Psychometric Analysis Report for the Michigan English Language Arts (ELA), Mathematics, Science, Social Studies, and SAT Student Growth Percentile Reporting

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Introduction

The use of student growth models is not uncommon in K-12 testing. The most commonly used approaches by states are conditional growth percentile models, which include student growth percentiles (Betebenner, 2008; 2009; 2011) or an alternative known as percentile rank residuals (Castellano & Ho, 2013). These models attempt to describe individual student growth relative to other students who are academically similar by using prior test scores as predictors. Individual level results from these models can be aggregated at a group level.

This was conducted for the M-Step, SAT, WIDA, and MI-Access assessments.

Methodology

Student Growth Percentiles (SGP)

For assessments with a sufficient sample size (M-STEP, SAT, and WIDA Access) student growth percentiles (SGPs) were calculated using the R SGP package (Betebenner et. al., 2015) version 1.5-0.0 as compiled from the master branch of the SGP GitHub repository. SGPs as defined this way take normative approach.

Specifically, let Y_t denote an assessment score at time $\boldsymbol{t}$, the expected value of Y_t at the $\boldsymbol{\tau}$-th quantile, $Q_{Y_t}(\tau|Y_{t-1},\ldots,Y_1)$ based on prior assessment scores $Y_{t-1},\ldots,Y_1$, is then given by (Betebenner, 2011, p17)

$$Q_{Y_t}(\tau|Y_{t-1},\ldots,Y_1) = \sum_{j=1}^{t-1}\sum_{i=1}^{3}\phi_{ij}(Y_j)\beta_{ij}(\tau) \qquad (1)$$

Where ϕ_{ij}, $i=1,2,3$ and $j=1,\ldots,t-1$ denote the B-spline basis functions for quantile τ. For instance, for τ =.5, Q_{Y_t} returns the estimated median expectation of Y_t for any combination of $Y_{t-1},\ldots,Y_1$. This analysis used the default parameters of the SGP package which generates 1+7*(number of pretest) parameters per quantile. For example, for a 3-pretest model we have 1+7*3 = 22 parameters per quantile and we estimate 100 quantiles independently (from 0.005 to 0.995 in 0.01 increments).

Calculating a SGP from equation 1 requires prior test score information to determine predicted scores. The SGP for a student is defined as the midpoint of the(ranked) two quantiles between which the student's score falls.

$$SGP_i = \left(\max\{\tau_i, \hat{Q}_\tau(Y|X=x_i) < y_i\} + \min\left(\{\tau_i, \hat{Q}_\tau(Y|X=x_i) > y_i\}\right) * \frac{100}{2} \qquad (2)$$

Where $_i$ is the student *i's* vector of prior test scores.

Note that while the SGP package can simulate the conditional standard error of measurement (CSEMs), for the initial Fall 2016 SGP student data files, the CSEM was not available.

Percentile Rank Residuals (PRR)

For assessments with small sample sizes (MI-Access), the PRR method (Castellano & Ho, 2013) was used to estimate the conditional student growth percentiles. This method uses an ordinary least squares (OLS) model, where the predictors consist of past student achievement data.

$$Y_{it} = \beta_0 + \beta_1 y_{i(t-1)} + \beta_2 y_{i(t-2)} + \beta_3 y_{i(t-3)} + \varepsilon_{it} \quad (5)$$

where Y_{it} is the observed score on the assessment at time *t* for student *i*, $Y_{i,\,t-1}$ is the observed score at prior time 1, $Y_{i,\,t-2}$ is the observed score at prior time 2, and $Y_{i,\,t-3}$ is the observed score at prior time 3, The βs are the regression coefficients, and ε_{it} is a residual error.

After estimating Equation 5, the residuals are calculated using Equation 6:

$$\hat{\varepsilon}_{it} = y_{it} - \hat{y}_{it} \quad (6)$$

where $\hat{\varepsilon}_{it}$ is the residual for student *i* at time *t*, $\hat{y}_{it}$ is the predicted score from equation 5.

Next, the residuals are rank ordered (Castellano & Ho, 2013, p. 195).

$$PRR_{it} = F(\hat{\varepsilon}_{it}) \times 100 = \frac{\#residuals \leq \hat{\varepsilon}_{it}}{n} \times 100 \quad (7)$$

where $\hat{\varepsilon}_{it}$ is the residual for student *i* at time *t* and *n* is the total sample size for all students with MI-Access FI results for a given posttest in 2015-16.

A standard error of measurement can be obtained by simulation for this method. Specifically, for a given posttest, y_{it}, and $CSEM(y_{it})$ 100 posttest were simulated such that they follow a normal distribution given by Equation 8:

$$y_{its} \sim N(mean = y_{it}, sd = CSEM(y_{it})) \quad (8)$$

For each simulated y_{its}, calculate the corresponding PRR using equations 5-7 while holding all other student data constant. Repeat this for each student.

Reporting Results

Results were reported at both the student and aggregate levels. This section provides a brief overview of the results provided to MDE. For more details, please see the *Requirement Document for the Spring 2016 Individual Growth Percentiles and Aggregated Growth Percentiles.*

For each assessment, results were reported for different content areas. Table 1 provides a list of the assessment and content areas combinations for which SGPs or PRRs were provided. Table 1 provides a list of the grades and domains for which results were reported.

Table 1: Applicable assessments by grade

Grade	M-STEP	SAT	MI-Access	WIDA
K				Overall Composite
1				Overall Composite
2				Overall Composite
3	ELA, Math		ELA, Math	Overall Composite
4	ELA, Math, Science		ELA, Math, Science	Overall Composite
5	ELA, Math, Social Studies		ELA, Math, SS	Overall Composite
6	ELA, Math		ELA, Math	Overall Composite
7	ELA, Math, Sci		ELA, Math, Science	Overall Composite
8	ELA, Math, Social Studies		ELA, Math, Social Studies	Overall Composite
11	Science,Social Studies	ELA, Math	ELA, Math, Science, Social Studies	Overall Composite
12				Overall Composite

Categorization of Individual (Level) Growth Percentiles

Individual (level) growth percentiles (either SGP or PRR) will be assigned one of the five categorical descriptors based on MDE accountability policies, which include:

- Significant Decline (SGP 0-19)
- Decline (SGP 20-39)
- Maintain (SGP 40-59)
- Improvement (SGP 60-79)
- Significant Improvement (SGP 80-99)

Valid Test Sequence Rules

Identified suitable pathways and their information can be found in Table 2 for the SGP method (M-STEP/SAT), Table 3 for the PRR approach (MI-Access FI), and Table 4 for the SGP method (WIDA Access).

Table 2: Valid Test Sequence for the SGP Method (M-STEP_SAT)

