Total Momentum after the Collision

Calculate the total momentum AFTER each collision using the data collected in Part 2 by completing the calculation table.

<table>
<thead>
<tr>
<th>Values Calculated from Experimental Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving Car (3M)</td>
</tr>
<tr>
<td><strong>Experiment Number</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>Average Momentum for all 5 collisions in Part 2</td>
</tr>
</tbody>
</table>

Questions

1) Part 1 established the momentum of the two cars before the collision (moving 3M car and stationary car). What is the percent difference between the average momentum calculated in Part 1 and the average momentum after the collision determined in Part 2?

2) What caused the differences between the momentum before and after the collision?
3) What conclusions can you draw about The Law of Conservation of Momentum based on the calculated momentum before and after the collision?

4) What would you do to improve this experiment to better observe conservation of momentum?

Discussion Notes
Activity 3: Collision Analysis

<table>
<thead>
<tr>
<th>Activity Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor Prep Time</td>
</tr>
<tr>
<td>Class Time</td>
</tr>
<tr>
<td>Grade/Class</td>
</tr>
<tr>
<td>Suggested Activity Grouping</td>
</tr>
<tr>
<td>Technology</td>
</tr>
<tr>
<td>National Science Education Standards</td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Introduction

Students will learn to calculate the initial or final velocity of a vehicle in a collision using the Law of Conservation of Momentum. Students will be presented three scenarios in which vehicles have collided, where they must determine either the initial or final velocities of the vehicles. Activity 3A presents this concept in one-dimensional motion, whereas activities 3B and 3C present this concept in two-dimensional motion.

Note: Activities 3B and 3C are both examples of two-dimensional collision analysis, with 3C being the more difficult of the two. As a result of 3B utilizing a right triangle and the Pythagorean Theorem, this problem is better suited for middle school students. On the other hand, Activity 3C uses trigonometry.

Objective

The objective of this activity is to use real-world applications of the Law of Conservation of Momentum. Students will learn to do the following:

- Activity 3A: Compare two differing scenarios using the Law of Conservation of Momentum.
- Activity 3B: Perform two-dimensional collision analysis using the Pythagorean Theorem.
- Activity 3C: Perform two-dimensional collision analysis using trigonometry.

Activity Expansion Ideas

One important take away from this activity is vectors and vector addition. This application will be used in physics, statistics, and other engineering courses. In order to better understand the process, students could explore the interactive vector game from Physics Classroom.
In this interactive vector game, students will be presented with two vectors that are summed to produce a resultant force, which the students construct in the large grid area. These vectors are summed together using one of two methods: the head-to-tail addition method or the parallelogram method.

Activity 3: Collision Analysis

General Activity Discussion
In practice, the Law of Conservation of Momentum and its associated calculations can assist law enforcement officers, transportation professionals, automobile designers, and medical professionals in gaining a better understanding of the events that transpired during the course of a crash, enabling them to learn how to prevent or reduce the severity of crashes in the future. Consider the design of airbags or bumpers for vehicles corresponding to a collision. If you note that particular angles of impact at high speeds do not result in injuries, then you can look closely at the design of the vehicle to understand why injuries did not occur (and how to replicate that design for other vehicles). Alternatively, if you find that certain angles of impact frequently occur (e.g. cars striking each other on the side at a 90-degree angle rather than straight on). From this you can consider other kinds of countermeasures, such as having side-impact airbags or improving sight distance at a particular intersection.

Activity 3A: Accident Analysis

Activity Solutions

Solution A: Mathematical Solution According to Driver of Sedan (Left Car, Figure 3-1)
Total momentum before collision ($\rho_i$) = Total momentum after collision ($\rho_f$)

\[
m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf}
\]

\[
\rho_i = (1500 \text{ kg} \times 4 \frac{\text{m}}{\text{s}}) + (1000 \text{ kg} \times -8 \frac{\text{m}}{\text{s}})
\]

\[
\rho_i = -2000 \text{ kg} \times \frac{\text{m}}{\text{s}}
\]

\[
\rho_f = (1500 \text{ kg} \times 2 \frac{\text{m}}{\text{s}}) + (1000 \text{ kg} \times 2 \frac{\text{m}}{\text{s}})
\]

\[
\rho_f = 5000 \text{ kg} \times \frac{\text{m}}{\text{s}}
\]

$\rho_f \neq \rho_i$

Solution B: Mathematical Solution According to Driver of Convertible (Right Car, Figure 3-1)

\[
m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf}
\]

\[
\rho_i = (1500 \text{ kg} \times 4 \frac{\text{m}}{\text{s}}) + (1000 \text{ kg} \times -2 \frac{\text{m}}{\text{s}})
\]

\[
\rho_i = 4000 \text{ kg} \times \frac{\text{m}}{\text{s}}
\]
\[ \rho_f = (1500 \text{ kg} \times 2 \frac{\text{m}}{\text{s}}) + (1000 \text{ kg} \times 2 \frac{\text{m}}{\text{s}}) \]

\[ \rho_f = 5000 \text{ kg} \times \frac{\text{m}}{\text{s}} \]

\[ \rho_f \approx \rho_i \]

The initial momentum based on the convertible driver’s story is much closer to the momentum after the collision. Although neither driver was completely correct, it’s clear that the estimate of 2 m/s is more plausible than the sedan driver’s estimate of 8 m/s.

Questions

1) Which driver’s account appears to be the most accurate? Why?

Most likely each driver believes their account to be accurate; however, the truth is more likely somewhere between their two accounts. The momentum equation for the information provided by the driver of the sedan shows a momentum before collision of -200 kg * m/s and a momentum after the collision of 5000 kg * m/s. We know these two values should be equal, and we are more certain of the accuracy of the final momentum because both drivers agree on that point. The information provided by the driver of the convertible results in a momentum of 4000 kg * m/s before the crash which is much closer to the final momentum. Therefore, the information provided by the driver of the convertible is most likely closer to the actual conditions present immediately prior to the accident.

2) How might the results of the problem change if the convertible were replaced with a semi-truck (larger mass)?

The sedan would likely have been pushed in the opposite direction due to the large difference in mass.

3) Would this collision be considered elastic or inelastic? Why?

While there may be some elastic properties, this collision would be considered inelastic because the cars moved together after the impact rather than moving apart.
Activity 3B: Two-dimensional Collision Analysis (Intermediate)

Activity Solution

1) Using vector notation \( \mathbf{v} = x \mathbf{i} + y \mathbf{j} \text{ m/s} \), describe the velocity of Car A immediately before the crash.

The equation for conservation of momentum should be used twice: once for components of velocity in the x-direction, and then for the components in the y-direction. Alternatively, the equation can be used once if the components in the x and y-direction are kept separate as shown below. The only unknown in the equation will be for the components of the initial velocity of Car A. Stated in vector notation:

\[
\left( v_{\text{Ai}} \frac{\text{m}}{\text{s}} \right) (1000 \text{ kg}) + \left( 0 \frac{\text{m}}{\text{s}} \right) (1500 \text{ kg}) = \left( 0 \frac{\text{m}}{\text{s}} \right) (1000 \text{ kg}) + \left( 2.4 \mathbf{i} + 3.2 \mathbf{j} \frac{\text{m}}{\text{s}} \right) (1500 \text{ kg})
\]

\[
v_{\text{Ai}} = 3.6 \mathbf{i} + 4.8 \mathbf{j} \text{ m/s}
\]

2) What was the magnitude of the velocity of Car A immediately before the crash?

Using the Pythagorean Theorem:

\[
|V_{\text{Ai}}| = \sqrt{3.6^2 + 4.8^2} = 6 \frac{\text{m}}{\text{s}}
\]
Activity 3C: Two-dimensional Collision Analysis (Advanced)

Activity Solution

First, realize that the sum of momentums in the x-direction (e.g., horizontally) will be equal. Before the crash, the angle of Car A from the horizontal line was 63 degrees and the angle of Car B was 90 degrees. Trigonometry can be used to determine the velocity of each car in vector notation.

To find the magnitude of the velocity, recall that velocity is a vector. Therefore, the magnitude can be found using Pythagorean’s Theorem.

