

A PROBABILITY MODEL FOR  
DETERMINING THE SAFETY BENEFITS  
OF MEDIAN BARRIER INSTALLATIONS

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The question of guardrail and median barrier installation at a particular location is complicated by the considerable doubts expressed in the literature as to net safety benefits. In general, it is acknowledged that any barrier, sufficiently strong to contain high velocity impact, is itself a hazard; therefore, engineers are cautioned as to the complex "trade-off" nature of decision making in this area.

Fixed-object accident models have been prepared in the past in an attempt to rationalize guardrail warrants (1). However, no moving object collision model is currently available in the literature. It is the purpose of this study to develop a median barrier model for handling the accident trade-off question.

Consider two roadways identified as Roadway 1 and 2, separated by a median of  $M$  ft. Roadway  $i$  has  $N_i$  lanes,  $i = 1, 2$ , as shown in Figure 1. In order to evaluate the net safety benefit of a median barrier, it is essential to know, for a given vehicle encroaching onto the median, the following probabilities:

a) The probability  $P_B$  that the encroaching vehicle will strike a barrier installed at  $M_i$  ft from the edge of Roadway  $i$ . Note that  $M_1 + M_2 = M$ .

b) The probability  $P_H$  that, if no barrier is installed between the roadways, the encroaching vehicle will cross over the median and collide with a vehicle in the opposite roadway.

Before proceeding with the development of the probability model for computing  $P_B$  and  $P_H$  defined in a) and b), respectively, we present the following notation used throughout this report.

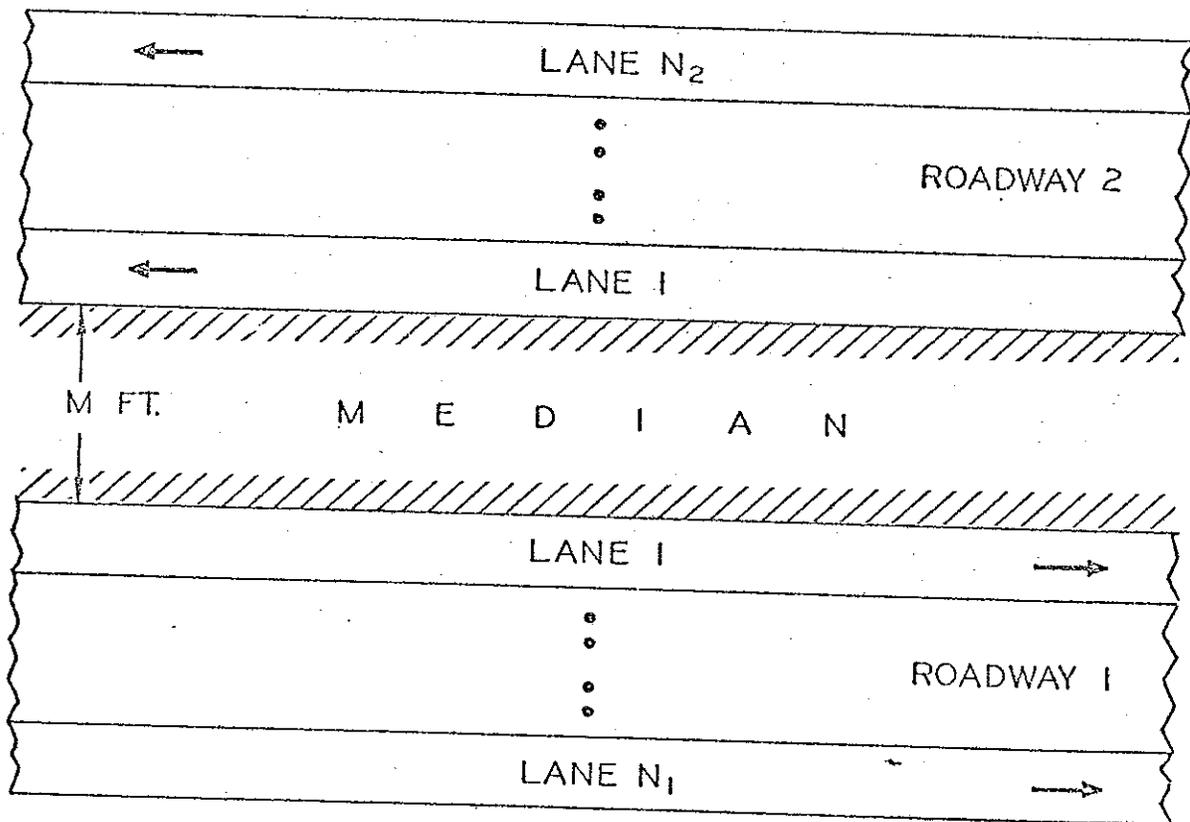


Figure 1. The roadway system in concern.

$\theta$  : the vehicle encroachment angle on the median

G : the probability distribution function of the encroachment angle

Y : the maximal lateral encroachment distance of a vehicle encroaching onto the median

X : the longitudinal distance, corresponding to Y, of a vehicle encroaching onto the median

$F_{\theta}$  : the conditional probability distribution function of Y, given the encroachment angle  $\theta$ .

$E_i$  : the probability that a vehicle traveling on Roadway i will encroach onto the median.

The definitions of  $\theta$ , Y, and X are illustrated in Figure 2.

#### The Models for Computing $P_B$ and $P_H$

##### Case 1: Barrier in Median

Given that a vehicle from Roadway i has encroached onto the median, this vehicle will strike the barrier installed at  $M_i$  ft from the edge of Roadway i if the maximal lateral encroachment distance is greater than  $M_i$ . The probability  $P_B^{(i)}$  that the encroaching vehicle will strike the barrier is:

$$P_B^{(i)} = \int_{\theta=0}^{\theta_0} \int_{y=M_i}^{\infty} dF_{\theta}(y) dG(\theta) \quad (1)$$