Subject	Model	DataElement	Unique String Identifier	Academic Year Id	Test Period Id	Assessment Id	Test Cycle Id	Content Area Id	SDS Grade Code
English Language Art	04th Grade 15/16	PostTest	S16_M-STEP_English Language Arts_04	16	117	18	129	3	04
		Prior1	S15_M-STEP_English Language Arts_03	15	106	18	112	3	03
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	05th Grade 15/16	PostTest	S16_M-STEP_English Language Arts_05	16	117	18	129	3	05
		Prior1	S15_M-STEP_English Language Arts_04	15	106	18	112	3	04
		Prior2	F13_MEAP_Reading_03	14	91	6	91	9	03
		Prior3	NA	--	--	--	--	--	--
	06th Grade 15/16	PostTest	S16_M-STEP_English Language Arts_06	16	117	18	129	3	06
		Prior1	S15_M-STEP_English Language Arts_05	15	106	18	112	3	05
		Prior2	F13_MEAP_Reading_04	14	91	6	91	9	04
		Prior3	F12_MEAP_Reading_03	13	80	6	80	9	03
	07th Grade 15/16	PostTest	S16_M-STEP_English Language Arts_07	16	117	18	129	3	07
		Prior1	S15_M-STEP_English Language Arts_06	15	106	18	112	3	06
		Prior2	F13_MEAP_Reading_05	14	91	6	91	9	05
		Prior3	F12_MEAP_Reading_04	13	80	6	80	9	04
	08th Grade 15/16	PostTest	S16_M-STEP_English Language Arts_08	16	117	18	129	3	08
		Prior1	S15_M-STEP_English Language Arts_07	15	106	18	112	3	07
		Prior2	F13_MEAP_Reading_06	14	91	6	91	9	06
		Prior3	F12_MEAP_Reading_05	13	80	6	80	9	05
	11th Grade 15/16	PostTest	S16_SAT_English Language Arts_11	16	117	24	136	3	11
		Prior1	F12_MEAP_Reading_08	13	80	6	80	9	08
		Prior2	F11_MEAP_Reading_07	12	69	6	69	9	07
		Prior3	F10_MEAP_Reading_06	11	61	6	61	9	06
Mathematics	04th Grade 15/16	PostTest	S16_M-STEP_Mathematics_04	16	117	18	129	7	04
		Prior1	S15_M-STEP_Mathematics_03	15	106	18	112	7	03
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	05th Grade 15/16	PostTest	S16_M-STEP_Mathematics_05	16	117	18	129	7	05
		Prior1	S15_M-STEP_Mathematics_04	15	106	18	112	7	04
		Prior2	F13_MEAP_Mathematics_03	14	91	6	91	7	03
		Prior3	NA	--	--	--	--	--	--
	06th Grade 15/16	PostTest	S16_M-STEP_Mathematics_06	16	117	18	129	7	06
		Prior1	S15_M-STEP_Mathematics_05	15	106	18	112	7	05
		Prior2	F13_MEAP_Mathematics_04	14	91	6	91	7	04
		Prior3	F12_MEAP_Mathematics_03	13	80	6	80	7	03
	07th Grade 15/16	PostTest	S16_M-STEP_Mathematics_07	16	117	18	129	7	07
		Prior1	S15_M-STEP_Mathematics_06	15	106	18	112	7	06
		Prior2	F13_MEAP_Mathematics_05	14	91	6	91	7	05
		Prior3	F12_MEAP_Mathematics_04	13	80	6	80	7	04
	08th Grade 15/16	PostTest	S16_M-STEP_Mathematics_08	16	117	18	129	7	08
		Prior1	S15_M-STEP_Mathematics_07	15	106	18	112	7	07
		Prior2	F13_MEAP_Mathematics_06	14	91	6	91	7	06
		Prior3	F12_MEAP_Mathematics_05	13	80	6	80	7	05
	11th Grade 15/16	PostTest	S16_SAT_Mathematics_11	16	117	24	136	7	11
		Prior1	F12_MEAP_Mathematics_08	13	80	6	80	7	08
		Prior2	F11_MEAP_Mathematics_07	12	69	6	69	7	07
		Prior3	F10_MEAP_Mathematics_06	11	61	6	61	7	06
Science	07th Grade 15/16	PostTest	S16_M-STEP_Science_07	16	117	18	129	10	07
		Prior1	F13_MEAP_Science_05	14	91	6	91	10	05
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	11th Grade 15/16	PostTest	S16_M-STEP_Science_11	16	117	18	129	10	11
		Prior1	F12_MEAP_Science_08	13	80	6	80	10	08
		Prior2	F09_MEAP_Science_05	10	52	6	52	10	05
		Prior3	NA	--	--	--	--	--	--
Social Studies	08th Grade 15/16	PostTest	S16_M-STEP_Social Studies_08	16	117	18	129	11	08
		Prior1	F13_MEAP_Social Studies_06	14	91	6	91	11	06
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	11th Grade 15/16	PostTest	S16_M-STEP_Social Studies_11	16	117	18	129	11	11
		Prior1	F13_MEAP_Social Studies_09	14	91	6	91	11	09
		Prior2	F10_MEAP_Social Studies_06	11	61	6	61	11	06
		Prior3	NA	--	--	--	--	--	--

Table 3: Valid Test Sequence for the PRR Approach (MI-Access FI)

Subject	Model	DataElemen	Unique String Identifier	Academic Year Id	Test Period Id	Assessment Id	Test Cycle Id	Content Area Id	SDS Grade Code
English Language Art	04th Grade 15/16	PostTest	S16_MI-Access FI_English Language Arts_04	16	117	9	131	3	04
		Prior1	S15_MI-Access FI_English Language Arts_03	15	106	9	114	3	03
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	05th Grade 15/16	PostTest	S16_MI-Access FI_English Language Arts_05	16	117	9	131	3	05
		Prior1	S15_MI-Access FI_English Language Arts_04	15	106	9	114	3	04
		Prior2	F13_MI-Access FI_Accessing Print_03	14	91	9	94	1	03
		Prior3	NA	--	--	--	--	--	--
	06th Grade 15/16	PostTest	S16_MI-Access FI_English Language Arts_06	16	117	9	131	3	06
		Prior1	S15_MI-Access FI_English Language Arts_05	15	106	9	114	3	05
		Prior2	F13_MI-Access FI_Accessing Print_04	14	91	9	94	1	04
		Prior3	F12_MI-Access FI_Accessing Print_03	13	80	8	82	1	03
	07th Grade 15/16	PostTest	S16_MI-Access FI_English Language Arts_07	16	117	9	131	3	07
		Prior1	S15_MI-Access FI_English Language Arts_06	15	106	9	114	3	06
		Prior2	F13_MI-Access FI_Accessing Print_05	14	91	9	94	1	05
		Prior3	F12_MI-Access FI_Accessing Print_04	13	80	8	82	1	04
	08th Grade 15/16	PostTest	S16_MI-Access FI_English Language Arts_08	16	117	9	131	3	08
		Prior1	S15_MI-Access FI_English Language Arts_07	15	106	9	114	3	07
		Prior2	F13_MI-Access FI_Accessing Print_06	14	91	9	94	1	06
		Prior3	F12_MI-Access FI_Accessing Print_05	13	80	8	82	1	05
	11th Grade 15/16	PostTest	S16_MI-Access FI_English Language Arts_11	16	117	9	131	3	11
		Prior1	F12_MI-Access FI_Accessing Print_08	13	80	8	82	1	08
		Prior2	F11_MI-Access FI_Accessing Print_07	12	69	8	70	1	07
		Prior3	F10_MI-Access FI_Accessing Print_06	11	61	8	62	1	06
Mathematics	04th Grade 15/16	PostTest	S16_MI-Access FI_Mathematics_04	16	117	9	131	7	04
		Prior1	S15_MI-Access FI_Mathematics_03	15	106	9	114	7	03
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	05th Grade 15/16	PostTest	S16_MI-Access FI_Mathematics_05	16	117	9	131	7	05
		Prior1	S15_MI-Access FI_Mathematics_04	15	106	9	114	7	04
		Prior2	F13_MI-Access FI_Mathematics_03	14	91	9	94	7	03
		Prior3	NA	--	--	--	--	--	--
	06th Grade 15/16	PostTest	S16_MI-Access FI_Mathematics_06	16	117	9	131	7	06
		Prior1	S15_MI-Access FI_Mathematics_05	15	106	9	114	7	05
		Prior2	F13_MI-Access FI_Mathematics_04	14	91	9	94	7	04
		Prior3	F12_MI-Access FI_Mathematics_03	13	80	8	82	7	03
	07th Grade 15/16	PostTest	S16_MI-Access FI_Mathematics_07	16	117	9	131	7	07
		Prior1	S15_MI-Access FI_Mathematics_06	15	106	9	114	7	06
		Prior2	F13_MI-Access FI_Mathematics_05	14	91	9	94	7	05
		Prior3	F12_MI-Access FI_Mathematics_04	13	80	8	82	7	04
	08th Grade 15/16	PostTest	S16_MI-Access FI_Mathematics_08	16	117	9	131	7	08
		Prior1	S15_MI-Access FI_Mathematics_07	15	106	9	114	7	07
		Prior2	F13_MI-Access FI_Mathematics_06	14	91	9	94	7	06
		Prior3	F12_MI-Access FI_Mathematics_05	13	80	8	82	7	05
	11th Grade 15/16	PostTest	S16_MI-Access FI_Mathematics_11	16	117	9	131	7	11
		Prior1	F12_MI-Access FI_Mathematics_08	13	80	8	82	7	08
		Prior2	F11_MI-Access FI_Mathematics_07	12	69	8	70	7	07
		Prior3	F10_MI-Access FI_Mathematics_06	11	61	8	62	7	06
Science	07th Grade 15/16	PostTest	S16_MI-Access FI_Science_07	16	117	9	131	10	07
		Prior1	F13_MI-Access FI_Science_05	14	91	9	94	10	05
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	11th Grade 15/16	PostTest	S16_MI-Access FI_Science_11	16	117	9	131	10	11
		Prior1	F12_MI-Access FI_Science_08	13	80	8	82	10	08
		Prior2	F09_MI-Access FI_Science_05	10	52	8	53	10	05
		Prior3	NA	--	--	--	--	--	--
Social Studies	08th Grade 15/16	PostTest	NA	--	--	--	--	--	--
		Prior1	NA	--	--	--	--	--	--
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	11th Grade 15/16	PostTest	NA	--	--	--	--	--	--
		Prior1	NA	--	--	--	--	--	--
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--

