# APPENDIX C

# DESIGN CALCULATIONS FOR SPECIMEN I LABORATORY BOX BEAMS

Department of Civil and Architectural Engineering

Lawrence Technological University

Southfield, MI 48075-0134

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**Design Selection**

- Design of the side by side box beam bridge followed requirements described in the MDOT Bridge Design Manual, MDOT Bridge Design Guides and the AASHTO LRFD Bridge Design Specifications.
- The proposed span length of the bridge is 60 feet. The width of the deck that was used in design was 45 feet. This allows for one lane of traffic in each direction and two shoulders with a width of 10 feet.
- A cross section of 36"W x 27"D was selected for use on this project. The beam is dimensioned according to the MDOT Bridge Design Guides 6.65.10A. Fifteen beams were used in the design of this bridge.
- According to the MDOT Bridge Design Manual 7.01.03 and 7.02.03.A.1, a concrete strength of 5000 – 7000 psi must be used in the construction of prestressed box beams. Concrete compressive strength specified and used in this design is 7,000 psi.
- In section 7.02.18.B.2 the center to center spacing of side by side prestressed box beams is the nominal width of the beam plus 1.5 inches.
- The steel prestressing strands are Gr. 270 low relaxed strands. This has been selected according to 7.01.03 of the MDOT Bridge Design Manual. Strand diameter used in the design is 0.6" and the equivalent area is 0.217 $in^2$.
- Mild steel longitudinal and stirrups for the prestressed box beam reinforcement is required to be Gr. 60. This has been selected according to 7.01.03 of the MDOT Bridge Design Manual.
- Side by side box beams shall have a wearing course of six inches as specified in 7.02.18.B.6.a of the MDOT Bridge Design Manual and 6.29.06A of the MDOT Bridge Design Guides. This bridge was designed with a 6 inch thick reinforced concrete deck. The compressive strength of concrete in the bridge deck is specified as 4000 psi.
- As stated in 7.02.18.B.5 of the MDOT Bridge Design Manual and 6.65.13A of the MDOT Bridge Design Guides, traverse post-tensioning ducts shall be placed at mid-depth of the beam if the box beam has a depth of less than 33 inches.

- Traverse post-tensioning tendons shall be placed at one (1) at each end of the beam, one (1) at center span and one (1) at each quarter point for beams between 50 and 62 feet in length. (MDOT Bridge Design Guides 6.65.13A) Post-tensioning tendons are arranged according to this specification.
- The size of the end block and intermediate diaphragms was determined in accordance with 6.65.12, 6.65.12A and 6.65.13 of the MDOT Bridge Design Guides. Each end block must have a minimum width of 2 feet (end block design was taken as 2 feet). The intermediate diaphragms have a width of 1'-2".
- The loading placed on the bridge followed specification of section 3.6.1.2.2 and 3.6.1.2.4 of the AASHTO. This section states that a truck with two 32 kip axels and one 8 kip axel spaced at 14 feet apart shall be placed on the bridge. A uniformly distributed load of 0.64 kips per linear foot in the longitudinal direction is also used in design.
- In this design, the barrier wall placed on the bridge was assumed to be 400 pounds per linear foot.
- Shear stirrups shall project from the beams into the slab to provide composite action as specified in section 7.02.18.B.6a. The stirrups will extend 2.75 inches above the top flange of the box beams to develop a composite section with the deck slab.

**Notation:**

$a$ = depth of the equivalent rectangular stress block

$A_b$ = area of the beam

$A_o$ = area enclosed by the centerline of the element

$A_{ps}$ = area of prestressing steel

$A_{pst}$ = total area of prestressing steel (all strands included)

$A_{tr}$ = transformed area (calculated by multiply by the modular ratio)

$b_{eff}$ = lateral dimension of the effective bearing area (effective flange width)

$b_v$ = effective web width

$c$ = distance from the extreme compression fiber and the neutral axis

$d_{beam}$ = depth of the beam

$d_e$ = effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement

$deck_{thick}$ = thickness of the bridge deck

$deck_{width}$ = width of the bridge deck

$DF_M$ = distribution factor for moment on the interior girders

$DF_S$ = distribution factor for shear on the interior girders

$d_p$ = depth from the extreme compression fiber to the centroid of the tension steel

$d_{strand}$ = diameter of the prestressing steel strand

$d_v$ = effective shear depth

$e$ = eccentricity of the prestressing steel to the centroid of the cross section

$E_{cb}$ = modulus of elasticity of the beam concrete

$E_{ci}$ = modulus of elasticity of the concrete at transfer

$E_{cs}$ = modulus of elasticity of slab concrete

$e_m$ = average eccentricity of the prestressing steel at midspan

$E_p$ = modulus of elasticity of the prestressing steel tendons

$f_{cb}$ = concrete compressive strength of the beam

$f_{cpe}$ = compressive stress in concrete due to effective prestress force only

$f_{cs}$ = concrete compressive strength of the slab

$f_{pe}$ = effective stress in the prestressing steel after losses

$f_{pi}$ = stress in prestressing steel immediately prior to transfer

$f_{ps}$ = average stress in prestressing steel at time in question

$f_{pt}$ = stress in prestressing steel immediately at transfer

$f_{pu}$ = specified tensile strength of prestressing steel

$f_{py}$ = yield strength of prestressing steel

$f_r$ = modulus of rupture of the concrete

$f_{ts/cs}$ = allowable stresses in the concrete at transfer and service

$f_{ys}$ = minimum yield strength of compression reinforcment

H = average ambient humidity

$I_{B3N}$ = moment of inertia resisting superimposed dead loads

$I_{beam}$ = moment of inertia for the beam cross section

$I_N$ = moment of inertia resisting live loads

J = St. Venant torsional inertia

$M_{cr}$ = cracking Moment

$M_r$ = factored flexural resistance of a section in bending

$M_u$ = factored moment at the section (applied moment)

n = modular ratio

$N_b$ = number of beams

$N_s$ = number of strands

$P_e$ = effective prestressing force at midspan after losses

$P_t$ = prestressing force at transfer

$S$ = average spacing of mild steel reinforcement

$S_b/S_t$ = section modulus of the beam (bottom or top, respectively)

$S_{B3N}/S_{T3N}$ = section modulus of the beam resisting superimposed dead loads (bottom or top, respectively)

$S_{BN}/S_{TN}$ = section modulus of the beam resisting live loads (bottom or top, respectively)

Span = span of the beam

$V_c$ = nominal shear resistance provided by the tensile stresses in the concrete

$V_p$ = applied shear of the effective prestressing force

$V_s$ = shear resistance provided by shear reinforcement

$V_u$ = factored shear force at section (applied shear)

$y_b/y_t$ = distance from the neutral axis to the extreme tension or compression fiber

$y_{tcs}$ = distance to the extreme top fiber of the composite section

$\Delta f_{pES}$ = loss in prestressing steel due to elastic shortening

$\Delta f_{pLT}$ = long term prestress losses due to creep of concrete, shrinkage of concrete and relaxation of steel strands

$\Delta f_{pR}$ = an estimate of relaxation loss (taken as 2.4 ksi for low relaxation strands)

## Design Calculations for a Box Beam used in Laboratory NDE Testing

***Design Selections:***

$f_{cb} := 7 \cdot ksi$ Compressive Strength of Concrete for the Beam

$f_{cs} := 4 \cdot ksi$ Compressive Strength of Concrete for the Bridge Deck

$d_{strand} := 0.6 \cdot in$ Diameter of Steel Prestressing Strand

$f_{pu} := 270 \cdot ksi$ Tensile Strength of Steel Prestressing Strand

$Span := 60 \cdot ft$ $deck_{width} := 45 \cdot ft$

$deck_{thick} := 6 \cdot in$ Side by Side Box Beam Bridges use a 6" Wearing Surface

Using a Box Beam with Dimensions of 36"W x 27"D the properties are as follows:

$d_{beam} := 27 \cdot in$ $w_{beam} := 36 \cdot in$ $W_{beam} := 530 \cdot \frac{lbf}{ft}$