Since the barrier can be struck by a vehicle from either roadway, we have

$$P_B = \sum_{i=1}^2 E_i P_B^{(i)} \quad (2)$$

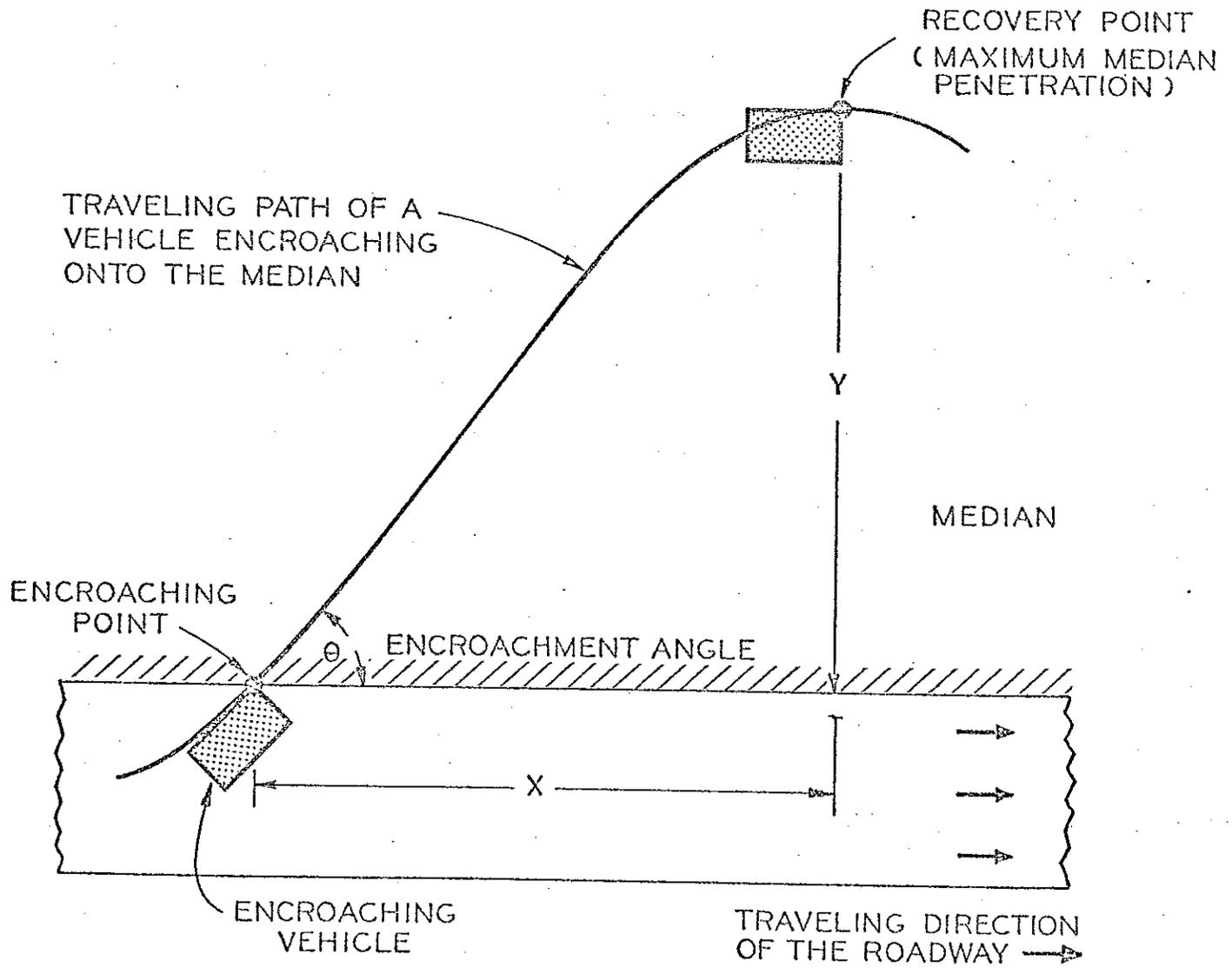


Figure 2. Illustration of a vehicle encroaching onto the median.

## Case 2: No Barrier in Median

Given that a vehicle from Roadway  $i$  has encroached onto the median, the probability  $P_H^{(i)}$  that this encroachment will result in a collision with a vehicle traveling on the opposite roadway is:

$$P_H^{(i)} = \int_{\theta=0}^{\theta_0} \int_{y=M}^{\infty} C(M, y, \theta) dF_{\theta}(y) d\bar{G}(\theta) \quad (3)$$

where  $C(M, y, \theta)$  is the probability that an encroaching vehicle of angle with  $y > M$  will collide with a vehicle traveling on the opposite roadway.

Again, because of encroachments from both roadways, we have

$$P_H = \sum_{i=1}^2 E_i P_H^{(i)} \quad (4)$$

In order to compute  $P_B$  and  $P_H$ ,  $G$ ,  $F$ ,  $E_i$  and  $C(M, y, \theta)$  must be determined. The computational methodology is presented in the following sections.

### Distribution of Encroachment Angle

The empirical distribution  $\bar{G}(\theta)$  of the encroachment angle  $\theta$  as reported by Hutchinson and Kennedy (2) is shown in Figure 3. It appears that  $\bar{G}(\theta)$  is a distribution of the gamma type. Thus,  $G$  was defined as:

$$G(\theta) = \int_0^{\theta} \frac{1}{\Gamma(\alpha) \beta^{\alpha}} \theta^{\alpha-1} e^{-y/\beta} dy \quad (5)$$

in an attempt to fit the empirical distribution  $\bar{G}$ . A non-linear least squares