1) Use trigonometry to determine the initial velocity of the cars in vector notation.

\[ V_{Ai} = 4 \cos(63) \hat{i} + 4 \sin(63) \hat{j} \frac{m}{s} \]

\[ V_{Ai} = 1.816 \hat{i} + 3.564 \hat{j} \frac{m}{s} \]

\[ V_{Bi} = 0 \hat{i} + 3.0 \hat{j} \frac{m}{s} \]

2) After the collision, what was the velocity, magnitude, and direction of Car B?

\[ (1500 \text{ kg}) \left( 1.816 \hat{i} + 3.564 \hat{j} \frac{m}{s} \right) + (0 \hat{i} + 3.0 \hat{j} \frac{m}{s}) = (1500 \text{ kg}) \left( 0.75 \hat{i} + 2.0 \hat{j} \frac{m}{s} \right) + \left( V_{Bf} \frac{m}{s} \right) \]

\[ V_{Bf} = 1.066 \hat{i} + 4.564 \hat{j} \frac{m}{s} \]

\[ |V_{Bf}| = 4.687 \frac{m}{s} \]

3) Clear communication is essential to engineers for relaying their findings to people who have an interest in the results. How would you communicate the direction of travel of Car B to a person who may not have a picture of the crash site?

Communicating in vector notation such as 1.066 m/s north and 4.564 m/s east or 4.687 m/s at a bearing of 13.1 degrees (measured clockwise of north on a compass) would clearly establish the direction of travel for the car.
Activity 3: Collision Analysis

Introduction

In this activity, you will learn to calculate the velocity of a vehicle in a collision using the Law of Conservation of Momentum. You will be given a variety of situations involving the mass, velocity, and direction of cars, and then you will determine information about the collisions between the two vehicles.

Objective

In these experiments, you will learn to do the following:

- Activity 3A: Compare two differing scenarios using the Law of Conservation of Momentum.
- Activity 3B: Perform two-dimensional collision analysis using the Pythagorean Theorem.
- Activity 3C: Perform two-dimensional collision analysis using trigonometry.

Background

Engineers study collisions to assess the integrity of a vehicle’s structure and to understand what events occurred before and during an accident. When two or more vehicles collide, some or all of the momentum of one vehicle is transferred to the other vehicle. This process yields information about the instant of the collision. It does not tell anything about what happens beyond this moment in time. To investigate the characteristics of an accident beyond this instant involves applying kinematics and Sir Isaac Newton’s laws of motion. Motion analysis after a collision is beyond the scope of this module.

In studying the various types of collisions, there are two extremes: elastic and inelastic. Before a collision occurs, one or both of the objects is in motion and, therefore, has kinetic energy. In a perfectly elastic collision, no kinetic energy is transformed into other forms (such as sound and heat), and the objects will bounce off each other. In reality, a perfectly elastic collision does not occur, because there will always be some transformed energy. Energy is required to make sound and bend metal. Even the compression of a spring generates heat energy. When the transformation of kinetic energy is significant, the objects will not fully rebound and may even stick together. We call this an inelastic collision. Momentum is conserved in both elastic and inelastic collisions, as the Law of Conservation of Momentum is independent of the energy relationships.

The Law of Conservation of Momentum applies to both elastic and inelastic collisions and can be stated many ways. One of those ways is to say the following: the sum of the momentum of all the vehicles before a collision equals the sum of the momentum of all vehicles after a collision. Mathematically, this can be stated as:

\[ m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf} \]

In this equation, the first vehicle is represented by the letter \( a \), and its mass and velocity before the collision are represented as \( m_a v_{ai} \); while its mass and velocity after the collision are represented as \( m_a v_{af} \). The second vehicle is represented by the letter \( b \).
If \( m_a = m_b \), then the equation simplifies to:

\[
    m(v_{ai} + v_{bi}) = m(v_{af} + v_{bf})
\]

Where:

- \( m \) = mass
- \( v \) = velocity
- \( i \) = initial value before collision
- \( f \) = final value after collision

The simplest collision occurs between two vehicles that **impact** bumper to bumper in an inelastic collision along a straight line.

We will apply this concept to the example problem as shown in Figure 3-1 below.

**Figure 3-1: Collision Analysis Example**

In this example, the following data is known before the collision occurs:

- Car A has a mass of 2100 kg and is moving to the right at a velocity of 3 m/s.
- Car B has a mass of 1400 kg and is moving to the left at a velocity of -1 m/s. (The velocity is negative because Car B is moving in the opposite direction as Car A.)

To find the total momentum of the cars before the collision, use the following equation:

\[
    \rho_i = m_a v_{ai} + m_b v_{bi}
\]

\[
    (2100 \text{ kg}) \times (3 \text{ m/s}) + (1400 \text{ kg}) \times (-1 \text{ m/s})
\]

\[
    = 4900 \text{ kg} \times \text{m/s}
\]

Similarly, to find the total momentum of the cars after the collision, use the following equation:

\[
    \rho_f = m_a v_{af} + m_b v_{bf}
\]

\[
    (2100 \text{ kg}) \times (1.4 \text{ m/s}) + (1400 \text{ kg}) \times (1.4 \text{ m/s})
\]

\[
    = 4900 \text{ kg} \times \text{m/s}
\]

The Law of Conservation of Momentum is clearly demonstrated here, as the momentum is the same before and after the collision. As previously discussed, kinetic energy may have been converted to
produce sound and heat and to bend the bumpers of the cars, but the momentum is still conserved independently of that energy transformation.

The previous example shows the Law of Conservation of Momentum for objects traveling in one-dimension, i.e. along a line. When objects are traveling along paths represented by two different lines, we need to consider a two-dimensional problem (i.e. objects on a flat surface). Here is where vectors become useful in the calculations.

A vector is defined as a quantity with a magnitude (represented by the length of a line) and a direction (represented by an arrow), like a car traveling down a road. The car is moving with a certain speed (as seen on the speedometer) in a direction defined by the road. Consider a vehicle (Car A) traveling northeast at 5 m/s with a component in the eastern direction of 3 m/s and a component to the north of 4 m/s, as shown in Figure 3-2. The magnitude of the car along its line of travel can be found from its components in the east/west and north/south directions by using the Pythagorean Theorem. Engineers often break measurements down into coordinates or components along reference lines. The most common way to do this is using directions, N/S, E/W or by using a graph with an x-y coordinate system.

![Figure 3-2: Viewing Velocity in Vector Quantities](image)

If a second vehicle (Car B) is traveling southwest with a western component of 5 m/s and a southern component of 3 m/s and collides with Car A, we can use the Law of Conservation of Momentum for the east/west components of travel and the north/south components to determine the outcome of the collision. If we graph the motion of these two vehicles on an x-y coordinate system, as shown in Figure 3-3, we will show Car A moving 3 m/s in the x-direction and 4 m/s in the y-direction. Car B will be shown moving -5 m/s in the x-direction and -3 m/s in the y-direction.

Generally, we will use an x-y coordinate system to represent problems like these, where the x-axis represents the east/west direction and the y-axis represents the north/south axis (see Figure 3-3).
Note: Engineers often use “vector notation” where “i” represents one “unit” in the x-direction and “j” represents one “unit” in the y-direction. This makes it easier to communicate with other engineers when working on a problem and simplifies the calculations. An engineer would describe the velocity of Car A and Car B as:

\[ v_{\text{Car A}} = 3i + 4j \, \left( \frac{m}{s} \right) \]

\[ v_{\text{Car B}} = -5i - 3j \, \left( \frac{m}{s} \right) \]

This simplifies calculations by allowing engineers to focus on motion along one line in either the x-direction or the y-direction, just as you saw in the previous example.

The calculations involved in applying the Law of Conservation of Momentum to two dimensions are the same as with one dimension, only we must consider both the x- and y-directions of travel individually. We can use the Pythagorean Theorem to determine the magnitude of the result; however, in order to understand the direction of the motion, we must keep the x- and y- components separate.