$A_b := 509 \cdot in^2$ $y_t := 13.43 \cdot in$ $y_b := 13.57 \cdot in$

$S_t := 3520 \cdot in^3$ $S_b := 3480 in^3$ $I_{beam} := 47300 \cdot in^4$

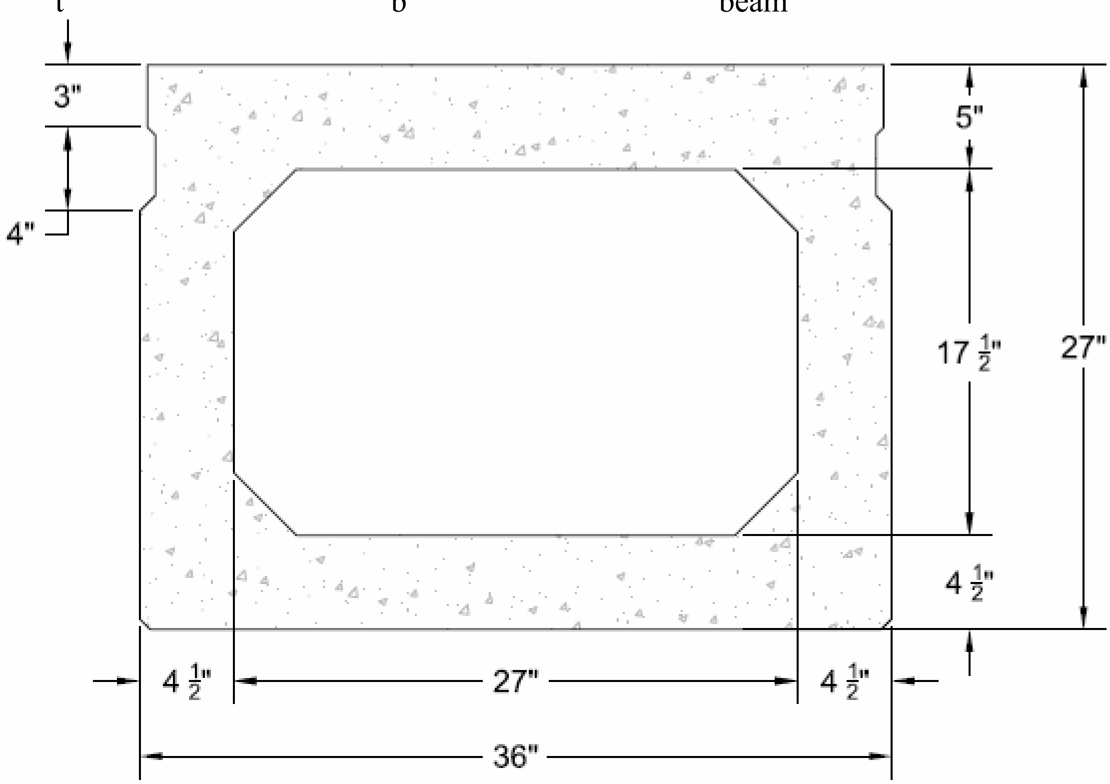


*Figure 1: 36" W x 27" D Box Beam Section*

***Check Maximum Span to Depth Ratio:***

Using Simple Span Adjacent Box Beams (AASHTO Table 2.5.2.6.3-1):

$Min_{depth} := 0.030 \cdot Span = 21.6 \cdot in$

$if\left(Min_{depth} < d_{beam} + deck_{thick}, "ok", "notok"\right) = "ok"$

***Composite Section Properties:***

For adjacent box beam, the effective flange width is equal to the width of the section.

$b_{spac} := 36 \cdot in$

$b_{eff} := min\left(b_{spac}, Span\right) = 36 \cdot in$ Section 4.6.2.6

*Modulus of Elasticity:*

Unitless values for use in elastic modulus equations

$w_c := 0.150$ $f_{cs1} := 4$ $f_{cb1} := 7$

$E_{cs} := 33000 \cdot w_c^{1.5} \cdot \sqrt{f_{cs1}} \cdot ksi = 3834.3 \cdot ksi$ Equation 5.4.2.4-1

$E_{cb} := 33000 \cdot w_c^{1.5} \cdot \sqrt{f_{cb1}} \cdot ksi = 5072.2 \cdot ksi$

*Modular Ratio:*

$$n := \frac{E_{cs}}{E_{cb}} = 0.756$$

*Slab Transformed Width:*

$s_{tr} := b_{eff} \cdot n = 27.213 \cdot in$

*Composite Section Resisting Superimposed Dead Loads:*

$k := 3$

$$A_{tr} := \frac{s_{tr} \cdot deck_{thick}}{k} = 54.427 \cdot in^2$$ Transformed area

$$I_{slab} := \frac{\frac{s_{tr}}{3} \cdot deck_{thick}^3}{12} = 163.281 \cdot in^4$$ Moment of Inertia of the slab

$$y_{slab} := d_{beam} + \frac{deck_{thick}}{2} = 30 \cdot in$$

Distance from bottom of section to the centroid of the slab

| Element | Area ($in^2$) | Y(in) | AY ($in^3$) | $AY^2$ ($in^3$) |
|---|---|---|---|---|
| Girder | $A_b = 509 \cdot in^2$ | $y_b = 13.57 \cdot in$ | $A_b \cdot y_b = 6907.1 \cdot in^3$ | $A_b \cdot {y_b}^2 = 93729.8 \cdot in^4$ |
| Slab | $A_{tr} = 54.427 \cdot in^2$ | $y_{slab} = 30 \cdot in$ | $A_{tr} \cdot y_{slab} = 1632.8 \cdot in^3$ | $A_{tr} \cdot {y_{slab}}^2 = 48984.2 \cdot in^4$ |

$$\Sigma A := A_b + A_{tr} = 563.427 \cdot in^2$$

$$\Sigma AY := A_b \cdot y_b + A_{tr} \cdot y_{slab} = 8.54 \times 10^3 \cdot in^3$$

$$\Sigma AY_2 := A_b \cdot {y_b}^2 + A_{tr} \cdot {y_{slab}}^2 = 142713.9 \cdot in^4$$

$$y_{bar3N} := \frac{\Sigma AY}{\Sigma A} = 15.157 \cdot in$$

$$I_z := I_{slab} + I_{beam} + \Sigma AY_2 = 190177.2 \cdot in^4$$

$$I_{3N} := I_z - \Sigma A \cdot {y_{bar3N}}^2 = 60736.3 \cdot in^4$$

Moment of Inertia of Composite Section

$$y_{tcs} := d_{beam} + deck_{thick} - y_{bar3N} = 17.843 \cdot in$$

Distance to Extreme Top Fiber of Composite Section

$$S_{B3N} := \frac{I_{3N}}{y_{bar3N}} = 4007.1 \cdot in^3$$

Bottom Section Modulus

$$S_{T3N} := \frac{I_{3N}}{y_{tcs}} = 3404 \cdot in^3$$

Top Section Modulus

*Composite Section Resisting Live Loads:*

$k := 1$

$A_{tr} := \dfrac{s_{tr} \cdot deck_{thick}}{k} = 163.281 \cdot in^2$ Transformed area

$I_{slab} := \dfrac{\dfrac{s_{tr}}{k} \cdot deck_{thick}^3}{12} = 489.842 \cdot in^4$ Moment of Inertia of the slab

$y_{slab} := d_{beam} + \dfrac{deck_{thick}}{2} = 30 \cdot in$ Distance from bottom of section to the centroid of the slab

| Element | Area ($in^2$) | Y(in) | AY ($in^3$) | AY² ($in^3$) |
|---|---|---|---|---|
| Girder | $A_b = 509 \cdot in^2$ | $y_b = 13.57 \cdot in$ | $A_b \cdot y_b = 6907.1 \cdot in^3$ | $A_b \cdot y_b^2 = 93729.8 \cdot in^4$ |
| Slab | $A_{tr} = 163.281 \cdot in^2$ | $y_{slab} = 30 \cdot in$ | $A_{tr} \cdot y_{slab} = 4898.4 \cdot in^3$ | $A_{tr} \cdot y_{slab}^2 = 146952.6 \cdot in^4$ |

$\Sigma A := A_b + A_{tr} = 672.281 \cdot in^2$

$\Sigma AY := A_b \cdot y_b + A_{tr} \cdot y_{slab} = 1.181 \times 10^4 \cdot in^3$

$\Sigma AY_2 := A_b \cdot y_b^2 + A_{tr} \cdot y_{slab}^2 = 240682.3 \cdot in^4$

$y_{barN} := \dfrac{\Sigma AY}{\Sigma A} = 17.56 \cdot in$

$I_z := I_{slab} + I_{beam} + \Sigma AY_2 = 288472.2 \cdot in^4$

$I_N := I_z - \Sigma A \cdot y_{barN}^2 = 81161.4 \cdot in^4$ Moment of Inertia of Composite Section

$y_{tcs} := d_{beam} + deck_{thick} - y_{barN} = 15.44 \cdot in$ Distance to Extreme Top Fiber of Composite Section

$S_{BN} := \dfrac{I_N}{y_{barN}} = 4621.8 \cdot in^3$ Bottom Section Modulus

$S_{TN} := \dfrac{I_N}{y_{tcs}} = 5256.7 \cdot in^3$ Top Section Modulus

***Determine Distribution Factors***

In order to determine the equations for the load distribution factors a common deck superstructure must be chosen from Table 4.6.2.2.1-1. For this design the typical cross section f was chosen: precast solid, voided or cellular concrete boxes with shear keys and traverse post-tensioning. The type of deck is cast in place overlay.