Table 4: Valid Test Sequence for the SGP Approach (WIDA Access)

Subject	Model	DataElem	Unique String Identifier	Academic Year Id	Test Period Id	Assessmen t Id	Test Cycle Id	Content Area Id	SDS Grade Code
Overall	01st Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_01	16	119	16	135	6	01
		Prior1	S15 WIDA_WIDA_Overall_00	15	109	16	120	6	00
		Prior2	NA	--	--	--	--	--	--
		Prior3	NA	--	--	--	--	--	--
	02nd Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_02	16	119	16	135	6	02
		Prior1	S15 WIDA_WIDA_Overall_01	15	109	16	120	6	01
		Prior2	S14 WIDA_WIDA_Overall_00	14	99	16	105	6	00
		Prior3	NA	--	--	--	--	--	--
	03rd Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_03	16	119	16	135	6	03
		Prior1	S15 WIDA_WIDA_Overall_02	15	109	16	120	6	02
		Prior2	S14 WIDA_WIDA_Overall_01	14	99	16	105	6	01
		Prior3	NA	--	--	--	--	--	--
	04th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_04	16	119	16	135	6	04
		Prior1	S15 WIDA_WIDA_Overall_03	15	109	16	120	6	03
		Prior2	S14 WIDA_WIDA_Overall_02	14	99	16	105	6	02
		Prior3	NA	--	--	--	--	--	--
	05th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_05	16	119	16	135	6	05
		Prior1	S15 WIDA_WIDA_Overall_04	15	109	16	120	6	04
		Prior2	S14 WIDA_WIDA_Overall_03	14	99	16	105	6	03
		Prior3	NA	--	--	--	--	--	--
	06th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_06	16	119	16	135	6	06
		Prior1	S15 WIDA_WIDA_Overall_05	15	109	16	120	6	05
		Prior2	S14 WIDA_WIDA_Overall_04	14	99	16	105	6	04
		Prior3	NA	--	--	--	--	--	--
	07th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_07	16	119	16	135	6	07
		Prior1	S15 WIDA_WIDA_Overall_06	15	109	16	120	6	06
		Prior2	S14 WIDA_WIDA_Overall_05	14	99	16	105	6	05
		Prior3	NA	--	--	--	--	--	--
	08th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_08	16	119	16	135	6	08
		Prior1	S15 WIDA_WIDA_Overall_07	15	109	16	120	6	07
		Prior2	S14 WIDA_WIDA_Overall_06	14	99	16	105	6	06
		Prior3	NA	--	--	--	--	--	--
	09th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_09	16	119	16	135	6	09
		Prior1	S15 WIDA_WIDA_Overall_08	15	109	16	120	6	08
		Prior2	S14 WIDA_WIDA_Overall_07	14	99	16	105	6	07
		Prior3	NA	--	--	--	--	--	--
	10th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_10	16	119	16	135	6	10
		Prior1	S15 WIDA_WIDA_Overall_09	15	109	16	120	6	09
		Prior2	S14 WIDA_WIDA_Overall_08	14	99	16	105	6	08
		Prior3	NA	--	--	--	--	--	--
	11th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_11	16	119	16	135	6	11
		Prior1	S15 WIDA_WIDA_Overall_10	15	109	16	120	6	10
		Prior2	S14 WIDA_WIDA_Overall_09	14	99	16	105	6	09
		Prior3	NA	--	--	--	--	--	--
	12th Grade 15/16	PostTest	S16 WIDA_WIDA_Overall_12	16	119	16	135	6	12
		Prior1	S15 WIDA_WIDA_Overall_11	15	109	16	120	6	11
		Prior2	S14 WIDA_WIDA_Overall_10	14	99	16	105	6	10
		Prior3	NA	--	--	--	--	--	--

Minimum Number of Students

A minimum of 5,000 students will be required for the SGP M-STEP & SAT run.
A minimum of 1,000 students is preferred for the MI-Access FI PRR run.
A minimum of 2,000 students will be required for the SGP WIDA Access for ELLs 2.0 run.

Repeat Test Takers

Students who repeated the grade immediately before the posttest will not be included in either the SGP or the PRR analysis, thus the SGPs were not calculated for these students. For instance, if posttest score (Y_t) and prior 1 year score (Y_{t-1}) are with the same grade, the student is not included in the analysis and does not receive an IGP. Similarly, if the grade repeat happened in prior 2 years score (Y_{t-2}) and

prior 3 years score (Y_{t-3}), then the valid model case for that student is posttest score on prior 1 year score and prior 2 years score.

Skipped Grades

Students who skipped the grade immediately prior to the posttest will not be included in the analysis (i.e. 5th grade posttest following skipping 4th grade in the previous example.) In addition, if a student has a test sequence with a skipped grade, only the grade prior will be used to calculate the SGP.

Gaps in Test Sequence

Some students in the dataset are missing certain years of test scores. This may be due to student mobility, missed test windows, or other factors (e.g., Grade 3 MEAP Reading in Fall 2012, followed by Grade 5 M-STEP ELA in Spring 2015). Students with a gap will not be included unless they have a recent, valid sequence leading up to the posttest.

Home School and Private School Exclusion

All home schooled and private school test records will be excluded from computing SGP. MDE will ensure that students were previously tested as home schooled or at a private school are also excluded from the data pull.

Student Level Results

Student level results provided to MDE included:

1. Demographic and assessment information
2. SGPs
3. SGP standard errors (where applicable)
4. SGP Growth Level Code
5. Estimation Method
6. Prior achievement information used

Aggregation

Results were aggregated by assessment and accountability at the state, district, and school level using a variety of subgroups specified in the *Requirement Document for the Spring 2016 Individual Growth Percentiles and Aggregated Growth Percentiles.* Aggregation results included:

1. Average (arithmetic mean) of the SGPs
2. Standard deviation of SGPs
3. Count of students included
4. Count of students at each of five growth levels (Significant Improvement, Improvement, Maintain, Decline, Significant Decline)
5. Percentage of students at each of these five levels as a percentage of total students with SGPs
6. Count of students at each of three growth levels (Low, Medium, High)
7. Percentage of students at each of these three levels as a percentage of total students with SGPs.
8. Building z-score

Quality Control

DRC's psychometric team verified the data coming from MDE was as outlined in the *Requirements Document for the Spring 2016 Individual Growth Percentiles and Aggregated Growth Percentiles*. Any issues around unexpected data or missing fields were addressed by MDE.