Materials

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper</td>
<td>As needed</td>
</tr>
<tr>
<td>Pencil</td>
<td>1 per student</td>
</tr>
<tr>
<td>Calculator</td>
<td>1 per student</td>
</tr>
</tbody>
</table>
Activity 3A: One Dimensional Accident Analysis

Set Up

You will work individually or in teams of two to find the solution to a story problem involving eyewitness observations of an automobile accident.

Procedure

Read the following story problem and solve for the convertible’s velocity in each scenario in the area provided in the Research Notes section.

You are a police officer who has been called to the scene of a collision. When you arrive and speak to the drivers, they each tell a different version of the story. Neither driver knows exactly how fast they were driving. As shown in the figures below, the collision happened as the driver in the sedan was moving forward and the driver in the convertible was backing up.

The driver of the sedan tells you they were moving forward at 4 m/s, while the driver of the convertible backed up twice as fast (Figure 3-4). The driver of the convertible tells you the sedan driver was moving at 4 m/s, but the convertible was only going half that speed at 2 m/s (Figure 3-4). After the collision, several bystanders observed the two vehicles moving at 2 m/s.

To find which driver’s version of the incident is more accurate, use the mathematical equation for the Law of Conservation of Momentum. Calculate the momentum before the collision based on each driver’s story, and then calculate the momentum after the collision based on bystander accounts. Use your Research Notes handout to conduct your calculations and determine the most likely scenario prior to the collision.

![Figure 3-4: Varying Accounts of Instance Prior to Collision](image-url)
Activity 3B: Two-dimensional Collision Analysis (Intermediate)

Set Up

You will work individually or in teams of two to find the solution to a story problem involving a collision between two vehicles while one was traveling at an angle to the other on a flat surface.

Procedure

Use the information from the following story problem and Figure 3-5 to solve for the initial velocity of Car A in the area provided in the Research Notes section.

Car B, which has a mass of 1500 kg, was sitting at rest when it was struck from behind by Car A, which has a mass of 1000 kg. After the collision, Car A was at rest, while Car B was sent down the road at 2.4 m/s and to the right 3.2 m/s. Using the equation for the Law of Conservation of Momentum, solve for the velocity of Car A BEFORE the collision.

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car A</td>
<td></td>
</tr>
<tr>
<td>1000 kg</td>
<td>1000 kg</td>
</tr>
<tr>
<td>( V_i = ? )</td>
<td>( V_f = 0 )</td>
</tr>
<tr>
<td>Car B</td>
<td></td>
</tr>
<tr>
<td>1500 kg</td>
<td>1500 kg</td>
</tr>
<tr>
<td>( V_i = 0 ) m/s</td>
<td>( V_f = 2.4i + 3.2j ) (m/s)</td>
</tr>
</tbody>
</table>

Figure 3-5: Solve for the Velocity of Car A before the Collision
Activity 3C: Two-dimensional Collision Analysis (Advanced)

Set Up
You will work individually or in teams of two to find the solution to a story problem involving a collision between two moving vehicles traveling along different paths on a flat surface.

Procedure
Use the information from the following story problem and Figure 3-6 and to solve for the velocity, magnitude, and direction of Car B in the area provided in the Research Notes section.

In a stock car race, two cars bump during the lap prior to the beginning of the race. Below there is a depiction of the collision. Onboard computers provide the velocities of the car before and after the collision, however, the onboard computer in Car B was broken during the collision and did not record the velocity after the collision. What is the velocity (magnitude and direction) of Car B after the collision?

Figure 3-6: Determine the Final Velocity of Car B

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car A</td>
<td>1500 kg</td>
</tr>
<tr>
<td></td>
<td>$v_i = *$</td>
</tr>
<tr>
<td></td>
<td>$v_f = 0.75i + 2j$ m/s</td>
</tr>
<tr>
<td>Car B</td>
<td>1500 kg</td>
</tr>
<tr>
<td></td>
<td>$v_i = *$</td>
</tr>
<tr>
<td></td>
<td>$v_f = ?$</td>
</tr>
</tbody>
</table>

* Use Figure 3-6 to determine the vector notation for the velocities before the collision.
Activity 3A: Accident Analysis

Equations

<table>
<thead>
<tr>
<th>Law of Conservation of Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf}$</td>
</tr>
</tbody>
</table>

Where:
- $m$ = mass (kg)
- $v$ = velocity (m/s)
- $i$ = initial value before collision
- $f$ = final value after collision

Activity Solution
Questions

1) Which driver’s account appears to be the most accurate? Why?

2) How might the results of the problem change if the convertible were replaced with a semi-truck (larger mass)?

3) Would this collision be considered elastic or inelastic? Why?

Discussion Notes
Activity 3B: Two-dimensional Collision Analysis

Equations

<table>
<thead>
<tr>
<th>Law of Conservation of Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf} )</td>
</tr>
</tbody>
</table>

Where:
- \( m \) = mass (kg)
- \( v \) = velocity (m/s)
- \( i \) = initial value before collision
- \( f \) = final value after collision

Activity Solution
Questions

1) Using vector notation \( \mathbf{v} = \_\hat{i} + \_\hat{j} \text{ m/s} \), describe the velocity of Car A immediately before the crash.

2) What was the magnitude of the velocity of Car A immediately before the crash?

Discussion Notes
Activity 3C: Two-dimensional Collision Analysis

Equations

<table>
<thead>
<tr>
<th>Law of Conservation of Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a v_{ai} + m_b v_{bi} = m_a v_{af} + m_b v_{bf}$</td>
</tr>
</tbody>
</table>

Where:
- $m = \text{mass (kg)}$
- $v = \text{velocity (m/s)}$
- $i = \text{initial value before collision}$
- $f = \text{final value after collision}$

Activity Solution
Research Notes
Activity 3: Collision Analysis

Questions

1) Use trigonometry to determine the initial velocity of the cars in vector notation.

2) After the collision, what was the velocity, magnitude, and direction of Car B?

3) Clear communication is essential to engineers for relaying their findings to people who have an interest in the results. How would you communicate the direction of travel of Car B to a person who may not have a picture of the crash site?

Discussion Notes
Activity 4: Impulse

**Activity Summary**

<table>
<thead>
<tr>
<th>Instructor Prep Time</th>
<th>30 minutes</th>
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<tr>
<td>Class Time</td>
<td>45 minutes</td>
</tr>
<tr>
<td>Grade/Class</td>
<td>6 - 8 Physical Science/ 9 - 12 Physics</td>
</tr>
<tr>
<td>Suggested Activity Grouping</td>
<td>Teams of two</td>
</tr>
<tr>
<td>Technology</td>
<td>Low Tech</td>
</tr>
</tbody>
</table>
| National Science Education Standards | Science as inquiry  
Identify problems for technological design  
Design a product  
Implement proposed design  
Evaluate product |

**Introduction**

In this activity, students will observe how various factors affect the impulse an object experiences in a collision. In the first activity, students will toss water balloons of two different masses (fully filled and partially filled) back and forth until each pops. Students will see firsthand how changing either the distance or the mass of the balloon affects the care with which the balloon must be caught to avoid breaking. In the second activity, students will toss an egg at a bed sheet to demonstrate how the sheet imparts an impulse that changes the momentum of the egg until it comes to a stop without breaking.

**Objective**

In these experiments, students will observe the following:

- Activity 4A: Relationship between mass and impulse in a collision.
- Activity 4B: Relationship between change in velocity and impulse in a collision.

**Activity Expansion Ideas**

**Throwing Balls**

The water balloons and eggs were selected to demonstrate impulse because of their fragile nature. Both of these activities require plenty of space; therefore, going outside is recommended. These activities could be demonstrated inside with a variety of balls. Activity 4A could be reproduced with two balls, a light ball and a heavier ball. The students will find that the lighter ball thrown at the same speed as the heavier ball will not require as much cradling to stop as the heavy ball. The effect of velocity on impulse could be explained by talking about how the balls are cradled during a catch so that impact against the hands and arms is lessened. Your hands don’t sting if a ball is properly cradled during a catch. This allows the ball to slow to a stop over a greater period of time.
Trajectory Motion

During the water balloon toss, some teams may toss the balloons high in the air. In addition to velocity and distance, Trajectory motion could be discussed. Students could estimate the distance the balloon traveled along a parabola and include the acceleration of gravity if the height of the balloon is known or can be approximated.
Activity 4: Impulse

General Activity Discussion
Impulse is a concept that most of us understand, although many of us have probably not thought of the contributing forces. Have you ever caught a ball only to have your hand sting? If you compensated on the next catch by cradling the ball, you have an understanding of the concept of impulse. Impulse shows how the effect of a force on an object is dependent on the amount of time the force is applied.