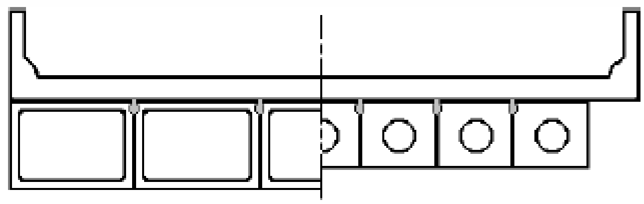

*Figure 2: Typical Deck Cross-Section F From Table 4.6.2.2.1-1*

*Determine the number of beams in the cross section:*

$deck_{width} = 45 \cdot ft$ $\quad\quad\quad\quad$ $w_{beam} = 36 \cdot in$

$$N_b := \frac{deck_{width}}{w_{beam}} = 15 \quad \text{beams}$$

*Live Load Distribution Factors for Moment in an Interior Girder*

*Note: Exterior girders were not considered in this design. They are not required for the scope of the Non-Destructive evaluation covered in this project.*

Using table 4.6.2.2.2b-1, determine if design criteria meets range of applicability requireme

$$\text{if}\left(35 \cdot in \le w_{beam} \le 60 \cdot in, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}(20 \cdot ft \le Span \le 120 \cdot ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$$

$$\text{if}\left(5 \le N_b \le 20, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$N := \frac{E_{cb}}{E_{cs}} = 1.323$$

Equation 4.6.2.2.1-2

$$k := \max\left[2.5 \cdot \left(N_b\right)^{-0.2}, 1.5\right] = 1.5$$

Equation in Table 4.6.2.2.2b-1 ( Under section, Concrete Beams used in Multi beam Decks as Type of Superstructure)

Determine $A_o$ which is defined as the area enclosed by the centerlines of the elements.

$$A_o := \left(w_{beam} - 4.5 \cdot in\right) \cdot \left(d_{beam} - \frac{4.5 \cdot in}{2} - \frac{5 \cdot in}{2}\right) = 700.875 \cdot in^2$$

$$J = \frac{4 \cdot A_o^{\,2}}{\Sigma \cdot \frac{s}{t}}$$

Equation C4.6.2.2.1-3

s = length of a side element (in) $s := 27 \cdot in$

t = thickness of plate-like element (in) $t := 4.5 \cdot in$

$$J := \frac{4 \cdot A_o^{\,2}}{2 \cdot \left(\frac{s}{t}\right)} = 163741.9 \cdot in^4$$

*One Lane Loaded (Live Load Distribution Factors for Moment):*

The equation for live load moment distribution factors is found in Table 4.6.2.2.2b-1.

$$DF_{M1Lane} = k \cdot \left(\frac{w_{beam}}{33.3 \cdot Span}\right)^{0.5} \cdot \left(\frac{I_{beam}}{J}\right)^{0.25}$$

For this equation given in AASHTO, the constants are developed around specific units (s
in feet and beam width in inches). Since Mathcad incorporates units, the equation below
been simplified to take out the effect of units and obtain the correct live load factors.

$$DF_{M1Lane} := k \cdot \left(\frac{w_{beam}}{2.75 \cdot Span}\right)^{0.5} \cdot \left(\frac{I_{beam}}{J}\right)^{0.25} = 0.148$$

*Two Lane or More Lanes Loaded (Live Load Distribution Factors for Moment):*

The equation for live load moment distribution factors is found in Table 4.6.2.2.2b-1

$$DF_{M2Lane} = k \cdot \left(\frac{w_{beam}}{305}\right)^{0.6} \cdot \left(\frac{w_{beam}}{12 \cdot Span}\right)^{0.2} \cdot \left(\frac{I_{beam}}{J}\right)^{0.06}$$

For this equation given in AASHTO, the constants are developed around specific units (sp
in feet and beam width in inches). Since Mathcad incorporates units, the equation below h
been simplified to take out the effect of units and obtain the correct live load factors.

$$DF_{M2Lane} := k \cdot \left(\frac{w_{beam}}{305 \cdot in}\right)^{0.6} \cdot \left(\frac{w_{beam}}{Span}\right)^{0.2} \cdot \left(\frac{I_{beam}}{J}\right)^{0.06} = 0.212$$

*Live Load Distribution Factors for Shear in an Interior Girder:*

Using table 4.6.2.2.3a-1, determine if design criteria meets range of applicability requiren

$$\text{if}\left(35\cdot in \le w_{beam} \le 60\cdot in, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(20\cdot ft \le Span \le 120\cdot ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(5 \le N_b \le 20, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(25000\cdot in^4 \le J \le 610000\cdot in^4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(40000\cdot in^4 \le I_{beam} \le 610000\cdot in^4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

*One Lane Loaded (Live Load Distribution Factor for Shear):*

The equation for live load shear distribution factors is found in Table 4.6.2.2.3a-1.

$$DF_{S1Lane} = \left(\frac{w_{beam}}{130\cdot Span}\right)^{0.15}\cdot\left(\frac{I_{beam}}{J}\right)^{0.05}$$

For this equation given in AASHTO, the constants are developed around specific units (s in feet and beam width in inches). Since Mathcad incorporates units, the equation below been simplified to take out the effect of units and obtain the correct live load factors.

$$DF_{S1Lane} := \left(\frac{w_{beam}}{10.833\cdot Span}\right)^{0.15}\cdot\left(\frac{I_{beam}}{J}\right)^{0.05} = 0.419$$

*Two Lane or More Lanes Loaded (Live Load Distribution Factors for Moment):*

$$DF_{S2Lane} = \left(\frac{w_{beam}}{156}\right)^{0.4}\cdot\left(\frac{w_{beam}}{12\cdot Span}\right)^{0.1}\cdot\left(\frac{I_{beam}}{J}\right)^{0.05}\cdot\left(\frac{w_{beam}}{48}\right)$$

$$\frac{w_{beam}}{48} \ge 1.0 \qquad \frac{w_{beam}}{48\cdot in} = 0.75$$

Since $w_{beam}/48$ is less than 1.0, use 1.0

For this equation given in AASHTO, the constants are developed around specific units (s in feet and beam width in inches). Since Mathcad incorporates units, the equation below been simplified to take out the effect of units and obtain the correct live load factors.

$$DF_{S2Lane} := \left(\frac{w_{beam}}{156\cdot in}\right)^{0.4}\cdot\left(\frac{w_{beam}}{Span}\right)^{0.1}\cdot\left(\frac{I_{beam}}{J}\right)^{0.05} = 0.387$$

*Live Load Distribution Factors Summary*

Moment Factors: $DF_{M1Lane} = 0.148$ $DF_{M2Lane} = 0.212$

$$DF_M := \max\left(DF_{M1Lane}, DF_{M2Lane}\right) = 0.212$$

Shear Factors: $DF_{S1Lane} = 0.419$ $DF_{S2Lane} = 0.387$

$$DF_S := \max\left(DF_{S1Lane}, DF_{S2Lane}\right) = 0.419$$

***Dead Load Moments on the Girder***

Moment due to the Self Weight of the Beam:

$$beam := \frac{W_{beam} \cdot Span^2}{8} = 238.5 \cdot ft \cdot kip$$

Moment due to the Weight of the Slab:

$$W_{slab} := 150 \cdot \frac{lbf}{ft^3}$$

$$slab := \frac{W_{slab} \cdot \left(w_{beam} \cdot deck_{thick}\right) \cdot Span^2}{8} = 101.25 \cdot kip \cdot ft$$

Load due to the Weight of the Wearing Surface:

$$W_{ws} := 0.025 \cdot \frac{kip}{ft^2}$$

$$WS := \frac{deck_{width} \cdot W_{ws}}{N_b} = 0.075 \cdot \frac{kip}{ft}$$

Load due to the Weight of Barrier Wall:

$$W_{barrier} := 400 \cdot \frac{lbf}{ft}$$

$$barrier := \frac{W_{barrier} \cdot 2}{N_b} = 0.053 \cdot \frac{kip}{ft}$$

Non-composite Dead Load Moments:

$$M_{DC1} := beam + slab = 339.75 \cdot kip \cdot ft$$

Composite Dead Load Moments:

$$M_{DC2} := \frac{barrier \cdot Span^2}{8} = 24 \cdot kip \cdot ft$$

Moment due to the Wearing Surface Load:

$$M_{DW} := \frac{WS \cdot Span^2}{2} = 135 \cdot kip \cdot ft$$

Moment due to Dead Load of Structural Components and Nonstructural attachments:

$$M_{DC} := M_{DC1} + M_{DC2} = 363.75 \cdot kip \cdot ft$$

***Live Load Moments on the Girders:***

Using the AASHTO Design truck from section 3.6.1.2.2 the truck has two 32 kip axles one 8 kip axle spaced at 14 feet apart. This truck is placed at midspan of the beam to determine the live load moment.