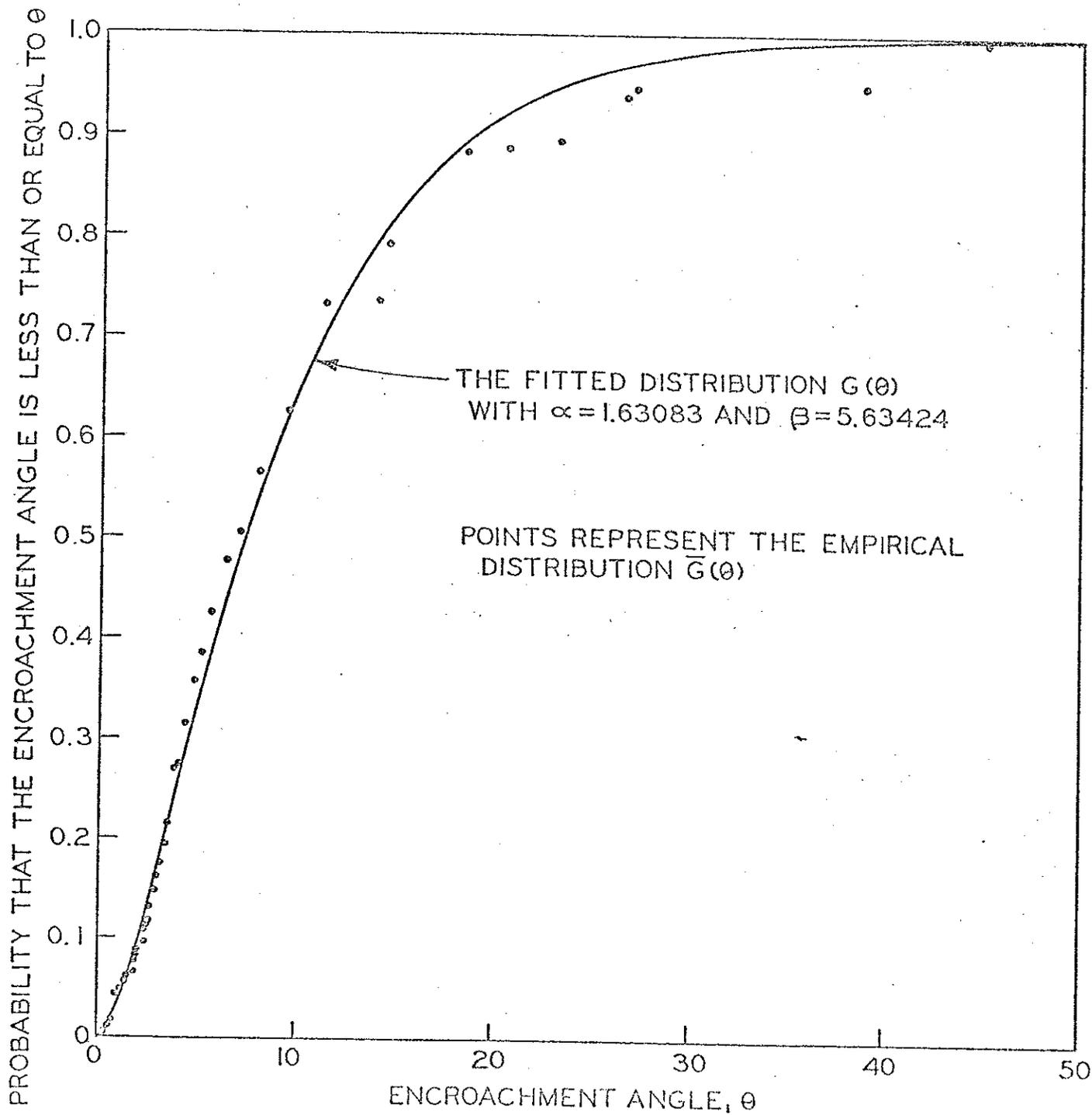


Figure 3. The empirical and fitted distributions of the encroachment angle  $\theta$ .

curve fitting computer program and a gamma distribution program (3) were used to determine  $\alpha$  and  $\beta$  in the sense that  $\sum_i [G(\theta_i) - \bar{G}(\theta_i)]^2$  is minimal. The resulting values were:  $\alpha = 1.63083$  and  $\beta = 5.63424$ . The fitted and empirical distributions of the encroachment angle  $\theta$  are presented in Figure 3.

This fit was statistically acceptable in the sense that the Kolmogorov-Smirnov test did not reject the null hypothesis that the encroachment angle was sampled from a population having the distribution  $G$  specified in Eq. (5) with parameters  $\alpha = 1.63083$  and  $\beta = 5.63424$ .

The Conditional Probability Distribution Function  $F_\theta$  of the Maximal Lateral Encroachment Distance, Given that the Encroachment Angle is  $\theta$

Since the number of observations on  $Y$  for a given encroachment angle was not large enough to obtain a reliable empirical distribution of  $Y$  for each value of  $\theta$ , observations on  $Y$  for a range of values for  $\theta$  given by an interval  $I = (\theta_1, \theta_2)$  were used to obtain the empirical distribution denoted by  $\bar{F}(y | \theta \text{ in } I)$ . Each interval was chosen as small as possible, but contained sufficient observations to obtain a reliable empirical distribution. This empirical distribution is then fitted by  $F(y | \theta \text{ in } I)$  defined as

$$F(y | \theta \text{ in } I) = \frac{1}{G(\theta_2) - G(\theta_1)} \int_{\theta_1}^{\theta_2} F_\theta(y) dG(\theta) \quad (6)$$

where  $G$  was defined in Eq. (5) with  $\alpha = 1.63083$  and  $\beta = 5.63424$  and  $F_\theta(y)$  is to be specified.

By the well-known mean-value theorem, there exists an angle  $\delta$ , depending on  $I$  and  $y$ , such that

$$F(y|\theta \text{ in } I) = F_S(y) \quad (7)$$

If the interval I is not large,  $S$  will not vary significantly. In this situation, the empirical distribution  $\bar{F}(y|\theta \text{ in } I)$  can be approximated by the empirical distribution of Y, given the encroachment angle  $S$ . By examining empirical distributions of Y for  $\theta$  in various intervals, it was found that  $\bar{F}(y|\theta \text{ in } I)$  takes the form of a normal distribution and is skewed when the end points of I are small. That is,  $\bar{F}(y|\theta \text{ in } I)$  can be approximated by  $F_S(y)$ , for some  $S$  in I, which is either a gamma distribution with parameters  $\alpha_S$  and  $\beta_S$  or is defined as

$$F_S(y) = \frac{N(y; \mu_S, \sigma_S^2) - N(0; \mu_S, \sigma_S^2)}{1 - N(0; \mu_S, \sigma_S^2)} \quad (8)$$

where  $N(y; \mu_S, \sigma_S^2)$  is a normal distribution with mean  $\mu_S$  and variance  $\sigma_S^2$ .

The distribution  $F_S(y)$  defined in Eq. (8) fits empirical distributions reasonably well. The fitted results for various chosen intervals are presented in Table 1. For the purpose of finding functional forms of  $\mu_\theta$  and  $\sigma_\theta$ , the average of angles in I was used as the angle  $S$  defined in Eq. (7). That is

$\mu_{\theta \text{ in } I} = \mu_S$  and  $\sigma_{\theta \text{ in } I} = \sigma_S$ . After examining the plots of  $(S, \mu_S)$ ,  $(S, \sigma_S)$ , and  $(\mu_S, \sigma_S)$ , it can be shown that  $\mu_S$  and  $\sigma_S$  will follow any one of the following functions:

$$\begin{aligned} \mu_S &= a_1 - a_2 e^{-a_3 S} \\ \mu_S &= a_1 + a_2 S + a_3 S^2 \\ \mu_S &= a_1 + a_2 S^{a_3} \end{aligned} \quad (9)$$

TABLE I  
 THE FITTED PARAMETERS OF  $F(y|\theta \text{ in } I)$   
 FOR VARIOUS INTERVALS  $I = (\theta_1, \theta_2)$