Impulse is why traffic safety devices are used on highway systems in areas where obstructions are near the roadway. Have you ever noticed what looks like a series of barrels in front of a concrete pier or an overpass? These are generally filled with water and are designed to slow a vehicle down over a greater period of time than if the vehicle crashed into the concrete pier directly. Damage is greatly reduced by slowing down over a period of time.

Activity 4A: Water Balloon Toss

Record your data from the water balloon toss in the table below. Place a ✓ where you completed a toss. Place an ✗ where the balloon popped. **Example data inserted below.**

<table>
<thead>
<tr>
<th>Mass of water balloon (kg)</th>
<th>Distance of Toss (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 m</td>
</tr>
<tr>
<td>Average-sized water balloon</td>
<td>1 kg</td>
</tr>
<tr>
<td>Small water balloon</td>
<td>0.5 kg</td>
</tr>
</tbody>
</table>

Questions

1) Which balloon popped more quickly? Explain why the mass of the balloon affected how you caught the balloon.

The average-filled balloon should have popped more quickly than the small balloon. The greater mass will require more time to slow down without causing adverse effects (popping).
2) Explain how other factors (aside from mass) impacted the number of tosses (and, therefore, the distance between tosses) it took to pop each balloon.

Surface tension of the balloon also comes into play, as well as any variations in the thickness or strength of the material used to make the balloon. In civil engineering, variations in material properties or dimensions are called construction tolerances. Some variation is expected and allowable, but too much variation, and the work may be rejected or replaced.

3) Think about which students were able to toss their balloons from the greatest distance. How did their technique change their results? Think in terms of impulse.

Students successful in tossing and catching their balloons a great distance were careful to bring the balloons up to speed over a period of time and likewise slow them to a stop over a period of time.

4) What would happen if you caught the balloon without cradling it?

The balloon would be more likely to break.

Discussion

Most students, when presented with this activity, realize that they needed to cradle the balloon in order to successfully catch it. These students were applying the concept of impulse by increasing the time in order to decrease the impulse force. Students could discuss ways in which the concept of impulse is used to increase the safety of both vehicles and highway structures.
Activity 4B: Egg Catch

Questions

1) If not for the sheet, the egg would have hit the wall. We know the egg would have broken from the impact with the wall. How was the collision different between the egg and the sheet?

The egg doesn’t break because the sheet increases the distance in which the egg has to come to a stop and, therefore, increases the time to stop the egg. As shown below, the collision between the egg and the wall happens instantaneously and breaks the egg, while the collision between the egg and the sheet occurs over a greater distance and takes more time. This means the impulse between the egg and the wall was greater than between the egg and the sheet.

![Diagram of egg and wall collision](Image)

A greater distance to stop means a greater time to come to a rest. This results in a smaller force acting on the egg.

2) Did an increase in velocity break the egg? Why or why not?

Even if they manage to throw the egg with greater velocity, the students should not be able to break the egg. The sheet will provide enough distance for the egg to stop over time and will result in the egg surviving the toss.

3) What would happen if the sheet were pulled very tight by the students holding it? Do you think the egg would break?

A tighter sheet will lead to a higher impulse, however as tight as the students are able to stretch the sheet, it will still not be enough to increase the impulse to a point where the egg will break.

Discussion

While some eggs may be broken due to poor aim by the students, eggs that hit the sheet will not break. Discuss how this activity relates to traffic attenuation devices, such as water-filled barrels placed in front of bridge piers.
Activity 4: Impulse

Introduction

In this activity, you will observe how various factors affect the impulse an object experiences in a collision. Activity 4A will demonstrate how the mass of an object affects impulse through different sized water balloons. Activity 4B will demonstrate how the velocity of an object affects the impulse of a collision by tossing an egg into a sheet at various speeds.

Objective

In these experiments, you will observe the following:

- Activity 4A: Relationship between mass and impulse in a collision
- Activity 4B: Relationship between velocity and impulse in a collision

Background

The Law of Conservation of Momentum states that the total momentum of two objects before a collision is equal to the total momentum of the two objects after a collision. As the previous activities in this module have demonstrated, the momentum of each object usually changes after the collision. That change in momentum is known as impulse.

Momentum and impulse are related by Newton’s First and Second Laws of Motion.

Momentum is calculated by multiplying the object’s mass by its velocity, which can be written mathematically as:

\[ \rho = mv \]

From Newton’s First Law, we know that an object will remain in motion unless acted upon by an unbalanced force. Therefore, an object’s momentum will remain constant if not influenced by outside factors. Assuming an object’s mass doesn’t change, then any change to an object’s momentum must come from a change in its velocity (an acceleration).

Acceleration is equal to a change in velocity over time, which can be written mathematically as:

\[ a = \frac{\Delta v}{t} \]

Newton’s Second Law states that the force applied by an object is equal to its mass multiplied by its acceleration. Mathematically, this is written as:

\[ F = ma \]

If the equation for acceleration is substituted in for “a,” the equation can be written as:

\[ F = m * \frac{\Delta v}{t} \]
If we look at Newton’s Second Law, \( F = m \times \frac{v}{t} \), and multiply both sides of the equation by “t,” we have the relationship for impulse.

<table>
<thead>
<tr>
<th>Impulse Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F \times t = m \times \Delta v )</td>
</tr>
</tbody>
</table>

Where:
- \( F = \) force
- \( t = \) time
- \( F \times t = \) impulse
- \( m = \) mass
- \( \Delta v = \) change in velocity

Because we already know that impulse is equal to force multiplied by time, we can also say that impulse is equal to mass multiplied by the change in velocity.

Have you ever heard the phrase “roll with the punches”? It usually means that you’re prepared to deal with issues as they arise. Although the phrase is applicable to a variety of situations, its origin has some scientific reasoning behind it, and that reasoning helps us to visualize impulse. When a boxer is about to be hit in the head, they relax their neck and move their head back with the hit, or they roll with the punch. One might think that the boxer is trying to move out of the way, but the velocity at which they move their head is insignificant compared to the incoming punch. Instead, the boxer is increasing the amount of time over which the punch is distributed and, therefore, decreasing the impulse and minimizing its effect. The relationship between time and force in a collision is a concept we will explore in the following activities.

Why is it necessary to know these equations? They help us to understand how the individual components can affect the resulting collision. In a car collision, we know that the mass of the car is a constant, meaning the value will not change. The change in velocity is also constant, which means that increasing the time will distribute the force applied during the collision. Like the boxer, an engineer wants to extend the amount of time over which a collision occurs to offer the least possible amount of injury to the occupants of the car. Knowing which variables won’t change allows us to see which variable to solve for and, therefore, how to manipulate the equation.

If mass and the change in velocity are constant, then any change in time will affect the impulse force. This is the backbone to understanding collisions and how to create safety barriers.
Activity 4A: Water Balloon Toss

Materials

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average-sized water balloon</td>
<td>1 per student</td>
</tr>
<tr>
<td>Small water balloon</td>
<td>1 per student</td>
</tr>
</tbody>
</table>

Setup

You will work in teams of two for this activity. Students will toss the balloon back and forth between each other.

Procedure

1. Fill two water balloons, one to an average size and one smaller than average.
2. Before beginning the balloon toss, measure the mass of each balloon. Record it in the Research Notes section.
3. Begin with the average-sized water balloon. Stand about 2 meters from your partner and toss the balloon to him or her, trying not to break it, as shown in Figure 4-1.