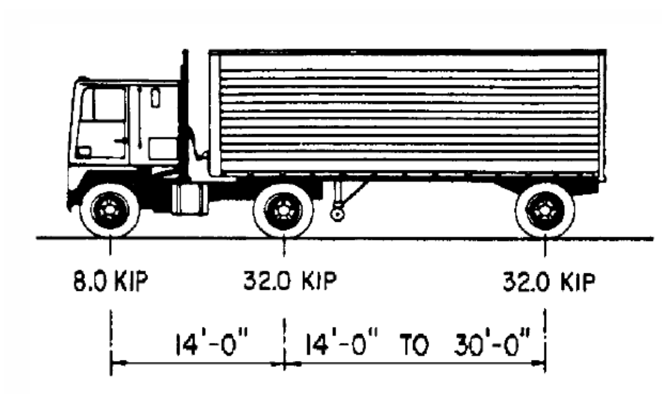


*Figure 3: AASHTO Design Truck*

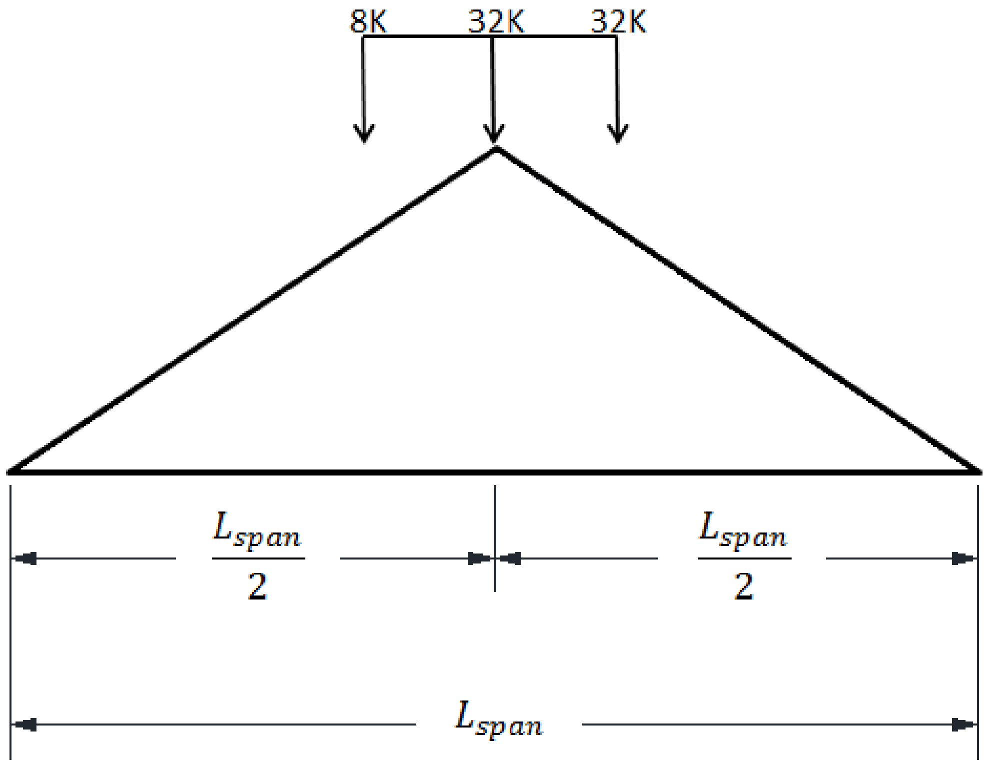


*Figure 4: Influence Line for Maximum moment at Midspan*

Design Lane Load Section 3.6.1.2.4:

This section states that the design lane load shall consist of a load of 0.64klf uniforn distributed in the longitude direction.

Dynamic Load Allowance Factors come from Table 3.6.2.1-1:

Fatigue and Fracture Limit State: $IM_{fatigue} := 1.15$

All other limit states: $IM := 1.33$

$$M_{DT1} := IM \cdot \left[ 8 \cdot kip \cdot \left( \frac{\frac{Span}{2} - 14 \cdot ft}{2} \right) + 32 \cdot kip \cdot \left( \frac{Span}{4} \right) + 32 \cdot kip \cdot \left( \frac{\frac{Span}{2} - 14 \cdot ft}{2} \right) \right]$$

$$M_{DT2} := \frac{0.64 \cdot \frac{kip}{ft} \cdot Span^2}{8}$$

$$M_{DT} := M_{DT1} + M_{DT2} = 1352 \, kip \cdot ft$$

Live Load Moment when distribution factors are taken into account.

$$M_{LLI} := M_{DT} \cdot DF_M = 286.884 \cdot kip \cdot ft$$

***Load Combinations for Moment:***

All load combinations are obtained from Section 3.4.1, specifically Table 3.4.1-1 and Table 3.4

Strength I: Basic load combination relating the normal vehicular use of the bridge without

$$LC_{ST1} := 1.25 \cdot M_{DC} + 1.50 \cdot M_{DW} + 1.75 \cdot M_{LLI} = 1159.2 \cdot kip \cdot ft$$

Strength IV: Load combination relating to very high dead load to live load force effect rati

$$LC_{ST4} := 1.25 \cdot M_{DC} + 1.5 \cdot M_{DW} = 657.2 \cdot kip \cdot ft$$

Service III: Load combination for longitudinal analysis relating to tension in prestressed concrete superstructures.

$$LC_{SV1} := 1.0 \cdot M_{DC} + 1.0 \cdot M_{DW} + 0.8 \cdot M_{LLI} = 728.3 \cdot kip \cdot ft$$

The governing load combination is Strength I with a design moment of 1159.2 kip-ft.

$$M_u := \max\left(LC_{ST1}, LC_{ST4}, LC_{SV1}\right) = 1159.2 \cdot kip \cdot ft$$

***Dead Load Shears on Girders***

Load due to Self Weight of the Beam:

$$beam := W_{beam} = 0.53 \cdot \frac{kip}{ft}$$

Load due to the Weight of the Slab:

$$W_{slab} := 150 \cdot \frac{lbf}{ft^3}$$

$$slab := W_{slab} \cdot \left(w_{beam} \cdot deck_{thick}\right) = 0.225 \cdot \frac{kip}{ft}$$

Load due to the Wearing Surface:

$$W_{ws} := 0.025 \cdot \frac{kip}{ft^2}$$

$$WS := \frac{deck_{width} \cdot W_{ws}}{N_b} = 0.075 \cdot \frac{kip}{ft}$$

Load due to the Weight of Barrier Wall:

$$W_{barrier} := 400 \cdot \frac{lbf}{ft}$$

$$barrier := \frac{W_{barrier} \cdot 2}{N_b} = 0.053 \cdot \frac{kip}{ft}$$

Non-composite Dead Loads:

$$DC1 := beam + slab = 0.755 \cdot \frac{kip}{ft}$$

Composite Dead Loads:

$$DC2 := barrier = 0.053 \cdot \frac{kip}{ft}$$

Wearing Surface Load:

$$DW := WS = 0.075 \cdot \frac{kip}{ft}$$

Shears due to Dead Load of Structural Components and Nonstructural attachments:

$$V_{DC} := \frac{(DC1 + DC2) \cdot Span}{2} = 24.25 \cdot kip$$

Shears due to Dead Load of the Wearing Surface:

$$V_{DW} := \frac{DW \cdot Span}{2} = 2.25 \cdot kip$$

***Live Load Shears on the Girders:***

Using the AASHTO Design truck from section 3.6.1.2.2 the truck has two 32 kip axles one 8 kip axle spaced at 14 feet apart. This truck is placed with the heavy axle at the en the beam to determine the live load shear for the girder. A lane load of 0.64 kips/ft is applied to the entire beam per 3.6.1.2.4. The distribution of loads on the girder is showı below in Figure 5.

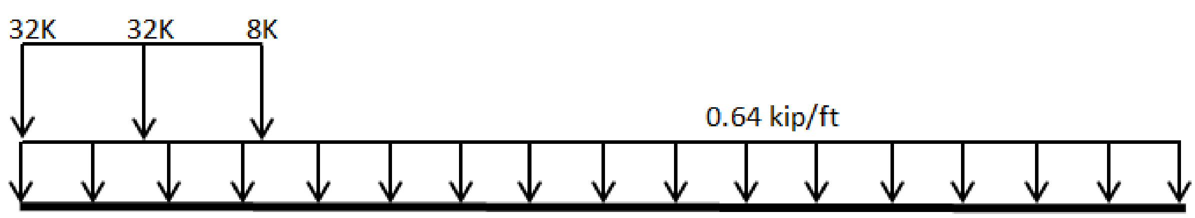


*Figure 5: Distribution of Loads to Determine Shear*

Design Lane Load Section 3.6.1.2.4:

This section states that the design lane load shall consist of a load of 0.64klf unifo distributed in the longitude direction.

Dynamic Load Allowance Factors come from Table 3.6.2.1-1:

Fatigue and Fracture Limit State: $IM_{fatigue} := 1.15$

All other limit states: $IM := 1.33$

$$V_{DT} := IM \cdot \left[ 32 \cdot kip + 32 \cdot kip \cdot \left( \frac{Span - 14 \cdot ft}{Span} \right) + 8 \cdot kip \cdot \left( \frac{Span - 28 \cdot ft}{Span} \right) \right] + \frac{0.64 \cdot \frac{kip}{ft} \cdot Span}{2}$$

$$V_{DT} = 100.064 \cdot kip$$

Live Load Moment when distribution factors are taken into account.

$$V_{LLI} := V_{DT} \cdot DF_S = 41.971 \cdot kip$$

***Load Combinations for Shear:***

All load combinations are obtained from Section 3.4.1, specifically Table 3.4.1-1 and Table 3.

Strength I: Basic load combination relating the normal vehicular use of the bridge without

$$LC_{ST1} := 1.25 \cdot V_{DC} + 1.50 \cdot V_{DW} + 1.75 \cdot V_{LLI} = 107.1 \cdot kip$$

Strength IV: Load combination relating to very high dead load to live load force effect rati

$$LC_{ST4} := 1.25 \cdot V_{DC} + 1.5 \cdot V_{DW} = 33.7 \cdot kip$$

Service III: Load combination for longitudinal analysis relating to tension in prestressed concrete superstructures.

$$LC_{SV1} := 1.0 \cdot V_{DC} + 1.0 \cdot V_{DW} + 0.8 \cdot V_{LLI} = 60.1 \cdot kip$$

The governing load combination is Strength I with a design shear of 107.1 kip.

$$V_u := \max\left(LC_{ST1}, LC_{ST4}, LC_{SV1}\right) = 107.1 \cdot kip$$

### *Prestressing Steel*

Check the theoretical number of strands using the Service III Load Combination:

For this process, the ECE 5783 Lecture 5: Prestressed Concrete Superstructure Desigr developed by Matt Chynoweth was followed.

$$f_b := -\left(\frac{M_{DC1}}{S_b} + \frac{M_{DC2} + M_{DW}}{S_{BN}} + \frac{0.8 \cdot M_{LLI}}{S_{BN}}\right) = -2.18 \cdot ksi$$

Tensile stress limit at service after losses:

Limit found from Table 5.9.4.2.2-1. It is considered "Other than Segmentally Construc in an area that is not worse than moderate corrosion conditions for bonded prestressing tendons.

$$T_{limit} := -\left(0.19 \cdot \sqrt{\frac{f_{cb}}{ksi}}\right) \cdot ksi = -0.503 \cdot ksi$$

Excess Tension in Bottom Fiber due to applied loads:

$$f_p := f_b - T_{limit} = -1.678 \cdot ksi$$

Assume a center of gravity location to be between 5% and 15% of beam depth. In this c 8% has been assumed in the process of determining the number of strands.