To ensure that the proper growth model was used, base R code was written by the psychometrician and verified by two statistical analysts. The code for each subject was reviewed and SGP or PRR values were internally checked for reasonability. Two statistical analysts verified aggregate results by independent replication, and MDE reviewed the reasonability of the aggregate and individual SGP/PRR results. Results went through several iterations of independent replication and MDE review until all discrepancies were resolved.

Summary of Results

A total of 1,692,424 individual results were computed in this analysis. Of those, 1,675,950 were computed using SGPs and 16,474 were PRRs. Tables 5 through 9 provide a summary of the number of students and median growth SGPs or PRR values by aggregate levels. Tables 5 and 6 provide the summary of number of students and median growth (SGP or PRR) by testing program, calculation method, content area, and grade. Table 7 provides the results by calculation method, content area, and grade. Table 8 provides the results by content area and grade and Table 9 provides the results by grade. As expected with these methods, the median values tend to be near 50.

Table 5: Number of cases and median SGP by testing program, content area, and grade.

Testing Program	Method	Content Area	Grade	N	Median
M-STEP	SGP	English Language Arts	4	102,648	50
			5	101,914	50
			6	104,657	50
			7	103,532	49
			8	104,132	49
		Mathematics	4	102,623	50
			5	101,789	50
			6	104,560	49
			7	103,364	49
			8	104,083	49
		Science	7	100,703	50
			11	93,015	50
		Social Studies	8	101,441	49
			11	97,141	50
SAT	SGP	English Language Arts	11	91,670	50
		Mathematics	11	91,820	50
WIDA	SGP	WIDA	1	8,203	50
			2	9,043	50
			3	8,760	51
			4	6,967	51
			5	5,714	51
			6	5,316	52
			7	5,124	51
			8	4,944	51
			9	4,401	52
			10	3,374	50
			11	2,676	51
			12	2,336	51

Table 6: Number of cases and median PRR by testing program, content area, and grade.

Testing Program	Method	Content Area	Grade	N	Median
MI-Access	PRR	English Language Arts	4	1,088	50
			5	1,247	53
			6	1,288	52
			7	1,249	50
			8	1,305	50
			11	848	50
		Mathematics	4	1,128	50
			5	1,312	52
			6	1,394	51
			7	1,387	52
			8	1,490	53
			11	896	49.5
		Science	7	923	50
			11	919	50

Table 7: Number of cases and median growth by method, content area, and grade.

Method	Content Area	Grade	N	Median
PRR	English Language Arts	4	1,088	50
		5	1,247	53
		6	1,288	52
		7	1,249	50
		8	1,305	50
		11	848	50
	Mathematics	4	1,128	50
		5	1,312	52
		6	1,394	51
		7	1,387	52
		8	1,490	53
		11	896	49.5
	Science	7	923	50
		11	919	50
SGP	English Language Arts	4	102,648	50
		5	101,914	50
		6	104,657	50
		7	103,532	49
		8	104,132	49
		11	91,670	50
	Mathematics	4	102,623	50
		5	101,789	50
		6	104,560	49
		7	103,364	49
		8	104,083	49
		11	91,820	50
	Science	7	100,703	50
		11	93,015	50
	Social Studies	8	101,441	49
		11	97,141	50
	WIDA	1	8,203	50
		2	9,043	50
		3	8,760	51
		4	6,967	51
		5	5,714	51
		6	5,316	52
		7	5,124	51
		8	4,944	51
		9	4,401	52
		10	3,374	50
		11	2,676	51
		12	2,336	51

Table 8: Number of cases and median growth by content area and grade.

Content Area	Grade	N	Median
English Language Arts	4	103,736	50
	5	103,161	50
	6	105,945	50
	7	104,781	49
	8	105,437	49
	11	92,518	50
Mathematics	4	103,751	50
	5	103,101	50
	6	105,954	49
	7	104,751	49
	8	105,573	50
	11	92,716	50
Science	7	101,626	50
	11	93,934	50
Social Studies	8	101,441	49
	11	97,141	50
WIDA	1	8,203	50
	2	9,043	50
	3	8,760	51
	4	6,967	51
	5	5,714	51
	6	5,316	52
	7	5,124	51
	8	4,944	51
	9	4,401	52
	10	3,374	50
	11	2,676	51
	12	2,336	51

Table 9: Number of cases and median growth by content area and grade.

Grade	N	Median
1	8,203	50
2	9,043	50
3	8,760	51
4	214,454	50
5	211,976	50
6	217,215	50
7	316,282	50
8	317,395	49
9	4,401	52
10	3,374	50
11	378,985	50
12	2,336	51

Goodness of Fit

To examine the fit of the growth models, the correlations between the outcome score (2016) and the prior achievement score was calculated. Tables 10 and 11 provide the correlations by program, content area, and grade. All correlations are acceptable and within the moderate range. For the M-STEP program, all correlations are consistent within content area. In Mathematics and English Language Arts, correlations are above 0.80, for Science and Social Studies they are above 0.70. With the SAT correlations are slightly lower; 0.72 for English Language Arts and 0.81 for Mathematics. WIDA correlations are fairly consistent but lower, ranging from 0.59 to 0.71. Finally, the correlations for MI-Access are consistent within content area but lower ranging from 0.52 to 0.61 for English Language Arts, from 0.53 to 0.61 for Mathematics, and 0.51 to 0.52 for Science.

Table 10: Correlation between current SS and prior SS by testing program, content area, and grade for SGP models.

Testing Program	Method	Content Area	Grade	N	Correlation
M-STEP	SGP	English Language Arts	4	102,648	0.81
			5	101,914	0.82
			6	104,657	0.82
			7	103,532	0.82
			8	104,132	0.84
		Mathematics	4	102,623	0.82
			5	101,789	0.83
			6	104,560	0.83
			7	103,364	0.81
			8	104,083	0.82
		Science	7	100,703	0.76
			11	93,015	0.73
		Social Studies	8	101,441	0.74
			11	97,141	0.75
SAT	SGP	English Language Arts	11	91,670	0.72
		Mathematics	11	91,820	0.81
WIDA	SGP	WIDA	1	8,203	0.65
			2	9,043	0.64
			3	8,760	0.64
			4	6,967	0.59
			5	5,714	0.64
			6	5,316	0.59
			7	5,124	0.68
			8	4,944	0.67
			9	4,401	0.70
			10	3,374	0.71
			11	2,676	0.66
			12	2,336	0.60

Table 11: Correlation between current SS and prior SS by testing program, content area, and grade for PRR model.

Testing Program	Method	Content Area	Grade	N	Correlation
MI-Access	PRR	English Language Arts	4	1,088	0.55
			5	1,247	0.52
			6	1,288	0.53
			7	1,249	0.59
			8	1,305	0.61
			11	848	0.49
		Mathematics	4	1,128	0.53
			5	1,312	0.58
			6	1,394	0.57
			7	1,387	0.61
			8	1,490	0.62
			11	896	0.54
		Science	7	923	0.51
			11	919	0.52

Distributions of SGPs and PRRs

The distributions of SGPs and PRRs are provided in Figure 1 through Figure 3, which shows that SGPs tend to uniformly range from 1 to 99. While the PRRs also range from 1 to 99, they are a bit less stable due to the small sample sizes used in the calculations. It should be noted that the differences in distributions of PRRs and SGPs across grade and content area tend to be relatively small given the scale of the density plots range from 0 to 0.012.