![Figure 4-1: Water Balloon Toss](image)

4. If your partner catches the balloon successfully, you should each step backward until you are about 4 meters apart. Toss the balloon again. Record the distance of the toss in the Research Notes section.
5. Step backward until an additional 2 meters separates you and your partner. Record the distance of the toss in the Research Notes section.
6. Repeat step 5 until the balloon pops.
7. Complete the table in the Research Notes section.
8. Repeat steps 2 through 7 using the small water balloon.
9. Proceed to the questions in the Research Notes section.
Activity 4B: Egg Catch

Materials

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large egg</td>
<td>1</td>
</tr>
<tr>
<td>Bed sheet</td>
<td>1</td>
</tr>
<tr>
<td>Meter stick</td>
<td>1</td>
</tr>
</tbody>
</table>

Setup

You will work in teams of three or as a class for this activity. Two students will hold the bed sheet about one meter in front of a wall, then one student will toss the egg into the sheet.

Procedure

1. With two students holding the bed sheet as shown in Figure 4-2, one student should toss the egg at the sheet. The student tossing the egg may choose how near or far away to stand, but the recommended distance is about 3-5 meters (10-15 feet) away from the sheet.

   ![Figure 4-2: Egg Toss](image)

2. Rotate positions so that each student has a chance to toss the egg.
3. Repeat step 3 until all students in the group have tossed the egg at least once.
4. Proceed to the questions in the Research Notes section.
Activity 4A: Water Balloon Toss

Record your data from the water balloon toss in the table below. Place a ✓ where you completed a toss. Place an ✗ where the balloon popped.

<table>
<thead>
<tr>
<th>Mass of water balloon</th>
<th>Distance of Toss (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average-sized water balloon</td>
<td></td>
</tr>
<tr>
<td>Small water balloon</td>
<td></td>
</tr>
</tbody>
</table>

Questions

1) Which balloon popped more quickly? Explain why the mass of the balloon affected how you caught the balloon.

2) Explain how other factors (aside from mass) impacted the number of times it took to pop each balloon.

3) Think about which students were able to toss their balloons from the greatest distance. How did their technique change their results? Think in terms of impulse.
4) What would happen if you caught the balloon without cradling it?

Discussion Notes
Activity 4B: Egg Catch

Questions

1) If not for the sheet, the egg would have hit the wall. We know that the egg would have broken from the impact with the wall. How was the collision different between the egg and the sheet?

2) Did either increased force or velocity break the egg? Why or why not?

3) What would happen if the sheet were pulled very tight by the students holding it? Do you think the egg would break?

Discussion Notes
Activity 5: Egg Drop

<table>
<thead>
<tr>
<th>Activity Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor Prep Time</td>
</tr>
<tr>
<td>Class Time</td>
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<td>Grade/Class</td>
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<td>Suggested Activity Grouping</td>
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</tbody>
</table>

Introduction

In this activity, students will design a structure that will protect an egg through a series of drops from increasing heights. Students will consider what they’ve learned about impulse and will visualize the collision before designing their structures. They will also apply their knowledge in a discussion on real-world safety devices implemented in automobiles.

Objective

Students will be able to:

- Demonstrate their understanding of impulse while building a protective structure for a series of collisions.
- Apply their knowledge of the factors of the collision when designing the structure.
- Perform the calculations for momentum and impulse during the collisions.

Activity Expansion Ideas

If class time is not sufficient for completing construction and testing of an egg drop structure, a virtual egg drop simulation could be used to model this activity. The questions and discussion activities would remain the same.

Activity 5: Egg Drop

Questions

1) What considerations did you take into account to ensure the safety of your egg?

If the egg drops directly onto the floor, it will break. The structure built around the egg must be such that the impact on the egg is minimized; therefore, students should think about ways to either fail sections of their structure or cushion the egg. Both of these approaches will increase the time in which the egg comes to a stop and, therefore, decreases the impulse applied to the egg.

2) In comparing your design to others, what did you notice about the most effective designs? How about the least effective? Explain.

Answers will vary, but the most effective designs should cushion the eggs, or portions of the structure should have been designed to fail so that the egg came to a stop more slowly than if the structure were not there. Therefore, the impulse force on the egg was lessened.

3) Based on the impulse equation (Impulse = F * t), in order for an egg to survive an impact, should the egg take a longer or shorter time to come to a complete stop? Compare this with the impact force estimated for direct contact with a hard surface.

Based on the impulse equation, a longer time of impact will result in a lesser impulse force. This can be seen from the equation for impact force, As time increases, force must be reduced to achieve the same impulse. A protective structure is designed to extend the time required to make a complete stop, as compared to the short time that would be experienced by dropping the egg onto a hard surface alone.

4) Compare the average velocity to the impact velocity, based on the acceleration of gravity. Why might these two values differ?

It is important to first realize that the average velocity is the average between no velocity at the beginning of the fall and the impact velocity at the end, so the average should be approximately half of the impact velocity. Human error in measuring the velocity will be a large contributor to the difference in values, especially over shorter free-fall distances. Air resistance is another large contributor.

Discussion

This activity explores the concepts of inertia and impulse and how these concepts are used in order to protect against excessive forces. These concepts are the basis for airbags and crumple zones in cars. One topic of discussion could be the proper inflation of an airbag. If it is overinflated the airbag will not distribute the impact force slowly enough; however, if it is underinflated it may not be able to adequately resist the force.
Activity 5: Egg Drop

Introduction
In this activity, you will design a structure that will protect an egg through a series of drops from increasing heights.

Objective
You will be able to:

- Demonstrate your understanding of impulse while building a protective structure for a series of collisions.
- Apply your knowledge of the factors in collisions when designing the structure.
- Perform the calculations for momentum and impulse during the collisions.

Background
When a car crashes, even though the car may stop, the driver continues to move forward due to inertia. Inertia is the resistance of an object to change its state of motion. Newton's First Law of Motion states that an object in motion (in this case the driver) will continue to move in a straight line unless acted upon by an outside force. Airbags are an outside force, they react to the collision by inflating and distributing the force over time. For our purpose, the egg in this activity will represent the driver and the support you build will behave similarly to an airbag.

Newton explained the relationship between force and inertia in his Second Law of Motion. Your egg will accelerate toward the ground due to gravity. Newton's Second Law of Motion states that the force applied to an object is equal to its mass multiplied by its acceleration. Mathematically, this is written as:

\[ F = ma \]

Because acceleration is a change in velocity over a period of time, Newton’s Second Law can also be written as:

\[ F = m \frac{\Delta v}{t} \]

If both sides of the equation are multiplied by time, the resulting equation is:

<table>
<thead>
<tr>
<th>Impulse Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F \times t = m \times \Delta v )</td>
</tr>
</tbody>
</table>

Where:
- \( F \) = force
- \( t \) = time
- \( F \times t \) = impulse
- \( m \) = mass
- \( \Delta v \) = change in velocity
Impulse is represented by the left side of this equation, or the force applied to an object multiplied by the duration of time the force is applied, which can be presented mathematically as:

\[ \text{Impulse} = F \times t \]

These equations represent one of the two primary principles to be used in the analysis of collisions. The physics of collisions are governed by the laws of momentum. In a collision, objects experience an impulse, which is a change in momentum.

Consider the following example: rock climbers attach themselves to steep cliffs using nylon ropes. Should they lose their grip on the rock and fall, the rope will ultimately halt their momentum and prevent them from falling to the ground. Nylon rope is chosen because of its ability to stretch—a rope that is unable to stretch would jolt the climber’s body and potentially hurt them in the case of a fall. The nylon rope stretches and increases the stopping time, which reduces the impulse force exerted on the falling climber. In this activity, you will learn how the design choices you make to protect your egg relate to the nylon ropes used by rock climbers.