$\theta_1$	$\theta_2$	Average $\theta$ in I	Fitted Parameters	
			$\mu_{\theta}$ in I	$\mu_{\theta}$ in I
0.00	1.10	0.854	8.75556	13.79640
0.00	1.90	1.186	8.80056	15.39680
0.00	2.30	1.367	8.72961	15.98530
1.10	2.30	1.843	4.62786	23.23010
1.90	2.90	2.521	15.39470	15.62440
2.30	2.90	2.650	18.15080	14.14130
2.30	3.40	2.925	22.27920	10.30130
2.90	3.40	3.200	22.91880	7.83612
2.90	3.80	3.533	17.76600	10.08370
3.40	4.10	3.761	12.81340	10.41560
3.80	5.20	4.728	20.54350	11.49310
4.10	4.80	4.617	20.73780	12.60750
4.10	5.20	4.748	21.14960	11.14460
4.40	6.40	5.600	17.08210	12.19330
4.80	5.70	5.516	15.73870	8.75378
5.20	6.40	6.089	13.67130	10.27850
5.70	6.40	6.400	9.95238	15.88370
5.70	7.10	6.643	12.81550	12.15330
6.40	8.10	7.780	19.30110	10.21000
6.40	9.50	8.476	20.60220	10.97330
7.10	11.30	9.992	21.68550	11.63890
7.10	18.40	12.721	22.36580	11.39600
7.10	20.60	12.794	22.38760	11.23690
7.10	23.20	12.981	22.42780	11.07010
7.10	27.00	14.609	23.21420	11.10810
8.10	9.50	9.500	20.69990	9.94794
8.10	18.40	13.585	22.26650	11.81360
8.10	20.60	13.661	22.29930	11.60900
8.10	23.20	13.864	22.35470	11.39270
8.10	27.00	15.624	23.29400	11.40930
9.50	9.50	9.500	20.69990	9.94794
9.50	14.50	12.423	21.25520	12.26010
9.50	18.40	14.523	22.27660	11.52090
9.50	20.60	14.604	22.30970	11.29090
9.50	23.20	14.827	22.36260	11.06370
9.50	27.00	16.755	23.43800	11.12620
11.30	14.50	14.471	21.35750	12.25840
14.00	20.60	17.000	22.87030	10.90850
14.50	23.20	18.807	23.37310	8.73342
14.50	27.00	21.482	25.06230	9.19946
18.40	45.00	34.078	26.85110	8.12058
10.60	45.00	34.513	27.11420	8.39423
20.60	90.00	36.247	27.12670	8.06347
23.20	27.00	26.653	28.06220	9.11606
23.20	45.00	35.293	27.51020	8.43546
23.20	90.00	37.117	27.49430	8.06662
26.60	45.00	42.356	26.25380	8.14392
26.60	90.00	45.159	26.60290	7.91612
27.00	90.00	47.580	26.65850	7.63441
45.00	45.00	45.000	25.41430	7.22221

and

$$\begin{aligned}\sigma_y &= b_1 e^{-b_2 y} \\ \sigma_y &= b_1 - b_2 \mu_y\end{aligned}\tag{10}$$

To estimate coefficients of  $\mu_y$  and  $\sigma_y$ , the  $\theta$ -data is first partitioned into nine mutually exclusive intervals in which sample sizes are large enough to obtain reliable empirical distributions of Y for  $\theta$  in various intervals. Then, the distribution defined in Eq. (6) was used to fit nine empirical distributions simultaneously. Among those functional forms of  $\mu_y$  and  $\sigma_y$  defined in Eqs. (9) and (10),  $F_\theta(y)$  with parameters  $\mu_\theta$  and  $\sigma_\theta$  defined as

$$\mu_\theta = 26.8844 - 16.9943e^{-0.103498\theta}\tag{11}$$

and

$$\sigma_\theta = 22.417 - 0.539499\mu_\theta\tag{12}$$

fit the empirical distributions best. Again, the fit was acceptable in the sense that the Kolmogorov-Smirnov test does not reject the null hypothesis at the 0.05 significance level that the data for maximal lateral encroachment distances was sampled from a population having the distribution  $F(y)$  defined

as

$$F(y) = \int_0^\theta F_\theta(y) dG(\theta)\tag{13}$$

where  $F_{\theta}(y)$  was defined in Eq. (8) with parameters  $\mu_{\theta}$  and  $\sigma_{\theta}$  defined in Eq. (11) and Eq. (12), respectively.

It can be shown that the mean and variance of  $Y$ , given the encroachment angle  $\theta$ , are respectively,

$$E_{\theta}(Y) = \mu_{\theta} + \sigma_{\theta} w(\theta) \quad (14)$$

and

$$\text{Var}_{\theta}(Y) = \sigma_{\theta}^2 \left[ 1 - w(\theta) \frac{\mu_{\theta}}{\sigma_{\theta}} - w^2(\theta) \right] \quad (15)$$

where

$$w(\theta) = \frac{1}{1 - N(0; \mu_{\theta}, \sigma_{\theta}^2)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\mu_{\theta}}{\sigma_{\theta}} \right)^2} \quad (16)$$

the results of  $\mu_{\theta}$  and  $\sigma_{\theta}$ , defined in Eqs. (11) and (12), and  $E_{\theta}(Y)$  and  $\text{Var}_{\theta}(Y)$ , defined in Eqs. (14) and (15), are presented in Figure 4.

Once  $G(\theta)$  and  $F_{\theta}(y)$  are defined,  $P_B^{(i)}$  defined in Eq. (1) can now be computed. However, we must still determine  $C(M, y, \theta)$  to compute  $P_H^{(i)}$  defined in Eq. (3). This is discussed in the following section.