$y_{bs} := 0.08 \cdot d_{beam} = 2.16 \cdot in$ from the bottom fiber of the beam

Determine the strand eccentricity:

$y_b = 13.57 \cdot in$ Beam Center of Gravity

$e := y_b - y_{bs} = 11.41 \cdot in$ Strand Eccentricity

Determine $P_e$ which is the effective final prestress force after all losses.

$$F_p = \frac{P_e}{A_b} + \frac{P_e \cdot e}{S_b}$$

$$P_e := -\left(\frac{A_b \cdot f_p \cdot S_b}{S_b + A_b \cdot e}\right) = 319.941 \cdot kip$$

Stress Limit Prior to Transfer:

This limit is found in Table 5.9.3-1. It is considered pretensioning low relaxed tendo immediately prior to transfer.

$$f_{pi} := 0.75 \cdot f_{pu} = 202.5 \cdot ksi$$

Assume 25% final prestressing losses per 0.6" diameter strand

$A_{psc} := 0.20 \cdot in^2$ Conservative value used when determining the number of strands

$A_{ps} := 0.217 \cdot in^2$ MDOT Standard Specifications Section 905.07

$f_{pe} := A_{psc} \cdot f_{pi} \cdot (1 - 0.25) = 30.375 \cdot kip$

Number of strands: $N_s := \dfrac{P_e}{f_{pe}} = 10.533$ Use 12-0.6" Diameter Strands

***Prestress Losses***

*Instantaneous Losses - Elastic Shortening*

$\Delta f_{pES} = \dfrac{E_p}{E_{ct}} \cdot f_{cgp}$ AASHTO Equation 5.9.5.2.3

$N_s := 12$

$A_{pst} := N_s \cdot A_{ps} = 2.604 \cdot in^2$ Total Area of Prestressing Steel

$f_{pi} := 0.75 \cdot f_{pu} = 202.5 \cdot ksi$ Stress in Prestressing Steel prior to Transfer

$e_m := y_b - 2 \cdot in = 11.57 \cdot in$ Prestressing Steel Eccentricity at Midspan

$E_{ci} := E_{cb} = 5072.2 \cdot ksi$ Concrete Elastic Modulus at Transfer

$E_p := 28500 \cdot ksi$ Elastic Modulus of Prestressing Steel Tendon

$$M_{beam} := \frac{W_{beam} \cdot Span^2}{8} = 2862 \cdot kip \cdot in$$

$$\Delta f_{pES} := \frac{A_{pst} \cdot f_{pi} \cdot \left(I_{beam} + e_m^{\,2} \cdot A_b\right) - e_m \cdot M_{beam} \cdot A_b}{A_{pst} \cdot \left(I_{beam} + e_m^{\,2} \cdot A_b\right) + \dfrac{A_b \cdot I_{beam} \cdot E_{ci}}{E_p}}$$ AASHTO Equation C5.9.5.2.3a-1

$\Delta f_{pES} = 9.599 \cdot ksi$

*Prestress Force at Transfer*

$f_{pt} := f_{pi} - \Delta f_{pES} = 192.901 \cdot ksi$

$P_t := N_s \cdot A_{ps} \cdot f_{pt} = 502.3 \cdot kip$

*Time Dependant Losses*

$H := 75$ AASHTO Figure 5.4.2.3.3-1

$\gamma_h := 1.7 - 0.01 \cdot H$ AASHTO Equation 5.9.5.3-2

$\gamma_h = 0.95$

$\gamma_{st} := \dfrac{5}{1 + \dfrac{f_{cb}}{ksi}} = 0.625$ AASHTO Equation 5.9.5.3-3

$\Delta f_{pR} := 2.4$

$\Delta f_{pLT} := 10 \cdot \dfrac{f_{pi} \cdot A_{pst}}{A_b} \gamma_h \cdot \gamma_{st} + 12 \cdot ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} \cdot ksi$ AASHTO Equation 5.9.5.3-1

$\Delta f_{pLT} = 15.676 \cdot ksi$

*Effective Prestressing Force at Midspan*

$f_{pe} := f_{pi} - \Delta f_{pLT} - \Delta f_{pES} = 177.225 \cdot ksi$

$P_e := N_s \cdot A_{ps} \cdot f_{pe} = 461.493 \cdot kip$

*Determine $f'_{ci}$*

$f'_{ci}$ is defined as the specified compressive strength of the concrete at time of transfer and is assumed to be $0.8f'_c$

$f_{ci} := 0.8 \cdot f_{cb} = 5.6 \cdot ksi$

$f_{ci1} := 0.8 \cdot \dfrac{f_{cb}}{ksi} = 5.6$ just a unitless value for ease of calculation

*Allowable Stresses for Concrete*

Initial allowable tensile stress is found in Table 5.9.4.1.2-1. This beam is considered other than segmentally constructed in areas with bonded reinforcement sufficient to resist the tensile force in the concrete computed assuming an uncracked section.

$f_{tsi} := 0.24 \cdot \sqrt{f_{ci1}} \cdot ksi = 0.568 \cdot ksi$

Initial allowable compressive stress is found in Section 5.9.4.1.1. It states that the compressive stress limit for pretensioned concrete components is $0.60f'_{ci}$

$f_{csi} := -0.6 \cdot f_{ci} = -3.36 \cdot ksi$

Final allowable tensile stress is found in Table 5.9.4.2.2-1. The bridge type is other t segmentally constructed bridges. The location is assumed to be for components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions.

$$f_{tsf} := 0.19 \cdot \sqrt{f_{ci1}} \cdot ksi = 0.45 \cdot ksi$$

Final allowable compressive stress is found in Table 5.9.4.2.1-1. The location is assumed as other than segmentally constructed bridges due to the sum of effective prestress and permanent loads.

$$f_{csf} := 0.45 \cdot f_{ci} = 2.52 \cdot ksi$$

*Initial Stresses at the End of the Beam*

Stress in the Top of the Beam at Beam End

(for checks made at the end of the beam, the moment is zero)

$M_{Bend} := 0 \cdot kip \cdot ft$ $\qquad$ $e := e_m = 11.57 \cdot in$

$$f_{tiend} := \frac{-P_t}{A_b} + \frac{P_t \cdot e}{S_t} - \frac{M_{Bend}}{S_t} = 0.664 \cdot ksi$$

$$\text{if}\left(f_{tiend} < f_{tsi}, \text{"ok"}, \text{"not ok"}\right) = \text{"not ok"}$$

Try debonding 2 strands to lower stress at the end of the beam.

$$N_{debond} := 2$$

$N_{send} := N_s - N_{debond} = 10$ $\qquad$ Number of bonded strands at end of beam

$$A_{ps} = 0.217 \cdot in^2$$

$$P_{t2} := N_{send} \cdot A_{ps} \cdot f_{pt} = 418.595 \cdot kip$$

$$f_{tiend2} := \frac{-P_{t2}}{A_b} + \frac{P_{t2} \cdot e}{S_t} - \frac{M_{Bend}}{S_t} = 0.554 \cdot ksi$$

$$\text{if}\left(f_{tiend2} < f_{tsi}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

Stresses in the Bottom of the Beam at Beam End

$$f_{ciend} := -\frac{P_t}{A_b} - \frac{P_t \cdot e}{S_b} + \frac{M_{Bend}}{S_b} = -2.657\,ksi$$

$$\text{if}\left(\left|f_{ciend}\right| < \left|f_{csi}\right|, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

Check to Ensure Stresses are okay at Bottom of Beam when Two Strands are Debonded

$$f_{ciend2} := -\frac{P_{t2}}{A_b} - \frac{P_{t2} \cdot e}{S_b} + \frac{M_{Bend}}{S_b}$$

$$f_{ciend2} = -2.214 \cdot \text{ksi}$$

$$\text{if}\left(\left|f_{ciend2}\right| < \left|f_{csi}\right|, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

*Initial Stresses at the Midspan of the Beam*

Stresses in top of the Beam at Midspan

$$M_{beam} = 238.5 \cdot \text{kip} \cdot \text{ft}$$

$$f_{timid} := \frac{-P_t}{A_b} + \frac{P_t \cdot e}{S_t} - \frac{M_{beam}}{S_t}$$

$$f_{timid} = -0.149 \cdot \text{ksi}$$

$$\text{if}\left(f_{timid} < f_{tsi}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

Stresses in Bottom of the Beam at Midspan

$$f_{cimid} := \frac{-P_t}{A_b} - \frac{P_t \cdot e}{S_b} + \frac{M_{beam}}{S_b}$$

$$f_{cimid} = -1.834 \cdot \text{ksi}$$

$$\text{if}\left(\left|f_{cimid}\right| < \left|f_{csi}\right|, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

*Final Stresses at the Midspan of the Beam*

$M_{DC1} = 339.75 \cdot \text{kip} \cdot \text{ft}$ $\qquad$ $M_{DW} = 135 \cdot \text{kip} \cdot \text{ft}$

$M_{DC2} = 24 \cdot \text{kip} \cdot \text{ft}$ $\qquad$ $M_{LLI} = 286.884 \cdot \text{kip} \cdot \text{ft}$

Stresses in the Top of the Beam at Midspan

$$f_{cfmid} := \frac{-P_e}{A_b} + \frac{P_e \cdot e}{S_t} - \frac{M_{DC1}}{S_t} - \frac{M_{DC2} + M_{DW}}{S_{B3N}} - \frac{M_{LLI}}{S_{BN}}$$

$$f_{cfmid} = -1.769 \cdot ksi$$

$$\text{if}\left(\left|f_{cimid}\right| < \left|f_{csf}\right|, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

Stresses in the Bottom of the Beam at Midspan

$$f_{tfmid} := \frac{-P_e}{A_b} - \frac{P_e \cdot e}{S_b} + \frac{M_{DC1}}{S_b} + \frac{M_{DC2} + M_{DW}}{S_{B3N}} + \frac{M_{LLI}}{S_{BN}}$$

$$f_{tfmid} = -0.048 \cdot ksi$$

$$\text{if}\left(f_{tfmid} < f_{tsf}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

***Check Flexural Resistance***

$$M_u = 1159.2 \cdot kip \cdot ft$$

Flexural Resistance Equation Considering just Prestressing Steel

$$M_n = A_{ps} \cdot F_{ps} \cdot \left(D_p - \frac{a}{2}\right)$$

Values needed to calculate the moment capacity.