**Materials**

**Per student or small group**

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large egg</td>
<td>1</td>
</tr>
<tr>
<td>Popsicle sticks</td>
<td>20</td>
</tr>
<tr>
<td>Straws</td>
<td>10</td>
</tr>
<tr>
<td>Rubber bands</td>
<td>6</td>
</tr>
<tr>
<td>Sheets of paper</td>
<td>2</td>
</tr>
<tr>
<td>Paper Dixie cups</td>
<td>4</td>
</tr>
<tr>
<td>String</td>
<td>100 cm</td>
</tr>
<tr>
<td>Masking tape</td>
<td>100 cm</td>
</tr>
<tr>
<td>Glue</td>
<td>1</td>
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<tr>
<td>Stopwatch</td>
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</tr>
<tr>
<td>Pencil</td>
<td>1 per student</td>
</tr>
<tr>
<td>Paper</td>
<td>As needed</td>
</tr>
</tbody>
</table>

**Setup**

For this activity, you will design and build a structure that will protect an egg from breaking when dropped from various heights, starting at one meter. You may use only the materials given to you by your instructor; however, you do not have to use all the materials in your design. When designing your structure, consider how it will change the impulse of the collision between the egg and the ground. Your goal is to decrease the amount of force that will be applied to the egg in order to minimize the damage.

**Procedure**

1. Create a detailed drawing of what you plan to build using the materials provided by your instructor.
2. Gather your materials from your instructor.
3. Build a structure that will protect the egg when it is dropped.
4. Measure the mass of the egg and the structure separately. Record both masses in the *Research Notes* section.

5. Start by dropping your structure with an egg from a height of one meter. Time the drop using a stopwatch, and record the time in the *Research Notes* section.

6. For structures where the egg survived the fall, move to the next drop point to test the structures again.

7. Continue increasing the drop height until all eggs have broken or there are no higher points from which to drop the eggs.

8. Calculate the **average velocity**, acceleration, force, and momentum of the final fall (which will have broken the egg or ended the contest), and record them in the *Research Notes* section.
Activity 5: Egg Drop

Equations

### Momentum

\[ \rho = mv \]

Where:
- \( m \) = mass (kg)
- \( v \) = velocity (m/s)

### Average Velocity

\[ v_{\text{ave}} = \frac{d}{t} \]

Where:
- \( d \) = distance (m)
- \( t \) = time (s)

### Impact Velocity

\[ v_{\text{imp}} = \sqrt{2gh} \]

Where:
- \( h \) = height of drop (m)
- \( g \) = acceleration of gravity (9.81 m/s²)

### Impact Force

\[ f = m \frac{\Delta v}{t} \]

Where:
- \( m \) = mass (kg)
- \( \Delta v \) = change in velocity (velocity at impact/motionless)
- \( t \) = stopping time (s)

What’s gravity got to do with it?

When you drop an object, the rate in which it falls (velocity) increases over time due to the acceleration of gravity. This can be seen in the figure above (right), which shows a ball being dropped along with the location and corresponding velocity of the ball in one second increments. Knowing the acceleration of gravity and the height of the drop, we can use physics, instead of a stop watch, to calculate the time of the fall and the impact velocity using the above equations.
**NOTE:** The force at impact can be estimated for an egg striking a hard surface by using an estimated stopping time of 0.003 seconds, during which the egg transitions from its impact velocity to zero. The impact force felt by your egg will be less than this force due to your structure increasing the stopping time over which the egg slows down.

### Data Table

<table>
<thead>
<tr>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td><strong>Mass of Egg (kg)</strong></td>
</tr>
<tr>
<td><strong>Mass of Structure (kg)</strong></td>
</tr>
<tr>
<td><strong>Height of Final Drop (m)</strong></td>
</tr>
<tr>
<td><strong>Values Calculated from Experimental Data</strong></td>
</tr>
<tr>
<td><strong>Drop Time (s)</strong></td>
</tr>
<tr>
<td><strong>Average Velocity (m/s)</strong></td>
</tr>
<tr>
<td><strong>Impact Velocity (m/s)</strong></td>
</tr>
<tr>
<td><strong>Impact Momentum (kg*m/s)</strong></td>
</tr>
<tr>
<td><strong>Impact Force (N)</strong></td>
</tr>
<tr>
<td><em>Assume contact with hard surface (t = 0.003s)</em></td>
</tr>
</tbody>
</table>
Questions

1) What considerations did you take into account to ensure the safety of your egg?

2) In comparing your design to others, what did you notice about the most effective designs? How about the least effective? Explain.

3) Based on the impulse equation (Impulse = F * t), in order for an egg to survive an impact, should the egg take a longer or shorter time to come to a complete stop? Compare this with the impact force estimated for direct contact with a hard surface.

4) Compare the average velocity to the impact velocity, based on the acceleration of gravity. Why might these two values differ?

Discussion Notes
Activity 6: Major Impacts

<table>
<thead>
<tr>
<th>Activity Summary</th>
</tr>
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<tbody>
<tr>
<td><strong>Instructor Prep Time</strong></td>
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<td><strong>Grade/Class</strong></td>
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<td><strong>Technology</strong></td>
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<tr>
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</tr>
<tr>
<td>Implement proposed design</td>
</tr>
<tr>
<td>Evaluate product</td>
</tr>
</tbody>
</table>

Introduction

This activity will combine the concepts learned throughout this module into a class competition. Students will design crash barriers to withstand the impact of a vehicle running down a track, while having a limited budget for materials. The vehicle will crash into a force sensor positioned at the end of the track that is connected to the LabQuest 2 which will display the impact force of the crash barrier collision. The most effective crash barriers will increase the amount of time it takes for a vehicle to come to a stop due to the impulse relationship, where increasing impact time will decrease impact force.

Objective

In this experiment, students will:

- Use their understanding of momentum and impulse to create an effective crash barrier.
- Learn about different crash barriers implemented on roadways.
- Allocate funds to make a cost effective design.

Activity Expansion Ideas

Round of Applause

Have students clap their hands as hard as they can. Can they feel the impact? Ask students what they could do to soften the impact felt as a result of this clap. They could wear mittens or gloves, but what should the mittens/gloves be made of (plastic, leather, knitted to different thicknesses, etc.)? Relating this to crash barriers, certain materials are selected to build crash barriers based on their ability to reduce impact force.
Video – Understanding Car Crashes: It’s Basic Physics

The Insurance Institute for Highway Safety (IIHS) has several excellent videos to teach middle and high school students about safety. “Understanding Car Crashes: It’s Basic Physics” is a 22 minute video showing the physics involved in car crashes and ways in which car manufacturers try to mitigate the harmful effects of automobile accidents. A DVD of this video is available to purchase from the IIHS website and is available to watch on their YouTube channel:

https://www.youtube.com/watch?v=yUpiV2I_IRI

The host of the video has also developed a Teacher’s Guide to accompany the video:


Trial and Error

In this activity students build and test a crash barrier. You could have students perform this activity, then repeat it to improve their design and make a more effective crash barrier. Therefore, students learn not only about applying impulse and momentum, but also the scientific process.

Look Around You

There are likely many types of crash barriers used on roadways in your area that students encounter on a regular basis. Have students identify different crash barriers that are introduced in the Background section of the Research Manual. Ask them to look and see how many they can find on their way home at the end of the day. Discuss the barriers they saw, and where they saw them. Were there any patterns to where these barriers were placed?
Activity 6: Major Impacts

Questions

1) Why would it cause issues for the crash barrier if you added more mass to the vehicle?

If the mass of a vehicle increases, the momentum of the vehicle also increases if the velocity remains constant. This means that the impulse force experienced during a collision is increased, and the crash barrier that was able to handle the lighter vehicle may not be able to handle the more massive vehicle. If the crash barrier does handle the crash, the impact force will be increased (assuming that the barrier still brings the vehicle to a stop in the same amount of time).

2) How would a crash barrier built for high speed collisions differ from one built for low speed collisions? Try to think of specific design choices you have seen in the real world.

A barrier built for higher speed collisions will need to absorb more impulse than a barrier for low speed collisions. To do this safely, high speed crash barriers must substantially increase the impact time in order to reduce impact force. This is often done by the use of moving components (read about sequential kinking terminals in the Background section) or by using highly compressible materials (read about crash cushion barriers in the Background section). A barrier for lower speed collisions does not need as much time to safely stop the car due to the lower overall impulse. Therefore, barrier designs for these scenarios are less complex.