#### Probability of a Collision with Vehicles Traveling on the Opposite Roadway

Given that a vehicle enters the opposite roadway, what is the probability that this encroaching vehicle will collide with a vehicle traveling on the opposite roadway? In general, the longer time the encroaching vehicle travels

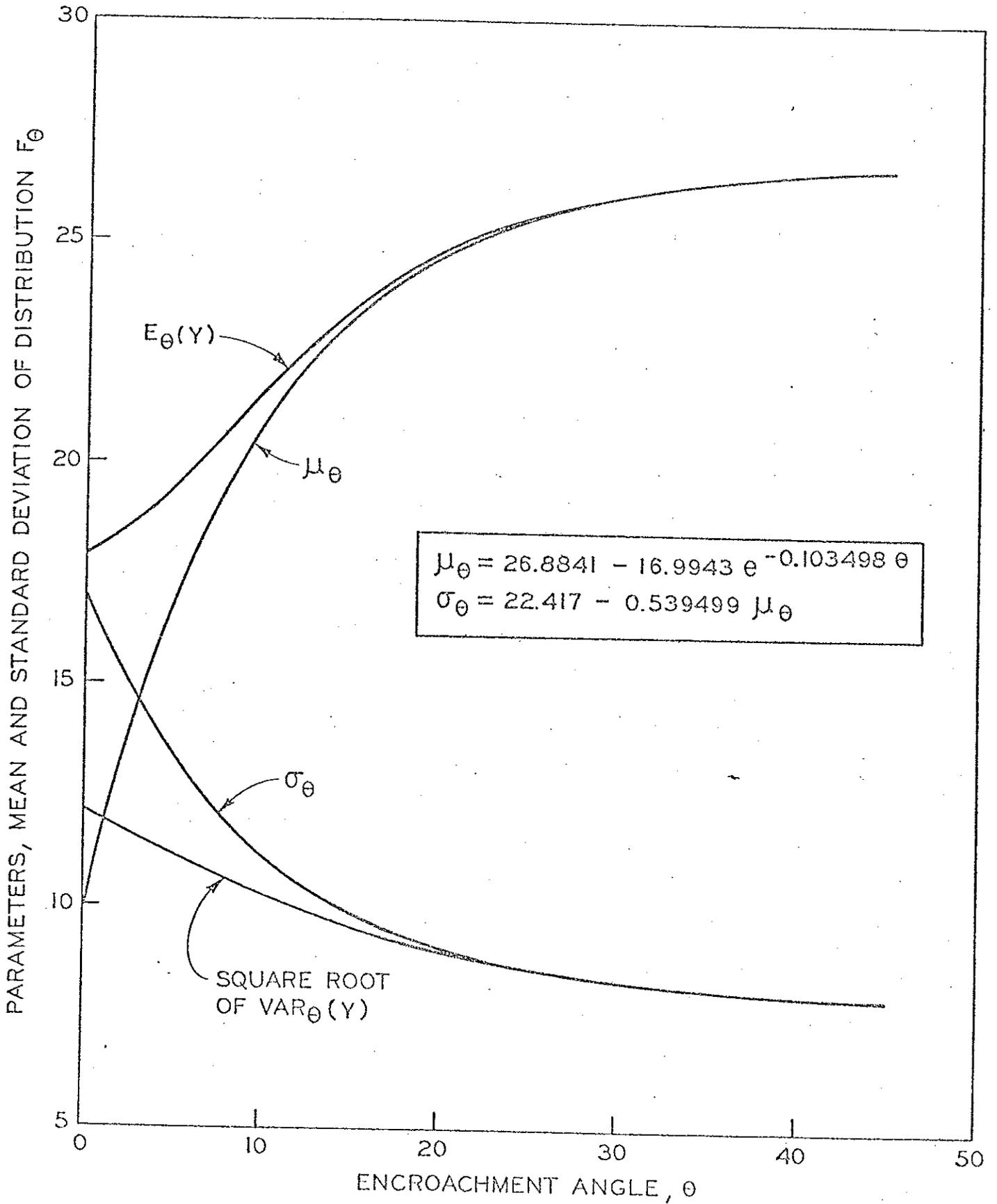


Figure 4. Parameters, mean and standard deviation of distribution  $F_\theta$ .

on the opposite roadway, the higher the probability that this vehicle will collide with another vehicle. If the traveling path of the encroaching vehicle is known, the total time  $t_j$  during which the encroaching vehicle travels on Roadway  $j$  can be determined.

As shown in Figure 5, a vehicle entering Roadway  $j$  will first intercept the traffic stream in the median lane. Then, if the path of the encroaching vehicle extends to the next lane, it will intercept the traffic stream of this lane plus the median lane, and so on, depending on  $Y$ . Thus,  $t_j$  can be broken down into time components  $t_{j,k}$ ,  $k = 1, 2, \dots$ , such that for each time component indexed by  $k$ , the encroaching vehicle conflicts with the opposing traffic stream characterized by the flow rates  $\lambda_{jk}$ . Note that  $\sum_k t_{jk} = t_j$ . The collision would not occur if no vehicle of the opposite roadway passes through the traveling path of the encroaching vehicle during time  $t_j$ . Thus, if we assume that the traffic pattern can be characterized as a Poisson process, the probability of no collision is  $\text{Exp}(-\sum_k \lambda_{jk} t_{jk})$ . Consequently, the probability of a collision can be expressed as:

$$C(M, Y, \theta) = 1 - \text{Exp}\left(-\sum_k \lambda_{jk} t_{jk}\right) \quad (17)$$

Note that  $t_{jk}$  is a function of median width, lane width, vehicle width and length, and the speed of the encroaching vehicle at various points along the encroachment path.

To evaluate this function, the path of the encroaching vehicle must be determined. This was estimated using the encroachment data presented in