$A_{ps} = 0.217 \cdot in^2$ $\qquad f_{pu} = 270 \cdot ksi$

$f_{cs} = 4 \cdot ksi$ $\qquad b := b_{eff} = 36 \cdot in$

Determine $ß_1$ based on AASHTO Section 5.7.2.2

The factor $ß_1$ shall be taken as 0.85 for concrete strengths not exceeding 4.0 ksi. Fo concrete strengths exceeding 4.0 ksi, $ß_1$ shall be reduced at a rate of 0.05 for each 1.0 ksi of strength in excess of 4.0 ksi, except $ß_1$ shall not be taken less than 0.65.

$\beta_1 := 0.85$ $\qquad$ for concretes less than or equal to 4ksi

$$k = 2 \cdot \left(1.04 - \frac{f_{py}}{f_{pu}}\right) \qquad \text{AASHTO Equation 5.7.3.1.1-2}$$

$$\frac{f_{py}}{f_{pu}} = 0.9$$ Table C5.7.3.1.1-1 for low relaxation strands

$k := 2(1.04 - 0.90) = 0.28$ Value can also be found on table C5.7.3.1.1-1

Determine the depth to the prestressing steel based on the beam depth plus the deck thickness

$$d_p := d_{beam} + deck_{thick} - 2 \cdot in = 31 \cdot in$$

Determine c: distance between the neutral axis and the compressive face for rectangular sections.

$$c := \frac{A_{ps} \cdot N_s \cdot f_{pu}}{0.85 \cdot f_{cs} \cdot \beta_1 \cdot b + k \cdot A_{ps} \cdot N_s \cdot \left(\frac{f_{pu}}{d_p}\right)}$$ AASHTO Equation 5.7.3.1.1-4

$$c = 6.369 \cdot in$$

Determine the depth of the equivalent rectangular stress block. a is defined in Section 5.7.3.2.2

$$a := \beta_1 \cdot c = 5.414 \cdot in$$

a is within the slab so the rectangular section assumption is valid.

Determine the average stress in the prestressing steel

$$f_{ps} := f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right)$$ AASHTO Equation 5.7.3.1.1-1

$$f_{ps} = 254.468 \cdot ksi$$

Determine the factored moment resistance

? is found based on Section 5.5.4.2.1. This section is considered a tension-controlled prestressed concrete sections.

$$\phi := 1.0$$

Nominal Moment Resistance

$$M_n := A_{pst} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right)$$ AASHTO Equation 5.7.3.2.2-1

$$M_n = 1562.3 \cdot kip \cdot ft$$

Factored Moment Resistance

$$M_r := \phi \cdot M_n = 1562.3 \cdot kip \cdot ft$$

*Calculate Minimum Reinforcement*

The amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, Mr, at least equal to the lesser of 1.2 time the cracking moment, Mcr, or 1.33 times the factored moment required by the applicable strength load combinations. This is described in Section 5.7.3.3.2. In equation form:

$$\phi M_n \geq \min\left(1.2 \cdot M_{cr}, 1.33 \cdot M_u\right)$$

Cracking Moment

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc} \cdot \left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c \cdot f_r$$ AASHTO Equation 5.7.3.3.2-1

Determine the Modulus of Rupture

Modulus of ruputure for normal weight concrete shall be taken as $0.37(f'_c)^{1/2}$ when calculating the cracking moment of a member in Article 5.7.3.3.2. This is describ in Section 5.4.2.6.

$$f_r := 0.37 \cdot \sqrt{f_{cb1}} \cdot ksi = 0.979 \cdot ksi$$

$$f_{cpe} := \frac{P_t}{A_b} + \frac{P_t \cdot e}{S_b} = 2.657 \cdot ksi$$

$$M_{cr1} := S_{BN} \cdot \left(f_r + f_{cpe}\right) - M_{DC1} \cdot \left(\frac{S_{BN}}{S_b} - 1\right) = 1288.9 \cdot kip \cdot ft$$

$$M_{cr2} := S_{BN} \cdot f_r = 377.037 \cdot kip \cdot ft$$

$$M_{cr} := \max\left(M_{cr1}, M_{cr2}\right) = 1288.9 \cdot kip \cdot ft$$

$$1.2 \cdot M_{cr} = 1546.7 \cdot kip \cdot ft$$

Determine the value for 1.33 times the factored moment required by the applicable strength load combinations specified in Table 3.4.1-1

$$M_{u133} := 1.33 \cdot M_u = 1541.8 \cdot kip \cdot ft$$

Minimum Reinforcement Check

$$\text{if}\left(M_r \geq \max\left(1.2 \cdot M_{cr}, 1.33 \cdot M_u\right), \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$M_r = 1562.3 \cdot kip \cdot ft$$

## ***Shear Design***

### *Determine the Critical Section for Shear*

Find $d_v$ which is defined as the effective shear depth. It is taken as the distance measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexture and can be taken less than the greater tof 0.9de c 0.72h. This is defined in Section 5.8.3.9.

$$d_v := \frac{M_n}{A_{pst} \cdot f_{ps}}$$ AASHTO Equation C5.8.2.9-1

$$d_v = 28.293 \cdot in$$

$$d_e := \frac{A_{pst} \cdot f_{ps} \cdot d_p}{A_{pst} \cdot f_{ps}} = 31 \cdot in$$ AASHTO Equation 5.8.2.9-2

$d_v$ can also be written as:

$$d_{v1} := d_e - \frac{a}{2} = 28.293 \cdot in$$

$$0.9 \cdot d_e = 27.9 \cdot in$$

$$0.72 \cdot \left(d_{beam} + deck_{thick}\right) = 23.76 \cdot in$$

$$d_v := \max\left[d_{v1}, 0.9 \cdot d_e, 0.72 \cdot \left(d_{beam} + deck_{thick}\right)\right] = 28.293 \cdot in$$

Assume that the shear at the end of the beam is equal to the shear in this section.

$$V_u = 107.136 \cdot kip$$

Determine $\phi_s$ value to use for shear and torsion

Found in section 5.5.4.2.1 under shear and torsion for normal weight concrete.

$$\phi_s := 0.9$$

### *Shear Stress on the Concrete*

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi \cdot b_v \cdot d_v}$$ AASHTO Equation 5.8.2.9-1

Vp is described as the component of prestressing force in the direction of the she force. This force is considered to be 0 for straight tendons. (AASHTO Section 5.8.2.4)

$$\phi V_p := 0 \cdot kip \qquad V_p := 0 \cdot kip$$

Bv is the effective web withd taken as the minimum web width, measured parallel the neutral axis, between the resultants of the tensile and compressive forces due tc flexure. This is found in Section 5.8.2.9 of the AASHTO Code.

$b_w := 4.5 \cdot in$

$b_v := 2 \cdot b_w = 9 \cdot in$

Shear stress on the concrete

$$v_u := \frac{\left|V_u - \phi V_p\right|}{\phi \cdot b_v \cdot d_v}$$

$v_u = 0.421 \cdot ksi$

Ratio of applied factored shear to concrete cpmpressive strength

$$\frac{v_u}{f_{cb}} = 0.06$$

*Determine ß and* θ

These angles are determined based the the general procedure found in Section 5.8.3.4.2. For sections containing the at least the minimum amout of reinforcement and θ can be found by.

$$\beta = \frac{4.8}{1 + 750 \cdot \varepsilon_s}$$ AASHTO Equation 5.8.3.4.2-1

$$\theta = 29 + 3500 \cdot \varepsilon_s$$ AASHTO Equation 5.8.3.4.2-3

Determine the net longitudinal tensile strain in the section at the centriod of the tension reinforcement, $\varepsilon_s$. $\varepsilon_s$ is required to determine ß and θ

$$\varepsilon_s = \frac{\dfrac{\left|M_u\right|}{d_v} + 0.5 \cdot N_u + \left|V_u - V_p\right| - A_{ps} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps}}$$ AASHTO Equation 5.8.3.4.2-4

Determine the $f_{po}$ according to Section 5.8.3.4.2

$f_{po} := 0.7 \cdot f_{pu}$

To be conserative, the contribution from the mild steel is ignored.