3) What would happen if we reduced the angle of the track?

The vehicle would have less velocity; therefore, the momentum of the vehicle would decrease and the impulse would be lower. This means that lower impact forces would be felt by the vehicle crashing into the same barrier.

4) What did the successful groups do to create an effective barrier?

Answers will vary depending on the designs the class comes up with. Generally, successful groups will show an understanding of impact by increasing the time needed to bring the vehicle to a stop, either by cushioning the blow or by strategic failure of the crash barrier components.

5) How could you improve your design?

Again, answers will vary. Adding more cushioning or implementing a strategic failure mechanism will allow increased performance.

Discussion

Students will need to combine their understanding of momentum and impulse in order to create an efficient barrier. As with many things, the quality of construction is critical. Some students may design parts of their attenuation device to fail, which is a good idea. Discuss how this type of system must be weak enough to fail yet strong enough to withstand most of the force. What happens if unforeseen conditions occur (i.e. an impact force lesser than or greater than what it was designed for)?
Activity 6: Major Impacts

Introduction

Imagine you are biking, lose control, and have to choose between crashing into a stone wall or a bale of hay. Hopefully, you would crash into the bale of hay because there would be less of an impact force acting upon you than there would be resulting from the stone wall. This is the case because the hay would slow you down over a greater amount of time, causing less damage than a sudden impact with the stone wall.

In this activity, you will apply your understanding of momentum and impulse, learned in previous activities, in attempt to safely stop a vehicle as it accelerates down an inclined track. To accomplish this, you will design a crash barrier. Crash barriers are devices implemented on roadways to protect vehicles from severe damage by reducing the impact force experienced by the vehicle. Your crash barrier will be tested in a class competition, where the barrier that results in the lowest impact force will be declared the most effective.

Objective

In this experiment, you will:

- Use your understanding of momentum and impulse to create an effective crash barrier.
- Learn about different crash barriers implemented on roadways.
- Allocate funds to make a cost effective design.

Background

There are many hazards that surround highways, some of which can cause serious injury to the occupants of the vehicle in the case of a collision. For example, if a car traveling on the highway was to lose control and crash into a bridge abutment, serious injuries could be expected. Not only would this collision be dangerous for the occupants of the crashed vehicle, but also for other motorists who may be affected by the resultant damage to the bridge. This is why we use impact attenuators, also known as crash barriers, to minimize impact forces and limit injury resulting from crashes. These barriers are placed in front of roadway hazards in order to distribute impact forces over a greater time period than if a direct collision was to occur. Let us look at a few crash barriers implemented on roadways.

Guardrails, as shown in Figure 6-1, are placed parallel to roadways and are designed to protect vehicles in the event of a side impact with the rail. The guardrail on the left side of Figure 6-1 is attached directly to the steel posts in the ground, and will not give or flex much in the event of a side impact. The guardrail on the right will flex upon impact, because of the wooden blocks between the rail and the posts. These wooden blocks will compact upon impact, and allow more room for the rail to flex. This extra room to flex means more time for the vehicle to stop, which, in turn, means a lower impact force is applied to the vehicle. Traffic engineers will preferably implement guardrails with these wooden blocks, especially when designing a guardrail around a curve. This is because more force is exerted on a guardrail during an angled crash by a vehicle failing to navigate a curve then a sideswipe hit by a vehicle driving while parallel to the roadway.
Figure 6-1: Guardrails Protect Against Side Impact with Rail

Figure 6-2 shows a blunt terminal attached to the end of a steel guardrail. A blunt terminal is a type of crash barrier that is implemented in slow-traffic areas, and can distribute small impact forces over a longer period of time than the end of the guardrail alone. However, if a fast-moving vehicle were to impact with this terminal, the rail could penetrate the vehicle or cause it to flip, because the impulse experienced is higher than the terminal is able to handle. For this reason, this type of crash barrier is only found in low-speed areas.

Figure 6-2: Blunt Guardrail Terminal

Figure 6-3 shows a sequential kinking terminal (SKT). An SKT works similar to a blunt terminal in that it is placed on the end of a guardrail to absorb the impulse resulting from a car collision. The SKT has a large square piece at its end, designed to be pushed and slid along the rail during a head-on impact. As the end piece slides, the wooden guardrail supports will break, resulting in the bending to absorb the impact, out of the way of the vehicle. Because the rail bends gradually upon impact, the time to slow down a vehicle is increased and, therefore, the force of impact is reduced. The wooden supports each have a hole drilled in them near the bottom ensuring the post will break on impact, but break off close enough to the ground as to not cause more damage to a vehicle. SKTs are found on roadways with moderate speed limits, and can thus handle moderate impacts.
Guardrails are permanent structures; they are anchored to the ground and are meant to move only in the event of an impact. To reduce injury from impact with a temporary hazard, such as construction equipment, a different solution is needed, one that is both effective and easily transportable. One solution is the use of crash cushion barrels, as shown in Figure 6-4. These cushions are placed directly in front of hazards, and work by compressing and deforming upon impact. This compression takes time, and, therefore, the impact force is reduced. You will often see these on construction sites, or near highway bridge abutments.

Figure 6-5 shows an energy absorption barrier. Usually implemented on trucks used on construction sites, this barrier is attached to a hinge at the rear of a semi-truck. In this picture, the stopped truck has the barrier in its lowered (functioning) position while work is being done on a nearby road. The absorption barrier acts as a super big bumper because if it is hit hard enough, the barrier will crumple and deform. This will give the impacting vehicle more time to slow down. More impact time means a lower impact force is exerted, not only on the impacting vehicle, but on the truck as well. In order for appropriate use, rules are made to govern the use of these barriers. If this truck is parked in a 45 mph speed zone or higher, the truck barrier is to be lowered into its functioning position. (The barrier shown in Figure 6-5 has sustained three impacts in less than one year’s time, and is still functioning!)
Materials

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vernier LabQuest 2</td>
<td>1</td>
</tr>
<tr>
<td>Vernier Dual-Range Force Sensor</td>
<td>1</td>
</tr>
<tr>
<td>Maglev Track</td>
<td>1</td>
</tr>
<tr>
<td>Pinewood Derby Car</td>
<td>1</td>
</tr>
<tr>
<td>Scissors</td>
<td>1 or more</td>
</tr>
<tr>
<td>Rulers</td>
<td>1 per group</td>
</tr>
<tr>
<td>Safety Glasses</td>
<td>1 per student</td>
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<tr>
<td><strong>Barrier Materials (see below)</strong></td>
<td>Enough for class</td>
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</tbody>
</table>

Barrier Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Cost</th>
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<tbody>
<tr>
<td>Straws</td>
<td>$6.00 each</td>
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<tr>
<td>Cardboard</td>
<td>$0.50 per in²</td>
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<tr>
<td>Popsicle Stick</td>
<td>$3.00 each</td>
</tr>
<tr>
<td>Toothpick</td>
<td>$1.00 each</td>
</tr>
<tr>
<td>Masking Tape</td>
<td>$0.50 per inch</td>
</tr>
<tr>
<td>Duct Tape</td>
<td>$1.00 per inch</td>
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</tbody>
</table>

Setup

Your instructor will set up the track at an angle to create an inclined surface. The difference between the two ends of the track should not exceed a height of 0.75 meters (2.5 feet) so that the impact of the vehicle will not surpass the sensor’s threshold (±50 N). The sensor should be secured to the wooden dowel and have the wood box placed around it (refer to Figure 6-7). You will then place your constructed crash barrier in front of the sensor. Set up of the LabQuest 2 system can be done by following along in Appendix B: Setting Up the LabQuest 2 & Vernier Dual-Range Force Sensor.

Your instructor will set up a store, containing the resources listed in the Barrier Materials table above. The goal is to use the building materials to create a crash barrier that will minimize the force of impact on the vehicle shown in Figure 6-6. The vehicle runs down the track and makes contact with the sensor, which will record the impact forces felt by the vehicle upon impact with the crash barrier. Keep in mind that a lower impact force is desired.