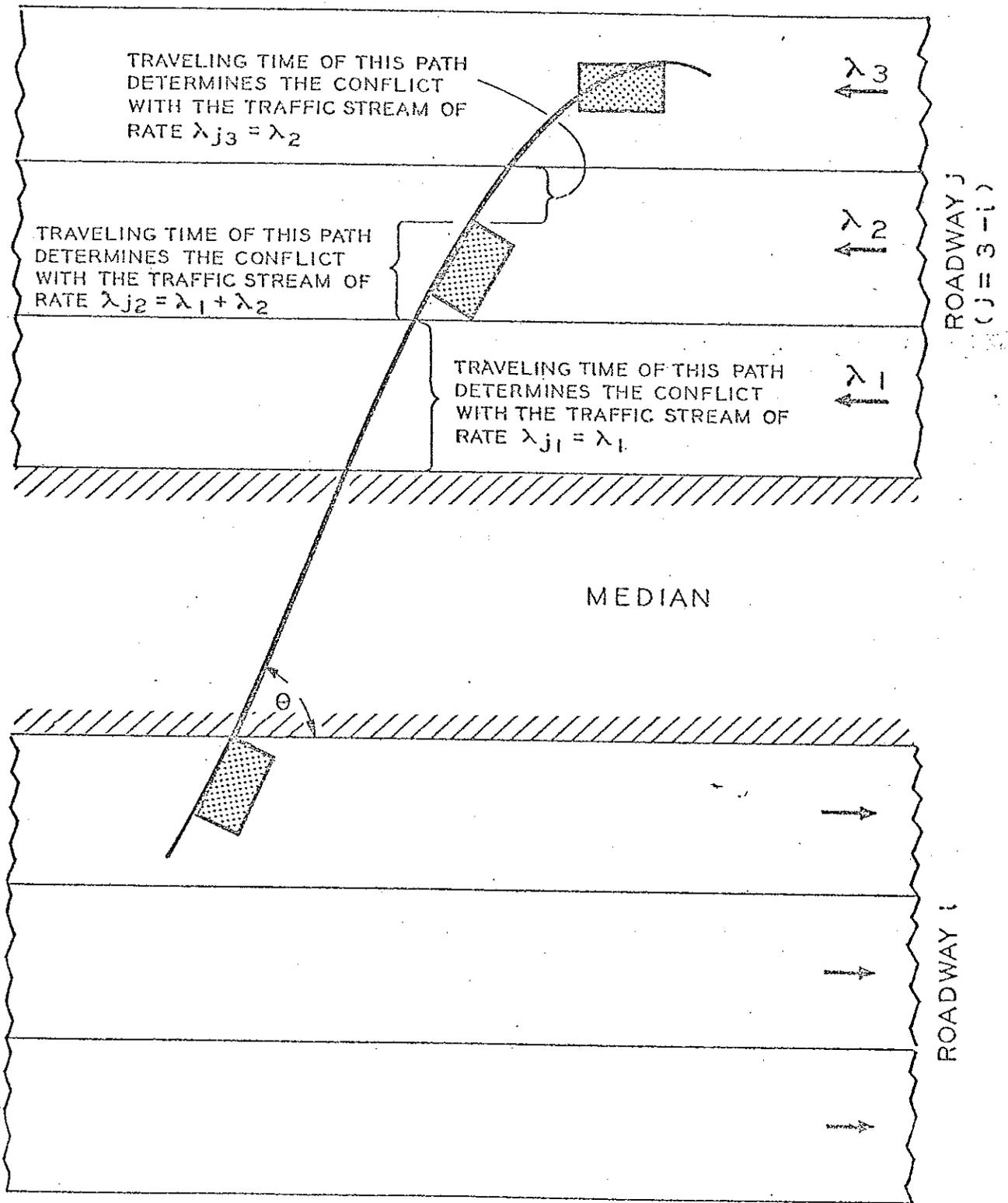


Figure 5. Relationship between the traveling path of an encroaching vehicle and the time components subject to a multi-vehicle collision. (Traveling time,  $t_j$ , of the path indicated by the bold line determines the collision probability.)

Hutchinson (2). For any encroachment angle  $\theta$ , we define

$$\Psi(\theta) = \tan^{-1}(Y/X) \quad (18)$$

Examining the plot of  $\theta$  versus  $\Psi(\theta)$  obtained from the Hutchinson data and using a non-linear least squares curve fitting technique, the relationship between the encroachment angle and  $\Psi(\theta)$  was found to be:

$$\Psi(\theta) = 1.10377 + 0.890244\theta - 0.0054231\theta^2 \quad (19)$$

This fitted equation implies that  $\Psi(\theta) \geq \theta$  for small  $\theta$  and  $\Psi(\theta) < \theta$  for large  $\theta$ . This indicates that the traveling path of a vehicle encroaching onto the median can be described by a polynomial  $T_\theta(z)$  of degree 3 satisfying the following conditions (see Fig. 2).

- a)  $T_\theta(z) = 0$  at the encroaching point (i.e.,  $z = 0$ ),
- b) Differentiation of  $T_\theta(z)$  with respect to  $z$ , evaluated at the encroaching point, is equal to  $\tan(\theta)$ ,
- c) Differentiation of  $T_\theta(z)$  with respect to  $z$ , evaluated at the recovery point, is equal to 0, and
- d)  $Y = T_\theta(X)$ .

Under these conditions, it can be shown that

$$T_\theta(z) = (\tan \theta) z + \frac{3Y - 2X(\tan \theta)}{X^2} z^2 + \frac{X(\tan \theta) - 2Y}{X^3} z^3 \quad (20)$$

This path equation can be used both to determine the probability of a collision, and the angle at which an encroaching vehicle would strike a fixed barrier. For example, for a barrier installed at  $M$  ft from the edge of Roadway  $i$ , the impact angle  $\epsilon(\theta)$  of an encroaching vehicle can be expressed as:

$$\epsilon(\theta) = \tan^{-1} \left[ \frac{dT_{\theta}(z)}{dz} \Bigg|_{\text{evaluated at } (T_{\theta}^{-1}(M_i), M_i)} \right] \quad (21)$$

#### An Example

To illustrate the use of these models, consider the following example as it might be applied to a freeway segment under design. First, assume that roadside encroachments will follow the  $G$  and  $F$  distributions observed in the Hutchinson-Kennedy study for the following roadway conditions:

- a) The median width is 29 ft,
- b) Each roadway has two 12-ft lanes,
- c) The ADT is split evenly between the two roadways,
- d) The estimated ADT is between 33,200 and 48,600.

In addition to the above conditions, we also assume the following:

- e) The width and length of the vehicle are 6 ft and 18 ft, respectively,
- f) All vehicles are traveling in the center of their respective lanes.
- g) Once two vehicles are on a collision course defined by the Poisson process, neither driver is able to take evasive action.
- h) No collision occurs at any time after the encroaching vehicle has reached the recovery point. At this point, the driver of the encroaching

vehicle is assumed to be in control and evasive action by all parties is possible.

- i) If the barrier is present, it is installed in the center of the median and is assumed to fully contain the encroaching vehicle. Thus, no barrier vaulting or breakthrough is possible.
- j) The speed of an encroaching vehicle entering the opposite roadway is 50 miles per hour.
- k) The proportions of total traffic traveling on the two lanes for various ADT are those shown in Table 2.