$E_s \cdot A_s = 0$

Axial load is assumed to be 0

$N_u := 0$

Determine $e_s$

$$\varepsilon_s := \frac{\frac{|M_u|}{d_v} + 0.5 \cdot N_u + |V_u - V_p| - A_{pst} \cdot f_{po}}{E_p \cdot A_{pst}}$$

$$\varepsilon_s = 0.00144$$

Determine ß

$$\beta := \frac{4.8}{1 + 750 \cdot \varepsilon_s} \qquad \beta = 2.31$$

Determine $\theta$

$$\theta := 29 + 3500 \cdot \varepsilon_s \qquad \theta = 34.03$$

*Nominal Shear Resistance*

Determine the nominal shear resistance provided by the tensile stresses in the concrete.

$$V_c := 0.0316 \cdot \beta \cdot \sqrt{f_{cb1}} \cdot ksi \cdot b_v \cdot d_v$$ AASHTO Equation 5.8.3.3-3

$$V_c = 49.182 \cdot kip$$

Determine the tensile stresses in the traverse reinforcement.

$$V_s = \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s}$$ AASHTO Equation C5.8.3.3-1

Determine if Shear Reinforcement is required in the cross-section of the beam. According to Section 5.8.2.4, reinforcement shall be provided where:

$$V_u \geq 0.5 \cdot \phi_s \cdot (V_c + V_p)$$ AASHTO Equation 5.8.2.4-1

$$\text{if}\left[V_u \geq 0.5 \cdot \phi_s \cdot (V_c + V_p), \text{"shear stirrups required"}, \text{"no stirrups"}\right] = \text{"shear stirrups required"}$$

*Determine Spacing of the Shear Reinforcement*

The nominal shear resistance $V_n$ must be greater than or equal to applied shear $V_u$

$$V_n \geq V_u$$

$$V_n = V_c + V_s + V_p$$ AASHTO Equation 5.8.3.3-1

Shear stirrups are Grade 60 #04 Bars.

$$f_{ys} := 60 \cdot ksi$$

$$A_s := 0.2 \cdot in^2$$

Determine the required spacing of shear stirrups.

$$V_s := \frac{V_u}{\phi_s} - V_c = 69.858 \cdot kip$$

$A_v := A_s \cdot 2 = 0.4 \cdot in^2$ There are two sections resisting shear

Reorganize the $V_s$ equation (Equation C5.8.3.3-1) to determine spacing

$$s := \frac{A_v \cdot f_{ys} \cdot d_v \cdot \cot(\theta \cdot deg)}{V_s}$$

$$s = 14.395 \cdot in$$

Determine the maximum spacing based on Section 5.8.2.7

If $v_u < 0.125 \cdot f_c$

$S_{max} = 0.8 d_v \le 24 \cdot in$ AASHTO Equation 5.8.2.7-1

If $v_u \ge 0.125 \cdot f_c$

$S_{max} = 0.4 d_v \le 12 \cdot in$ AASHTO Equation 5.8.2.7-2

$$\text{if}\left(v_u < 0.125 \cdot f_{cb}, \min\left(0.8 \cdot d_v, 24 \cdot in\right), \text{"use Equation 5.8.2.7-2"}\right) = 22.635 \cdot in$$

$$\text{if}\left(v_u \ge 0.125 \cdot f_{cb}, \min\left(0.4 \cdot d_v, 12 \cdot in\right), \text{"use Equation 5.8.2.7-1"}\right) = \text{"use Equation 5.8.2.7-1"}$$

Therefore use spacing determined by $V_s$ equation or the Maximum MDOT spacing of 12" as defined in MDOT Bridge Design Guides 6.65.10A

$$s := 12 \cdot in$$

$$V_s := \frac{A_v \cdot f_{ys} \cdot d_v \cdot \cot(\theta \cdot deg)}{s} = 83.799 \cdot kip$$

$$\text{if}\left(A_v \ge 0.0316 \cdot \sqrt{f_{cb1}} \cdot ksi \cdot \frac{b_v \cdot s}{f_{ys}}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$V_n := V_s + V_c = 132.981 \cdot kip$$

### *Bursting Reinforcement (Splitting Resistance)*

The splitting resistance of pretensioned anchorage zones provided by reinforcement i the ends of pretensioned beams shall be taken as:

$P_r = f_s \cdot A_s$ AASHTO Equation 5.10.10.1-1

The resistance shall not be taken less than 4% of the prestressing force at transfer.

$$P_r := 0.04 \cdot P_t = 20.093 \cdot kip$$

Determine the stress in the steel. (not to exceed 20 ksi)

$$f_s := 20 \cdot ksi$$

Find the area of steel required to meet the minimum resistance.

$$A_{sb} := \frac{P_r}{f_s} = 1.005 \cdot in^2$$

Determine the number of stirrups

$$N_{stb} := \frac{A_{sb}}{A_v} = 2.512 \qquad \text{use} \qquad N_{stb} := 3$$

For pretensioned box or tub girders, $A_s$ shall be taken as the total area of vertical reinforcement or horizontal reinforcement located within a distance h/4 from the end of the member, where h is the lesser of the overall width or height of the member (in).

$$x := \frac{d_{beam}}{4} = 6.75 \cdot in$$

Spacing of the bursting reinforcement

$$s_b := \frac{x}{N_{stb}} \qquad s_b = 2.25 \cdot in$$

### *Confinement Reinforcement*

Confinement reinforcement must be provided for a distance 1.5d from the end of the beam. This is stated in Section 5.10.10.2. For box beams, traverse reinforcement shall be provided and anchored by extending the leg of the stirrup into the web of the girder.

$$L_c := 1.5 \cdot d_{beam} = 40.5 \cdot in \qquad L_c = 3.375 \cdot ft$$

Space Stirrups at 6.0 in for 3.5 ft along the beam

# APPENDIX D

# SAMPLE CALCULATIONS FOR RESIDUAL STRENGTH OF MDOT SALVAGED BOX BEAM (J11)

**Department of Civil and Architectural Engineering**

**Lawrence Technological University**

**Southfield, MI 48075-0134**

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**1. Residual Ultimate Loa**

$L = 42 \text{ feet} \quad b = 36 \text{ in} \quad d = 21 \text{ in} \quad f_c = 6100 \text{ psi}$

Information from chart

Area, A = 467 in$^2$

Self-weight, $w = \dfrac{486}{1000} = 0.486 \ \dfrac{\text{kip}}{\text{feet}}$

$y_t$ = 10.6 in  $y_b$ = 10.4 in  $S_T$ = 2320 in$^3$  $S_B$ = 2360 in$^3$  I = 24600 in$^4$

Modulus of Elasticity for beam, $E_b = 5700\sqrt{f_c} = 4.452 \times 10^5$ psi

Loads (Beam dead load moment)

$$M_{D.beam} = \frac{w \times L^2}{8} = 107.163 \text{ kip - ft}$$

**Calculating $M_n$: assuming all 10 strands are in good condition**

$A_{ps}$ = 1.53 in$^2$  $f_{pu}$ = 270 ksi  $d_p$ = 19 in  $A_{s.prime}$ = 1.323 in$^2$  $f_{yprime}$ = 60 ksi

$d_{sprime}$ = 2 in  k = 2(1.04 - 0.9) =0.28  $f_c$ = 6100 psi  β = 0.75  b = 36 in

$A_s$ = 0  $f_y$ = 60 ksi

$$c = \frac{A_{ps} \times f_{pu} \times 1000 + A_s \times f_y \times 1000 - A_{sprime} \times f_{yprime} \times 1000}{0.85 \times f_c \times \beta \times b + k \times A_{ps} - \dfrac{f_{pu} \times 1000}{d_p}} = 2.6531 \text{ in}$$

a = 0.8 × c = 2.122 in

$$f_{ps} = f_{pu} \times \left(1 - \frac{k \times c}{d_p}\right) = 295.443 \text{ ksi}$$

$$M_n = A_{ps} \times f_{ps} \times \left(d_p - \frac{a}{2}\right) - A_{sprime} \times f_{yprime} \times \left(d_{sprime} - \frac{a}{2}\right) = 7.046 \times 10^3 \text{ kip - in}$$

$$M_{LL} = Mn - \left(M_{Dbeam} \times 12\right) = 5.76 \times 10^3 \text{ kip - in}$$

For 4-point loading: $P = \dfrac{M_{LL} \times 2}{19 \times 12} = 50.529\,\text{kips}$

**Calculating M$_n$: assuming all 8 strands are in good condition**

N = 8  $A_{1ps} = N \times 1.53 = 1.224\ in^2$

$$c_1 = \dfrac{A_{1ps} \times f_{pu} \times 1000 + A_s \times f_y \times 1000 - A_{sprime} \times f_{yprime} \times 1000}{0.85 \times f_c \times \beta \times b + k \times A_{1ps} - \dfrac{f_{pu} \times 1000}{d_p}} = 1.9963\,\text{in}$$

$a = 0.8 \times c_1 = 1.597$ in

$$f_{ps} = f_{pu} \times \left(1 - \dfrac{k \times c_1}{d_p}\right) = 262.057\,\text{ksi}$$

$$M_n = A_{1ps} \times f_{ps} \times \left(d_p - \dfrac{a}{2}\right) - A_{sprime} \times f_{yprime} \times \left(d_{sprime} - \dfrac{a}{2}\right) = 5.743 \times 10^3\ \text{kip - in}$$

$$M_{LL} = Mn - \left(M_{Dbeam} \times 12\right) = 4.457 \times 10^3\ \text{kip - in}$$

For 4-point loading: $P = \dfrac{M_{LL} \times 2}{19 \times 12} = 39.096\,\text{kips}$