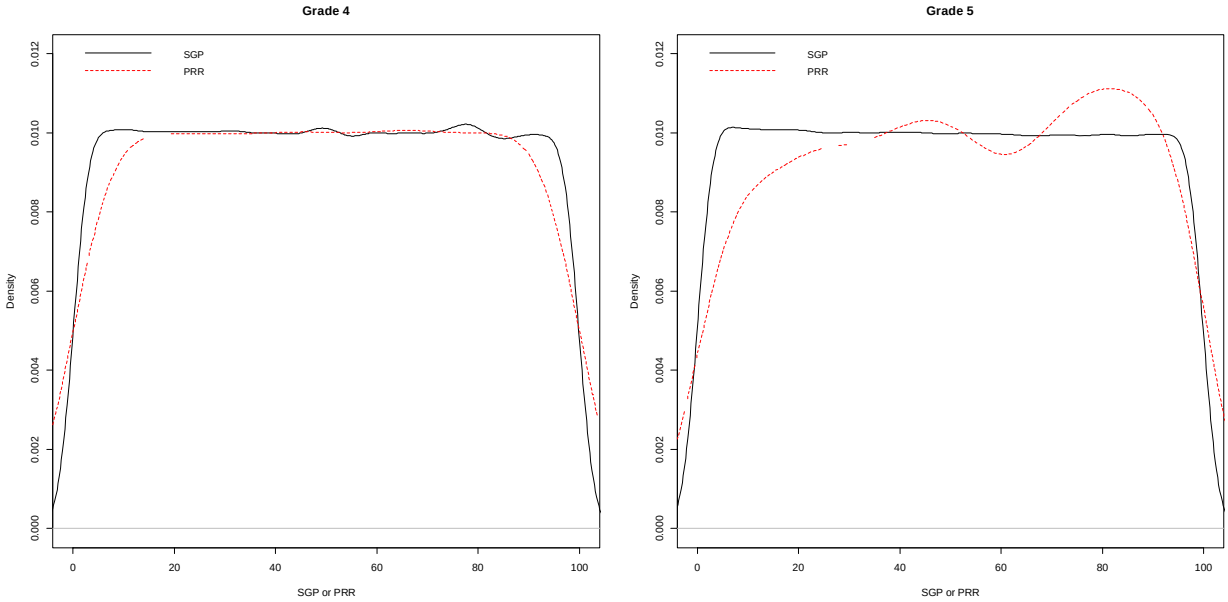

Figure 1. Distribution of SGP/PRR for Mathematics Grades, 4 and 5.

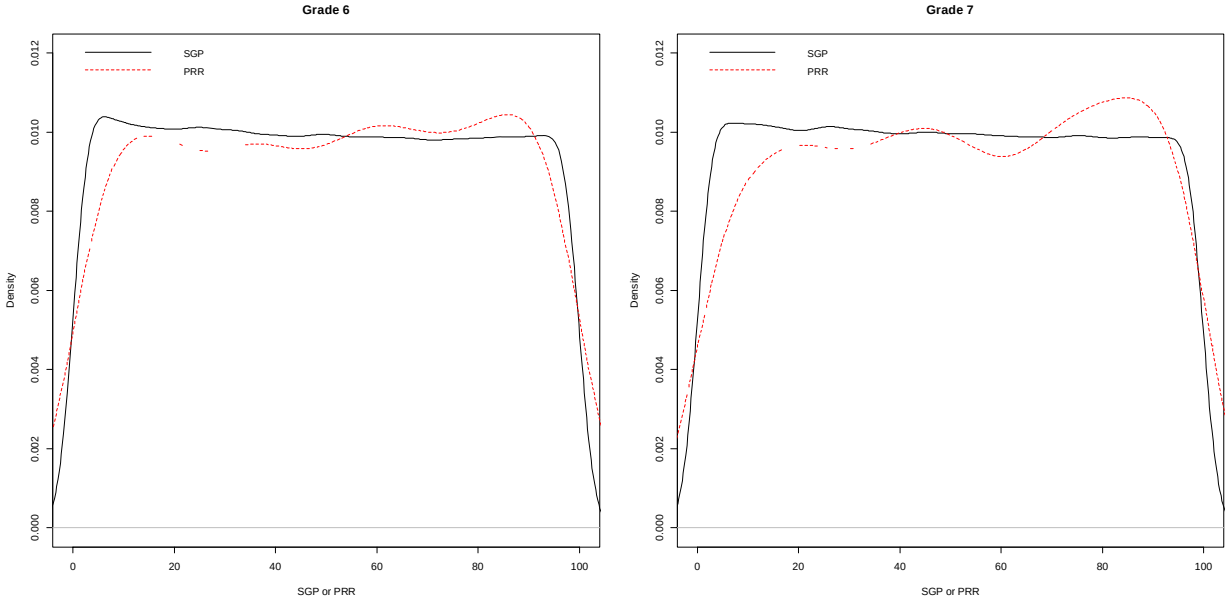

Figure 2. Distribution of SGP/PRR for Mathematics Grades, 6 and 7.

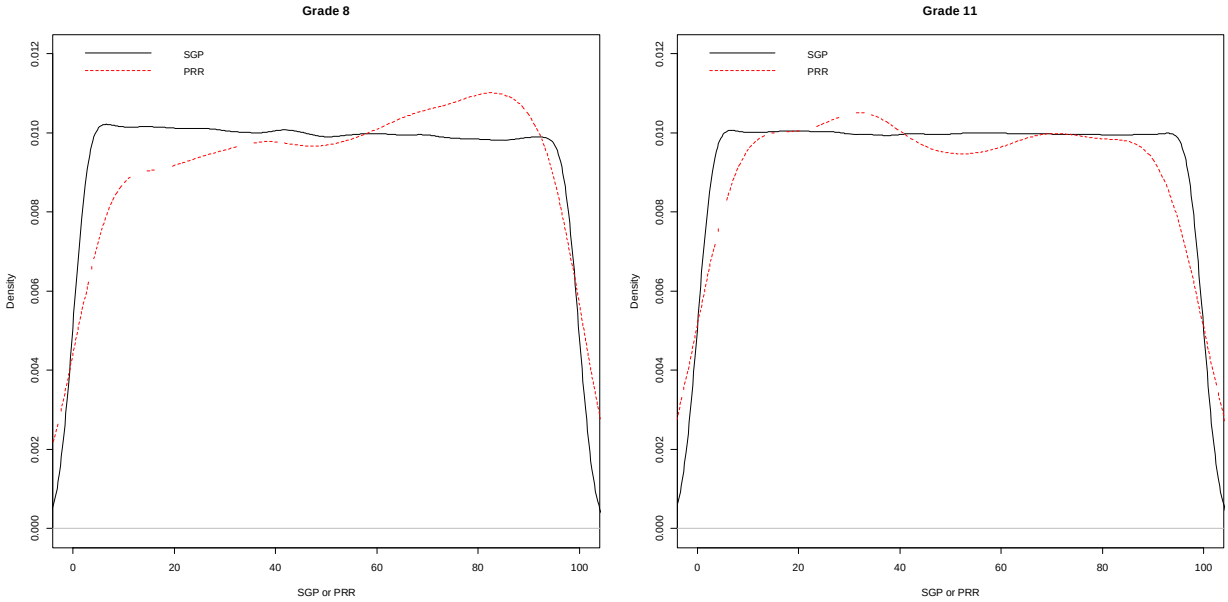

Figure 3. Distribution of SGP/PRR for Mathematics Grades, 8 and 11.