The force felt by the occupants of the car as it crashes into the barrier is the same as the force observed behind the barrier. For every force applied, there must be an equal and opposite reaction. Therefore the force could be measured with a sensor placed on the car or by a sensor placed behind the crash barrier. Mounting the sensor to the track behind the barrier helps protect it from potential damage, such as being dropped, which could occur if mounted to the car.
You have a budget of $30 to purchase supplies from the *Barrier Materials* table. You will work individually or in teams of two, depending on the size of the class, to build a crash barrier, as determined by your instructor. **The barrier must fit entirely within the track. In order to do so, the barrier cannot exceed a length of 4 inches, a height of 3 ½ inches, or a width of 2 ¾ inches.**

**Procedure**

1. Review the available materials, and make a list of what you plan to purchase with your allotted budget.
2. Create a detailed drawing of the barrier you plan to build.
3. Purchase your materials from your instructor.
4. Build your crash barrier.
5. Once the class is finished creating their barriers, it is time for the competition. First, send the vehicle down from the top of the ramp to impact against the wall without a crash barrier in place (see Figure 6-7). Record the impact force (see Figure 6-8 & Appendix B). You may want to repeat this three times to achieve an average impact force. This will be used as a baseline value to determine how effective your barrier is at reducing the impact force.
6. Repeat step five, only this time place your crash barrier in front of the wall at the end of the track. Perform only one trial rather than three, as your barrier will likely become less effective the more impacts it experiences. Record your resulting impact force in the Research Notes section (see Figure 6-7).

7. Repeat step 6 for every barrier in the class. The winning crash barrier has the lowest force at impact.

8. Continue on to the calculations and questions in the Research Notes section.
Activity 6: Major Impacts

Calculating Crash Barrier Score

Record the force given by the LabQuest 2 for each student/team, and determine which barrier reduced the max force by the largest amount.

Max Force without Barrier = ________________

<table>
<thead>
<tr>
<th>Team</th>
<th>Force with Barrier</th>
<th>Reduction of Max Force</th>
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<tbody>
<tr>
<td></td>
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Questions

1) Why would it cause issues for the crash barrier if you added more mass to the vehicle?

2) How would a crash barrier built for high-speed collisions differ from one built for low-speed collisions? Try to think of specific design choices you have seen in the real world.
3) What would happen if we reduced the angle of the track?

4) What did the successful students/groups do to create an effective barrier?

5) How could you improve your design?

Discussion Notes
Appendix A: Document Links

NCHRP 20-52
The NCHRP 20-52 final report details the completion of the original TRAC PAC 2 program, including the original manual.

TRAC/Michigan Education Standards
Link: http://www.michigan.gov/mdot/0,4616,7-151-9623_38029_38059_41397-184233--,00.html
The Michigan Education Standards are outlined in terms of how TRAC meets the benchmark goals. This page includes the standards for 6th, 7th, and 8th grades as well as high school standards. Both the TRAC modules and bridge building competition are listed.

Manuals
Appendix B: Setting Up the LabQuest 2 & Vernier Dual-Range Force Sensor

1. Turn on the LabQuest 2.
2. On the Dual-Range Force Sensor, set the force reading to ±50 N.
3. The LabQuest 2 should be connected to the Sensor by a cord plugged into the CH1 port on the LabQuest 2.
4. In the top right corner are options allowing you to change the mode, sample rate, and duration. These may need to be changed depending on the setup in order to show all of the collected data.
5. Be sure to set the sample rate to 200 samples per second in order to get accurate and consistent data collection. This can be done by clicking on the sample box and entering 200 into the rate text box. (Note: The sample rate will reset to 50 samples per second each time it is turned off.)
6. When you are ready to run the experiment press play in the bottom left corner.
7. After a moment a graph will display showing the force data collected by the sensor. Keep an eye on the screen, when an impact force is collected you will see a downward spike in the data. After impact has occurred press the stop button in the lower left corner of the screen to stop collecting data.
8. The impact force is recorded as a negative number because the force is in compression. Go to the Analyze menu and select Statistics and then check the button by Force to view the maximum and minimum values in the collected data.
9. The minimum value is the greatest compressive force measured during the test and corresponds to the force at impact.
10. Repeat procedure starting at step 6 for additional trials.

Note: If further instruction is needed, turn to the Appendix A: Document Links. Here you will find links that will take you to the user manual for the LabQuest 2 and/or the Dual-Range Force Sensor.
Appendix C: Glossary of Terms

Acceleration (a): Physical vector quantity that is defined as the rate at which an object changes its velocity. (velocity / time)

Acceleration of Gravity: Acceleration of an object due to the effects of gravity.

Air Resistance: A force resisting motion that acts on an object as it moves through air. This is a unique type of friction force. Also known as drag or wind resistance.

Average Velocity: The distance from a start to an end location over a period of time. This is not necessarily equal to the distance travelled over a period of time.

Component: A part or element of a larger whole. (For example: A vector can be separated into a vertical component and a horizontal component that represent the force’s effect in those two directions)

Coulomb’s Law: The force of attraction or repulsion on two particles due to their electrical charges (acting along a straight line) is directly proportional to the product of the particles’ charges and inversely related to the square of the distance between them.

Deformation: The process of an object changing in shape or distorting, especially through the application of forces.

Elastic Collision: A collision in which there is no loss of kinetic energy or momentum.

Force: A push or pull acting upon an object as a result of the object’s interaction with another object.

Friction Forces: Forces that resist motion, generated when two surfaces are in contact and slide against each other.

Gravity (Fg): Universal force of attraction acting between all matter. Gravity is a force pulling objects towards the center of a mass.

Impact Velocity: Velocity of an object in motion just before its motion is stopped by another object.

Impact: The action of one object coming forcibly into contact with another. Generates a large force over a short period of time.

Impulse: Quantity representing the overall effect of a force acting over time. (force * time)

Inelastic Collision: A collision in which a portion of the kinetic energy is lost from the object in motion and converted into some other form of energy.

Inertia: An object’s tendency to resist changes in the state of motion.

Kinetic Energy (KE): Energy of motion. (Forms of KE include: vibrational, rotational, and translational motion)

Law of Conservation of Momentum: Total linear momentum of an object within closed system remains constant through time, regardless of other possible changes within the system. A closed system is a system that isn’t subject to external forces.
Appendix C: Glossary of Terms

**Maglev:** A vehicle that travels along a magnetic guideway which lifts the vehicle off of the ground during motion, reducing friction to a negligible level and allowing higher travel velocities. (i.e. maglev trains)

**Magnitude:** Absolute size or extent of a quantity; how large or small a quantity is.

**Mass:** Quantity of matter in a body regardless of its volume or of a forces acting on it.

**Momentum (p):** Measurement of mass in motion; how much mass is in how much motion. (mass * velocity)

**Negligible:** so small or unimportant that its effects may safely be ignored. In this module, to simplify a problem a force may have been chosen to be neglected.

**Newton’s First Law (Law of Inertia):** An object at rest stays at rest, while an object in motion stays in motion with a constant velocity unless acted upon by a force.

**Newton’s Second Law (Law of Motion):** The magnitude of a resultant force acting on an object is equal to the mass of the object multiplied by the acceleration of the object. This resultant force is the sum of all forces acting on the object.

**Pythagorean Theorem:** Fundamental relation in trigonometry between the three sides of a right triangle. It states that the square of the hypotenuse (the longest side opposite the right angle) is equal to the sum of the squares of the other two sides. \( c^2 = a^2 + b^2 \)

**Relative Velocity:** Vector difference between the velocities of two bodies – the velocity of a body with respect to another regarded as being at rest – compare relative motion.

**Trigonometry:** Branch of mathematics dealing with the relationship between the size of a triangle’s sides and angles.

**Vector:** A physical quantity that has both a magnitude and a direction. Typically represented as an arrow pointing in the vector direction with a length based upon the vector quantity.

**Velocity (v):** Physical vector quantity that is defined as the speed and direction of an object in motion. (distance / time)
This manual was updated and revised in 2016 by the Center for Technology and Training (CTT) at Michigan Technological University for the Michigan Department of Transportation (MDOT).