TABLE 2  
THE PROPORTIONS OF TOTAL TRAFFIC  
TRAVELING ON THE TRAFFIC AND PASSING  
LANES FOR VARIOUS ADT

ADT	Proportion	
	Traffic Lane	Passing Lane
33,200	0.59	0.41
38,600	0.58	0.42
48,300	0.58	0.42

With this information, we would like to know whether median guardrail is desirable for this highway segment. The decision-making process for this hypothetical problem would be as follows:

The first step is to compute  $P_B$  and  $P_H$  defined in Eqs. (2) and (4), and  $R$  defined as

$$R = \frac{P_B}{P_H} \tag{22}$$

The results for  $P_B$ ,  $P_H$ , and  $R$  are presented in Figures 6, 7, and 8,

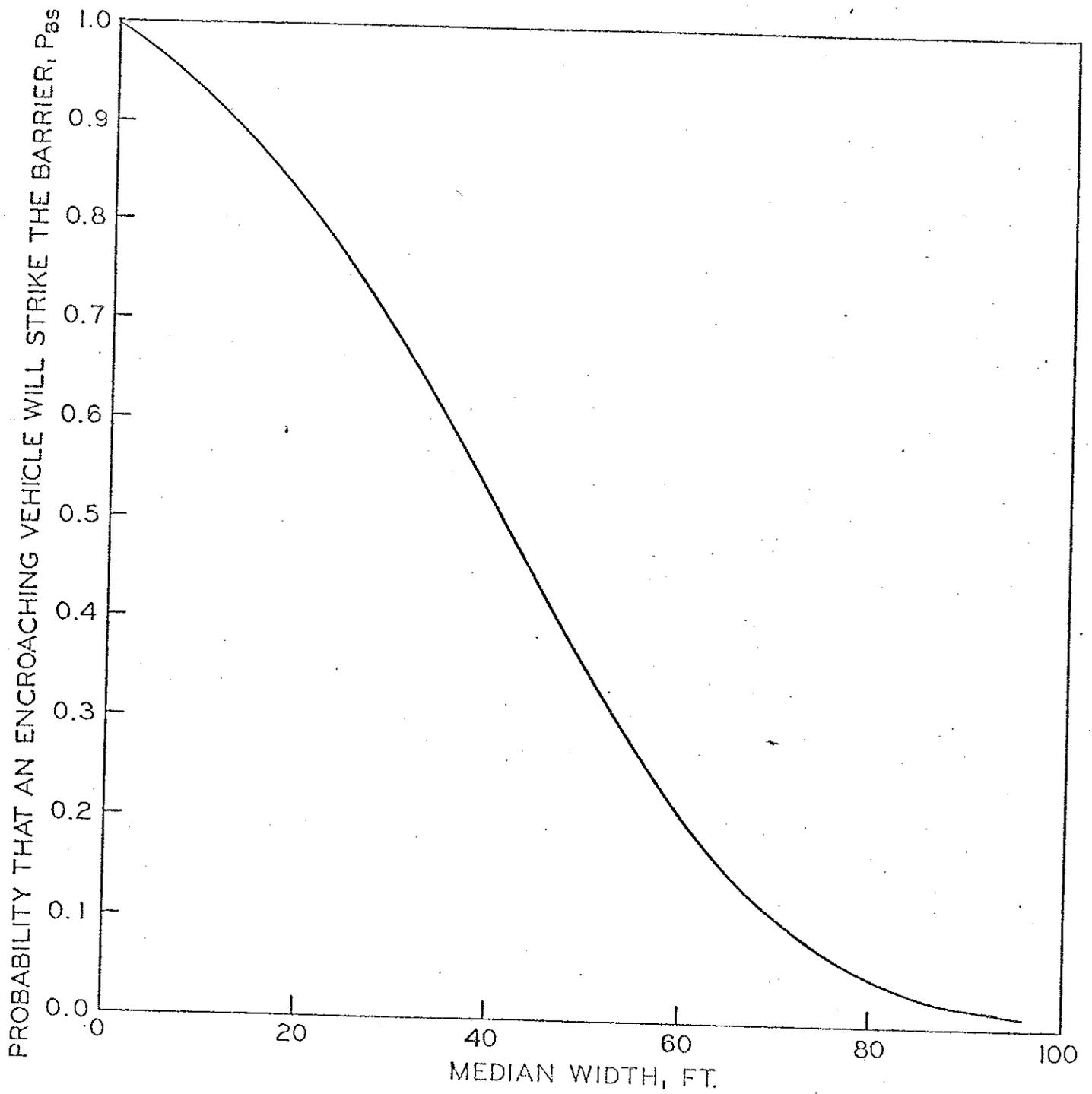


Figure 6. The probability that an encroaching vehicle will strike the median barrier.