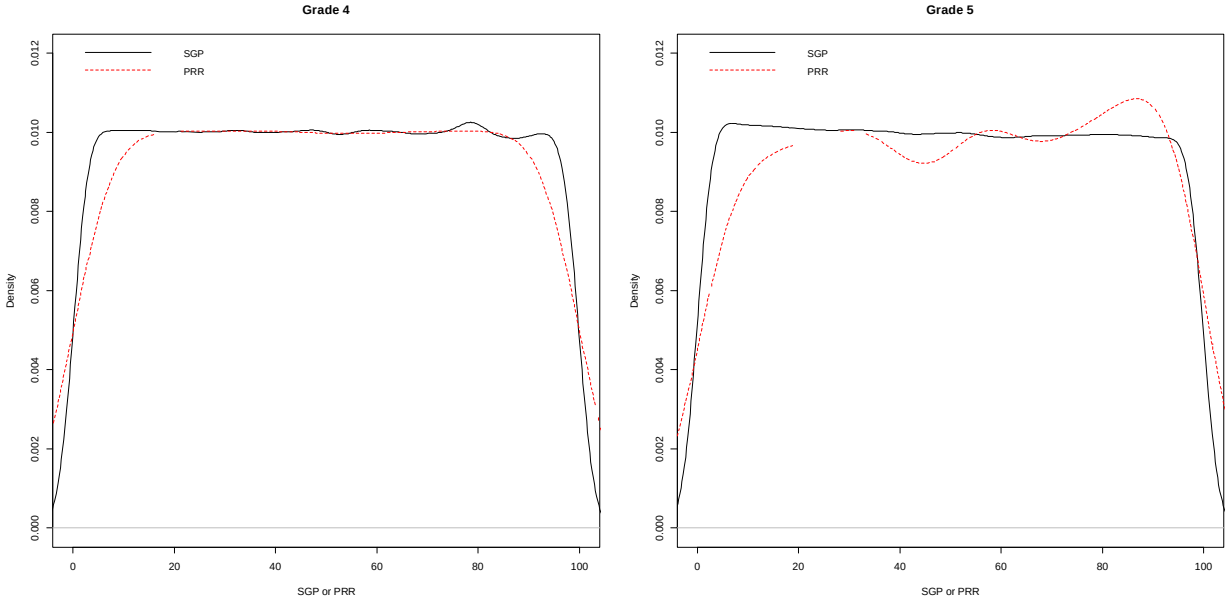

Figure 4. Distribution of SGP/PRR for English Language Arts Grades, 4 and 5.

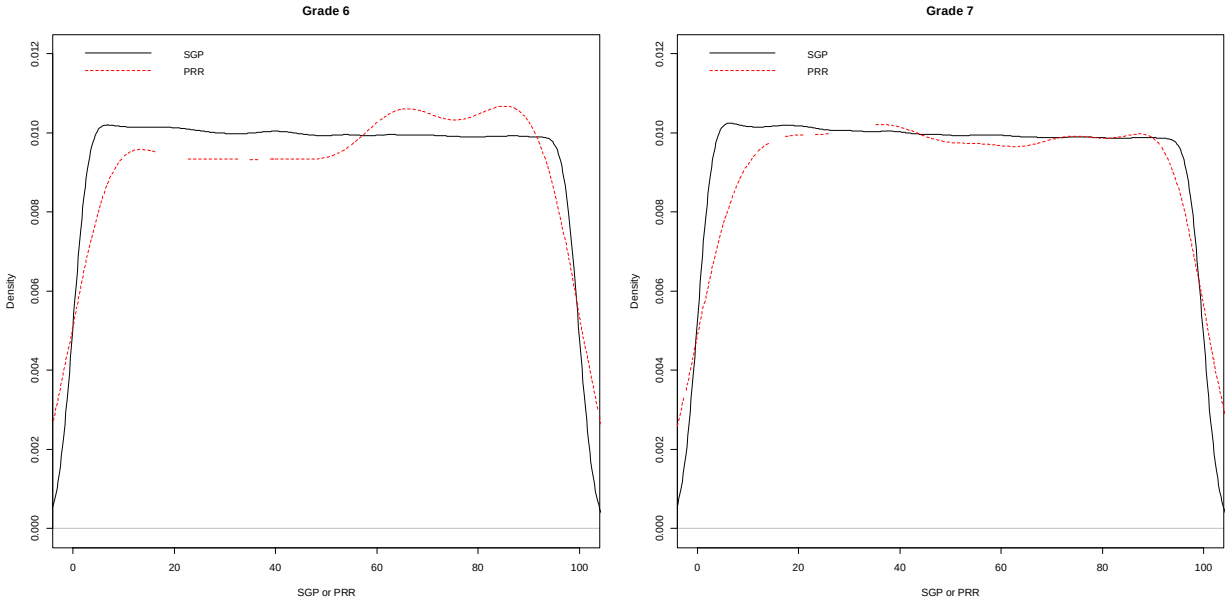

Figure 5. Distribution of SGP/PRR for English Language Arts Grades, 6 and 7.

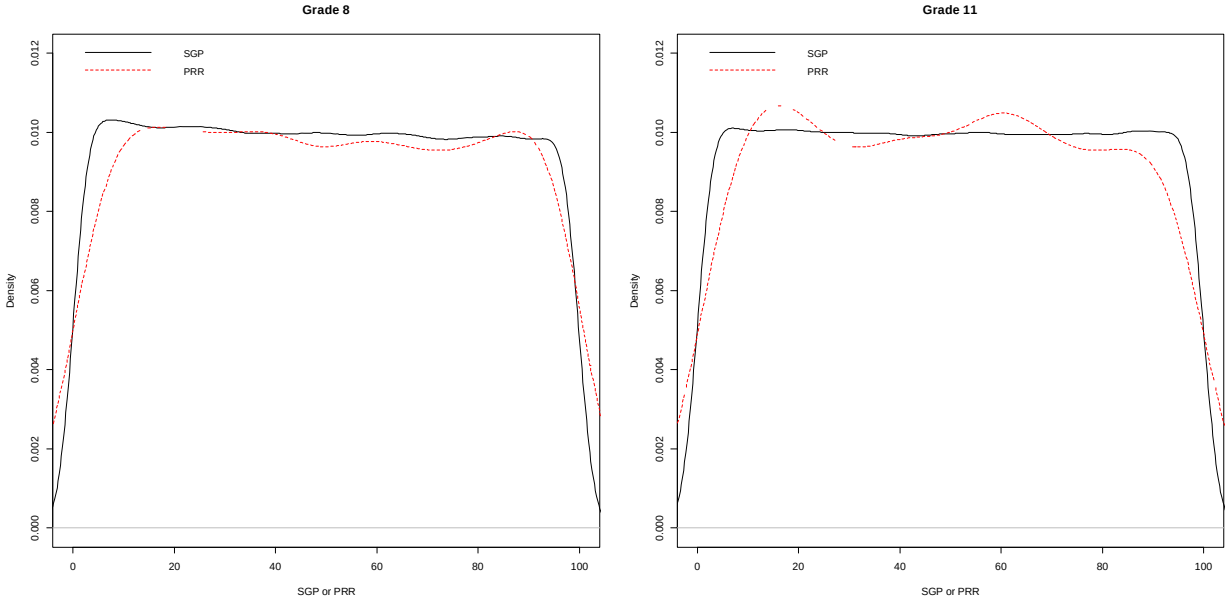

Figure 6. Distribution of SGP/PRR for English Language Arts Grades, 8 and 11.