**Calculating M$_n$: assuming all 8 strands with 5% loss in cross-sectional area**

N = 8  $A_{1ps} = N \times 1.53 = 1.224\ in^2$  $A_{2ps} = 0.95 \times N \times 1.53 = 1.165\ in^2$

$$c_2 = \dfrac{A_{2ps} \times f_{pu} \times 1000 + A_s \times f_y \times 1000 - A_{sprime} \times f_{yprime} \times 1000}{0.85 \times f_c \times \beta \times b + k \times A_{2ps} - \dfrac{f_{pu} \times 1000}{d_p}} = 1.8649\,\text{in}$$

$a = 0.8 \times c_2 = 1.492$ in

$$f_{ps} = f_{pu} \times \left(1 - \dfrac{k \times c_2}{d_p}\right) = 262.58\,\text{ksi}$$

$$M_n = A_{2ps} \times f_{ps} \times \left(d_p - \frac{a}{2}\right) - A_{sprime} \times f_{yprime} \times \left(d_{sprime} - \frac{a}{2}\right) = 5.4.74 \times 10^3 \text{ kip - in}$$

$$M_{LL} = Mn - (M_{Dbeam} \times 12) = 4.188 \times 10^3 \text{ kip - in}$$

For 4-point loading: $P = \frac{M_{LL} \times 2}{19 \times 12} = 36.737 \text{ kips}$

**Calculating $M_n$: assuming all 8 strands with 10% loss in cross-sectional area**

N = 8   $A_{1ps}$ = N×1.53 = 1.224 in$^2$   $A_{3ps}$ = 0.90×N×1.53 = 1.102 in$^2$

$$c_3 = \frac{A_{3ps} \times f_{pu} \times 1000 + A_s \times f_y \times 1000 - A_{sprime} \times f_{yprime} \times 1000}{0.85 \times f_c \times \beta \times b + k \times A_{3ps} - \frac{f_{pu} \times 1000}{d_p}} = 1.7335 \text{ in}$$

a = 0.8 × $c_3$ = 1.387 in

$$f_{ps} = f_{pu} \times \left(1 - \frac{k \times c_3}{d_p}\right) = 263.102 \text{ ksi}$$

$$M_n = A_{3ps} \times f_{ps} \times \left(d_p - \frac{a}{2}\right) - A_{sprime} \times f_{yprime} \times \left(d_{sprime} - \frac{a}{2}\right) = 5.202 \times 10^3 \text{ kip - in}$$

$$M_{LL} = Mn - (M_{Dbeam} \times 12) = 3.916 \times 10^3 \text{ kip - in}$$

For 4-point loading: $P = \frac{M_{LL} \times 2}{19 \times 12} = 34.353 \text{ kips}$

**Calculating $M_n$: assuming all 8 strands with 20% loss in cross-sectional area**

N = 8   $A_{1ps}$ = N×1.53 = 1.224 in$^2$   $A_{4ps}$ = 0.80×N×1.53 = 0.979 in$^2$

$$c_4 = \frac{A_{4ps} \times f_{pu} \times 1000 + A_s \times f_y \times 1000 - A_{sprime} \times f_{yprime} \times 1000}{0.85 \times f_c \times \beta \times b + k \times A_{4ps} - \frac{f_{pu} \times 1000}{d_p}} = 1.4708 \text{ in}$$

a = 0.8 × $c_4$ = 1.177 in

$$f_{ps} = f_{pu} \times \left(1 - \frac{k \times c_4}{d_p}\right) = 264.148\,\text{ksi}$$

$$M_n = A_{4ps} \times f_{ps} \times \left(d_p - \frac{a}{2}\right) - A_{sprime} \times f_{yprime} \times \left(d_{sprime} - \frac{a}{2}\right) = 4.65 \times 10^3 \text{ kip - in}$$

$$M_{LL} = Mn - (M_{Dbeam} \times 12) = 3.364 \times 10^3 \text{ kip - in}$$

For 4-point loading: $P = \dfrac{M_{LL} \times 2}{19 \times 12} = 29.511\,\text{kips}$

**2. Residual Cracking Load.**

$L = 42$ feet $\quad f_c = 6100$ psi $\quad E_s = 29000$ ksi $\quad f_{up} = 270$ ksi

$E_b = 5700\sqrt{f_c} = 4.452 \times 10^5$ psi $\qquad \varepsilon_p = \dfrac{f_{up}}{E_s} = 0.0093$

$fr = 7.5\sqrt{f_c} \times \dfrac{1}{1000} = 0.586\,\text{ksi}$ $\quad h_1 = 21$ in $\quad b_1 = 36$ in $\quad h_2 = 11$ in $\quad b_2 = 26$ in

$$A_1 = [(h_1 b_1) - (h_2 b_2)] = 470\,\text{in}^2$$

$$I_1 = \left[\left(\frac{b_1 h_1^{\,3}}{12}\right) - \left(\frac{b_2 h_2^{\,3}}{12}\right)\right] = 2.49 \times 10^4 \text{ in}^4$$

Beam centroid from top.

$y_t = 10.5$ in

Effective depth

$d_p = 21 - 2 = 19$ in

$y_b = h_1 - yt = 10.5$ in

Eccentricity of prestressing from centroid

$e_0 = d_p - y_t = 8.5$ in

$\rho = 0.150$ kip/ft$^3$

Dead load, $w = \rho \dfrac{A_1}{144} = 0.49 \dfrac{kip}{ft}$

$$M_D = w\frac{(42)^2}{8} \times 12 = 1.295 \times 10^3 \text{ kip - in}$$

Considering all 10 strands in good condition, N = 10

Prestressing force per strand, $P_f = 30.75$ ksi

Total prestressing force, $P = N \times P_f = 307.5$ ksi

Calculating the cracking moment for original design beam as at the time of design

$$\sigma_{bot} = \frac{-P}{A} - \frac{P \times e_0 \times y_b}{I_1} + \frac{M_D \times yb}{I_1} + \frac{M_{LL} \times y_b}{I_1} \quad , \qquad \sigma_{bot} = fr = 0.586 \text{ ksi}$$

$$M_{LL} = \left( \sigma_{bot} + \frac{P}{A_1} + \frac{P \times e_0 \times y_b}{I_1} - \frac{M_D \times y_b}{I_1} \right) \times \frac{I_1}{y_b} = 4.259 \times 10^3 \text{ kip - in}$$

$M_{cr} = M_{LL} \times 1000 = 4.259 \times 10^6$ Ib-in

Calculating the cracking load for original design beam as at the time of design

$$P_{cr} = 2.\frac{Mcr}{288} = 2.958 \times 10^4 \text{ Ib}$$

Calculating the cracking moment after assumed 20% loss per strand with only 8 remaining strands with 2 strands tone by corrosion.

N = 8, Prestressing force per strand, $P_f = 30.75$ ksi

Total prestressing force, $P = 0.8 \times 8 \times P_f = 196.80$ ksi

$$\sigma_{bot} = \frac{-P}{A} - \frac{P \times e_0 \times y_b}{I_1} + \frac{M_D \times yb}{I_1} + \frac{M_{LL} \times y_b}{I_1} \quad , \qquad \sigma_{bot} = fr = 0.586\,\text{ksi}$$

$$M_{LL} = \left( \sigma_{bot} + \frac{P}{A_1} + \frac{P \times e_0 \times y_b}{I_1} - \frac{M_D \times y_b}{I_1} \right) \times \frac{I_1}{y_b} = 2.759 \times 10^3 \text{ kip - in}$$

$M_{cr} = M_{LL} \times 1000 = 2.759 \times 10^6$ Ib-in

Calculating the cracking load for original design beam as at the time of design

$$P_{cr} = 2.\frac{Mcr}{288} = 1.916 \times 10^4 \text{ Ib}$$

Calculating the Yielding Moment

Considering, $f_{up} = 270$ ksi

Yield stress, $\sigma_{yield} = 230$ ksi

Total strain, $\varepsilon_{yield} = \dfrac{\sigma_{yield}}{E_s} = 7.931 \times 10^{-3}$ ksi

Total strain = strain due to prestressing +strain due to loading

Prestressing force per strand, $P_{fs} = \dfrac{P}{8} = 24.6$ ksi $\qquad A_p = 0.153$ in$^2$

Strain due to prestressing, $\varepsilon_{pre} = \dfrac{P_{fs}}{A_p E_s} = 5.544 \times 10^{-3}$

Strain due to loading, $\varepsilon_{load} = \varepsilon_{yield} - \varepsilon_{pre} = 2.387 \times 10^{-3}$

Compressive Force = Tension Force + Prestressing Force

For 6100 psi concrete, $\beta_1 = 0.75$ $\quad$ b = 36 in

$F_{comp} = 0.85 \times f_c \times \beta_1 \times c \times b$, $\quad E_s = 2.9 \times 10^7$ psi

$$F_{ten} = \varepsilon_{load} \times E_s \times 8 \times A_p = 8.472 \times 0^{-3}$$

$$F_{pres} = P \times 1000 = 1.968 \times 0^{5}$$

Therefore, $c = \dfrac{(F_{ten} + F_{pres})}{0.85 \times f_c \times \beta_1 \times b} = 2.011\,\text{in}$

$a = \beta_1 \times c = 1.508$ in

$$M_{yield} = (F_{ten} + F_{pres}) \times \left( d_p - \frac{a}{2} \right) = 5.137 \times 10^{6}\ \text{kip - in}$$

Yield Moment due to 2-point load

$$M_{yield2pt} = M_{yield} - M_D \times 1000 = 3.841 \times 10^{6}\ \text{kip - in}$$

$$P_{yield} = \frac{M_{yield2pt}}{144} = 2.667 \times 10^{4}\ \text{Ib}$$