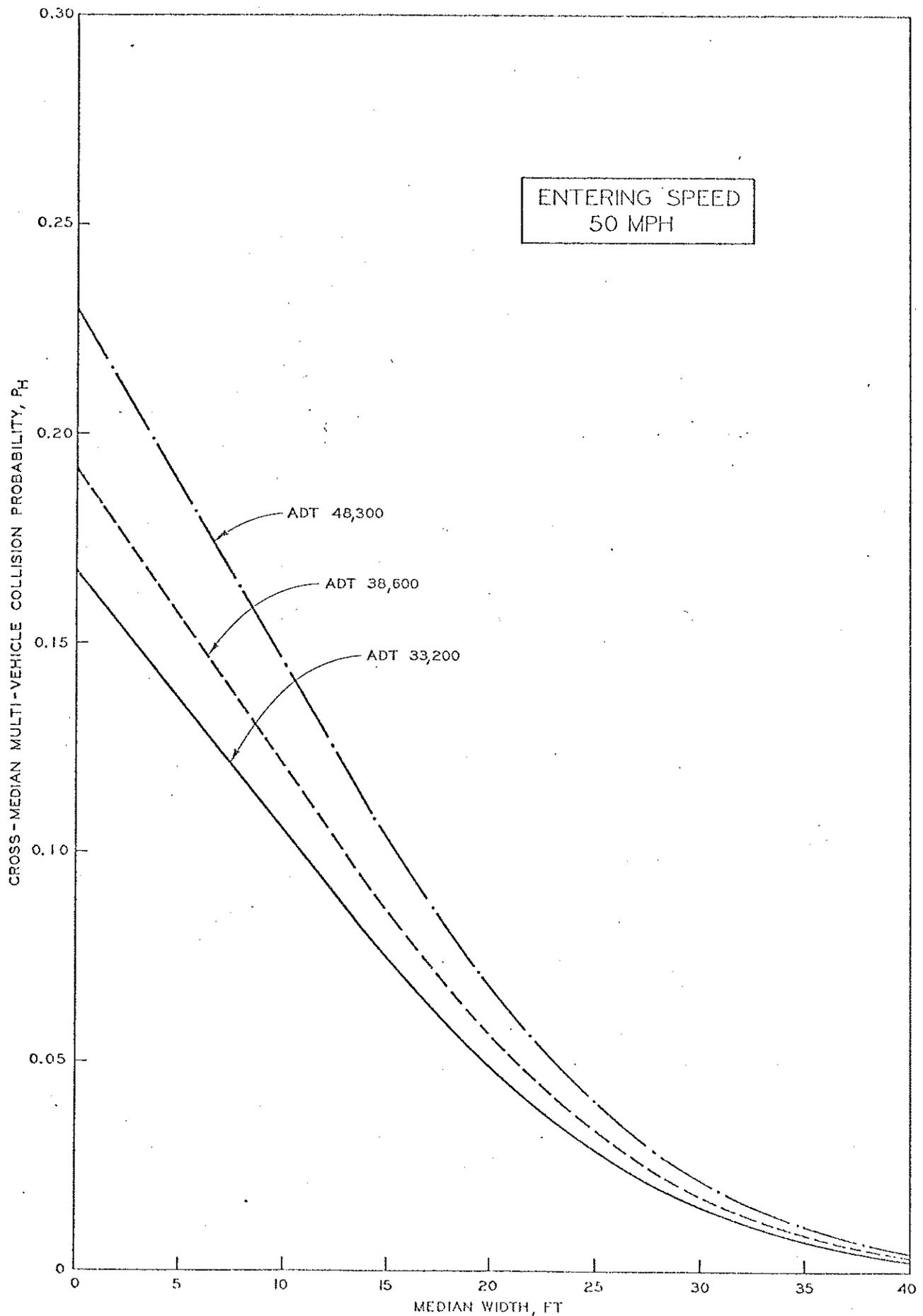


Figure 7. The probability that an encroaching vehicle will cross over a barrier-free median and collide with a vehicle traveling on the opposite roadway.

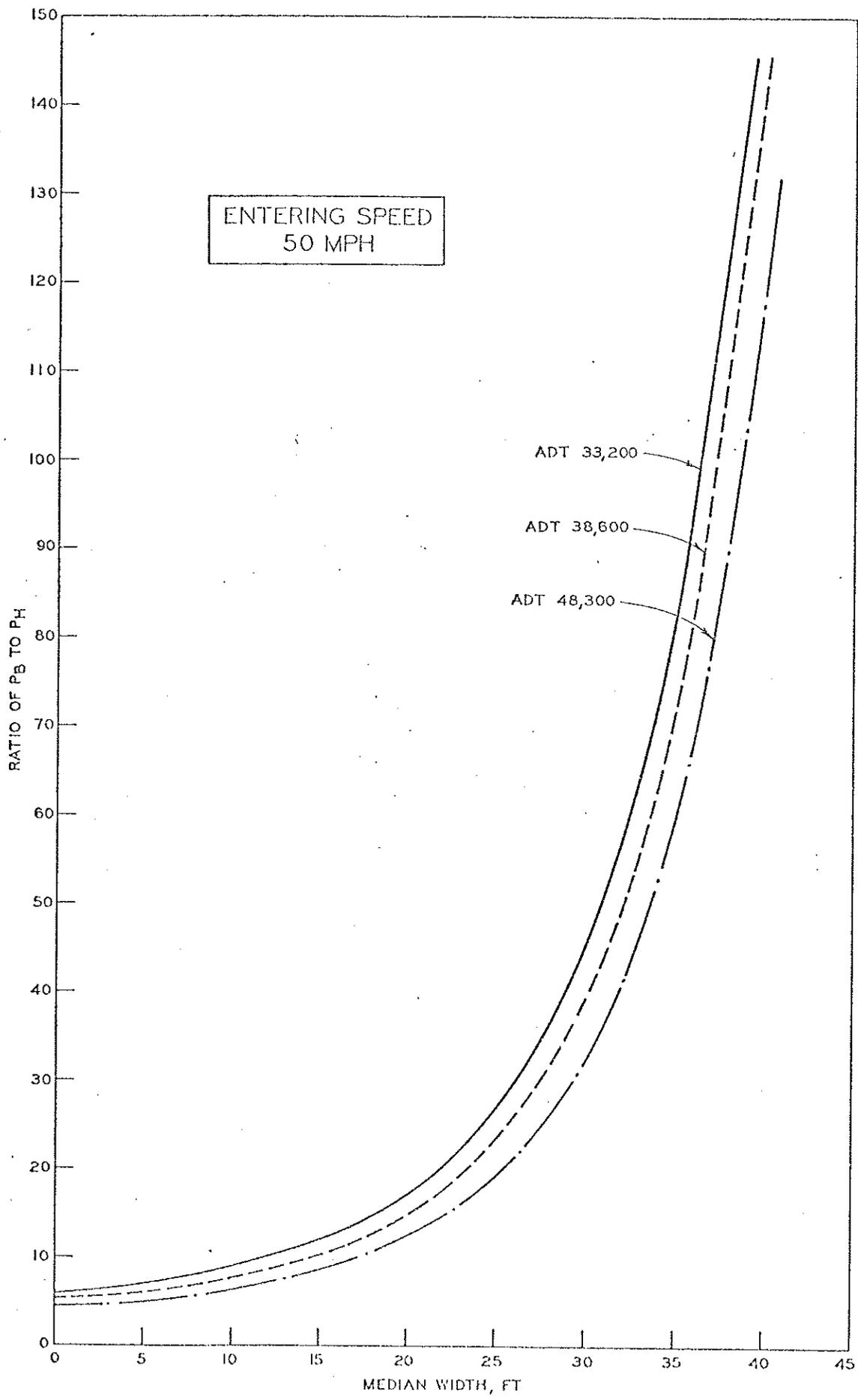


Figure 8. Ratio of  $P_B$  to  $P_H$ .

respectively. The R defined in Eq. (22) can be interpreted as the ratio of barrier accidents to cross-median multi-vehicle collisions.

Let B be the probability that a vehicle striking the guardrail will produce an injury or fatal accident. Similarly, define H to be the probability that a cross-median collision will produce an injury or fatal accident. If we denote S to be the ratio of H and B, i.e.,

$$S = \frac{H}{B} \quad (23)$$

then, S is the ratio of cross-median multi-vehicle severity to barrier accident severity. Based on various accident sources (1, 4), we can assume that  $B = 0.33$  and  $H = 1$ . Since B and H are obtained from reported accidents and the accident reporting level is a function of accident severity, the true ratio, S, is not  $1/0.33$ , but rather  $(1.0 \times r_H)/(0.33 \times r_B)$  where  $r_B$  and  $r_H$  are the reporting levels of guardrail and multi-vehicle collision accidents, respectively. By assigning maximum possible severity to cross-median collisions ( $r_H = 1$ ), and assuming that  $r_B = 0.15$  (5), the ratio S is 20.2. Thus, the median guardrail installation would be justified if R is less than or equal to 20.2. We see from Figure 8 that the R for ADT = 48,600 is 28.74 when the median width is 29 ft. Thus, we conclude that barrier-free median for this highway segment would be the better safety policy.

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