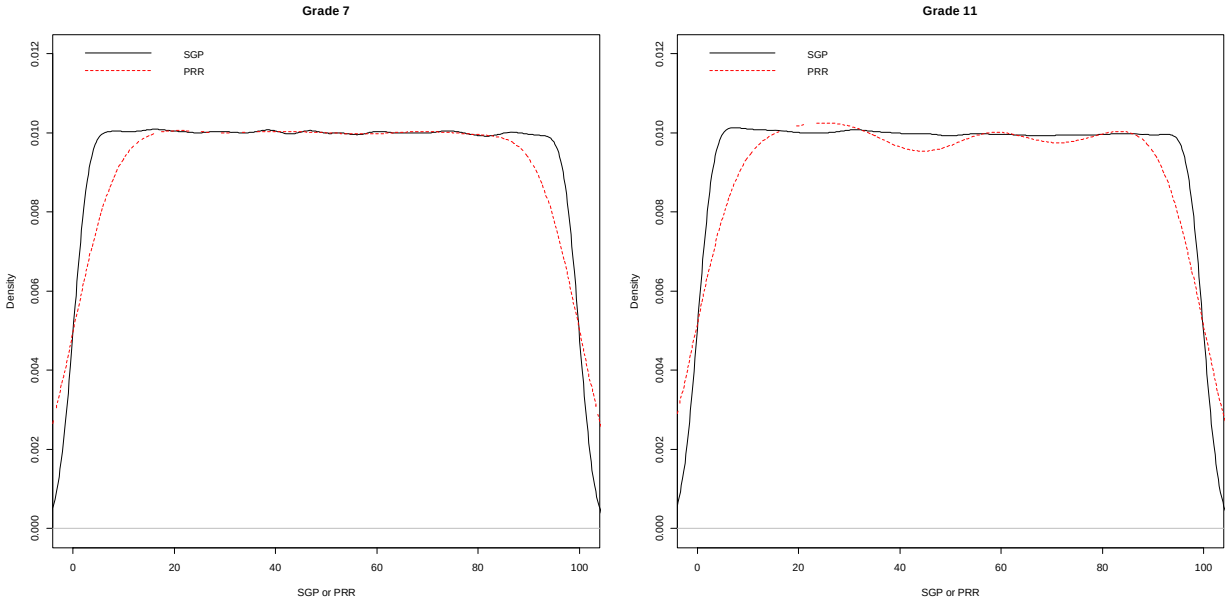

Figure 7. Distribution of SGP/PRR for Science, Grades 7 and 11.

Checks for Neutrality

Since the growth models used in this analysis do not control for demographic variables, particularly those that may have some impact on student growth rates and trajectories, it is unknown whether the results are biased, especially when aggregated at the school or district level (Education Analytics, 2015). Thus it is important to look at the relationship between the aggregated growth measure, in this case median SGP and the variables of interest that were not controlled for in the growth models. It is important to note that it is unknown what the correlations "should be." Tables 10 and 11 provide the correlations between the median SGP for a school or a district (with more than 20 students) related to the percentage of each demographic for that building or district. Graphs of these relationships can be found in the appendix.

Table 10: Correlations between Median SGP and Demographic at the school level.

Content Area	ED	SE	LEP	Non-White
English Language Arts	-0.41	-0.15	-0.01	-0.29
Mathematics	-0.45	-0.18	-0.03	-0.33
Science	-0.46	-0.21	-0.07	-0.45
Social Studies	-0.41	-0.23	-0.01	-0.33
WIDA	-0.44	-0.12		-0.06

Table 11: Correlations between Median SGP and Demographic at the district level.

Content Area	ED	SE	LEP	Non-White
English Language Arts	-0.36	-0.16	0.00	-0.17
Mathematics	-0.44	-0.20	-0.03	-0.33
Science	-0.43	-0.17	-0.04	-0.38
Social Studies	-0.34	-0.22	0.10	-0.24
WIDA	-0.46	-0.15		-0.09

When aggregating growth model outcomes, it is also important to note that growth model, as with most regression models, have issues (more variability or less precision) when sample sizes are small. This is also true when aggregating growth model results at the school level. There is more noise or variability. Figure 8 provides the relationship between the number of students and SGP. This shows that there is less variability in median SGP as the number of students increase.

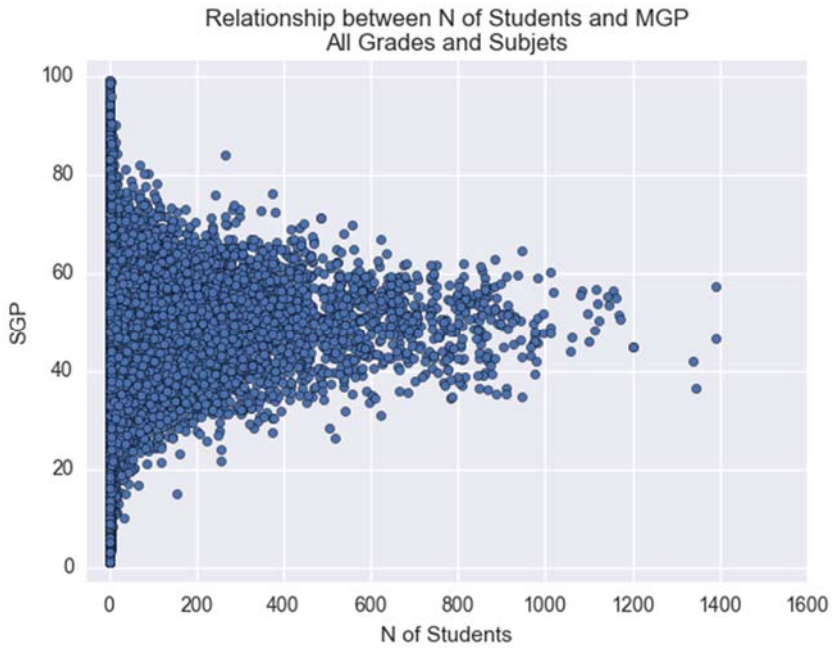

Figure 8. Number of Students versus SGP

References

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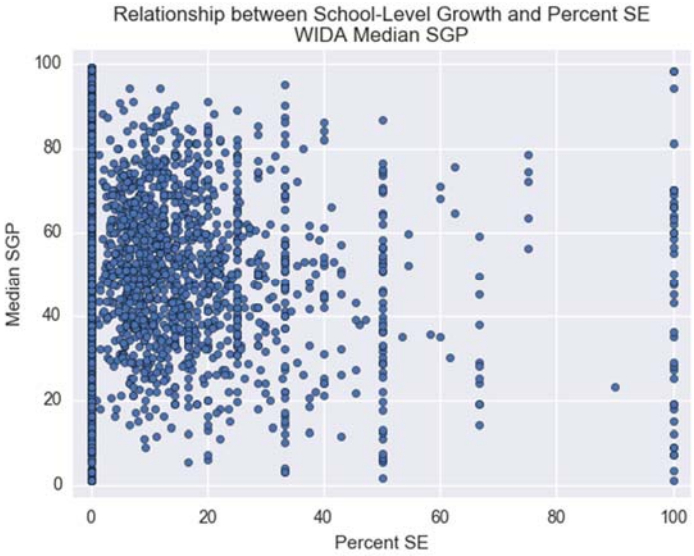
Castellano, K.E., and Ho, A.D. (2015). Practical Differences Among Aggregate-Level Conditional Status Metrics: From Median Student Growth Percentiles to Value-Added Models. Journal of Educational and Behavioral Statistics, 40(1), 35-68. doi:10.3102/107699861454848

Castellano, K.E., and Ho, A.D. (2013). Contrasting OLS and Quantile Regression Approaches to Student "Growth" Percentiles. Journal of Educational and Behavioral Statistics, 38(2), 190-215. doi:10.3102/1076998611435413

Education Analytics (2015). Michigan Department of Education Technical Report.

Appendix


Relationship between School-Level Growth and Percent SE
WIDA Median SGP
Median SGP
Percent SE
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20
40
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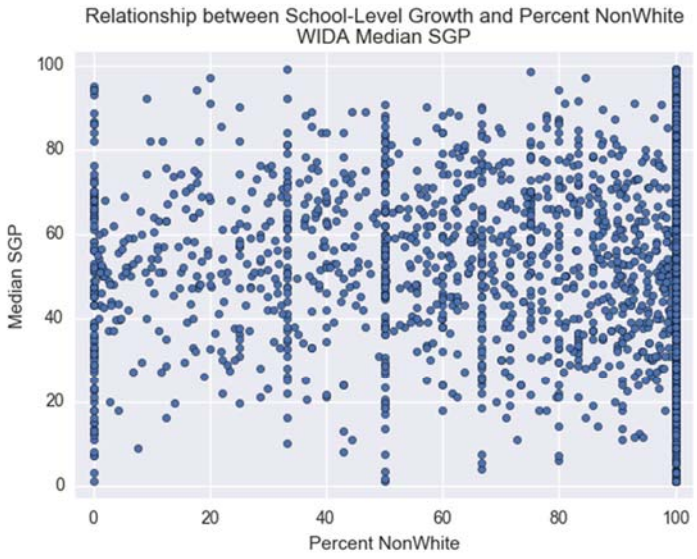
Relationship between School-Level Growth and Percent NonWhite
WIDA Median SGP
Median SGP
Percent NonWhite
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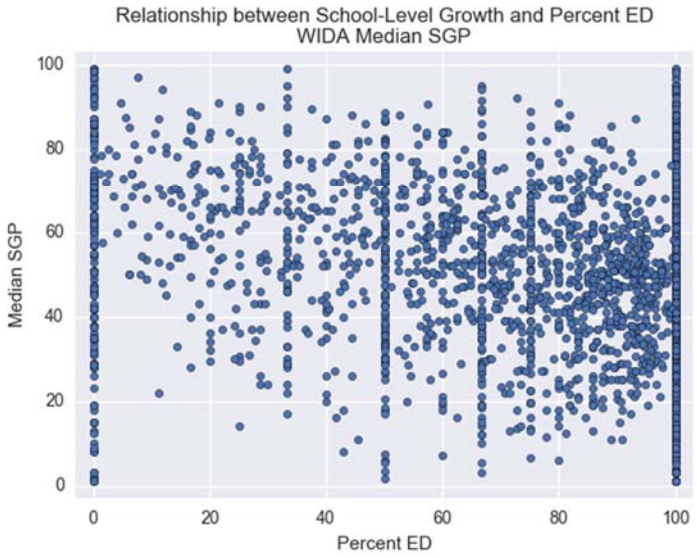
Relationship between School-Level Growth and Percent ED
WIDA Median SGP
Median SGP
Percent ED
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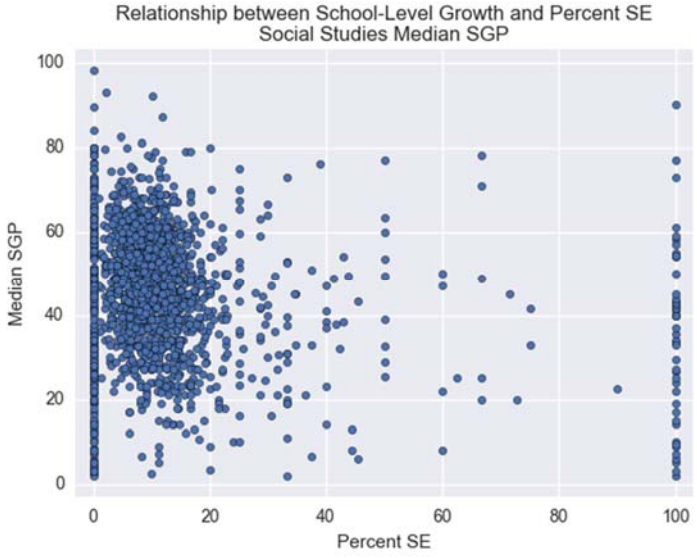
Relationship between School-Level Growth and Percent SE
Social Studies Median SGP
Median SGP
Percent SE
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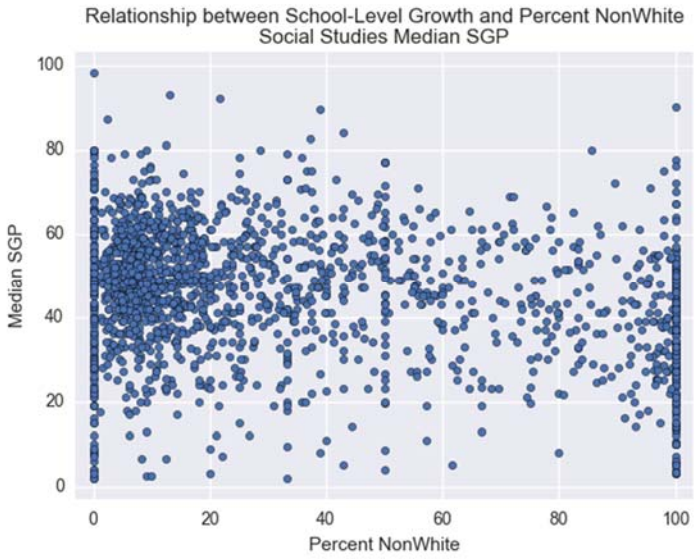
Relationship between School-Level Growth and Percent NonWhite
Social Studies Median SGP
Median SGP
Percent NonWhite
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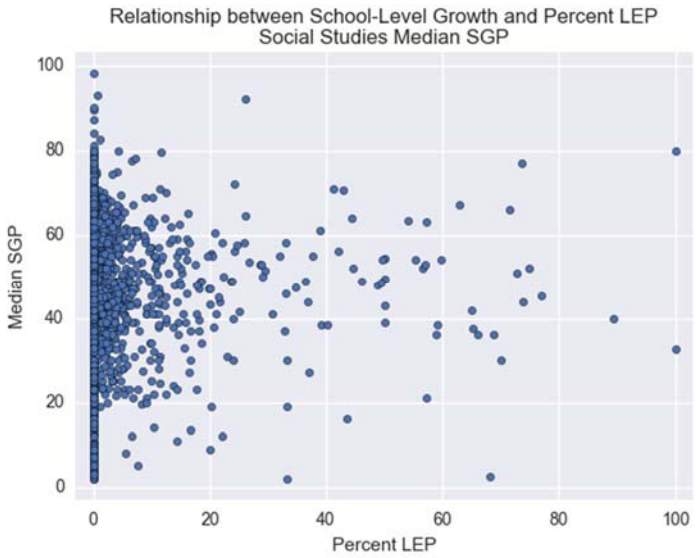
Relationship between School-Level Growth and Percent LEP
Social Studies Median SGP
Median SGP
Percent LEP
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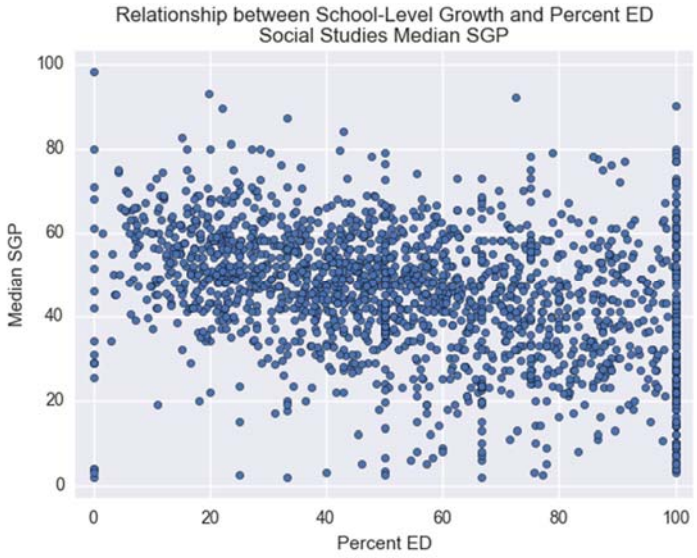
Relationship between School-Level Growth and Percent ED
Social Studies Median SGP
Median SGP
Percent ED
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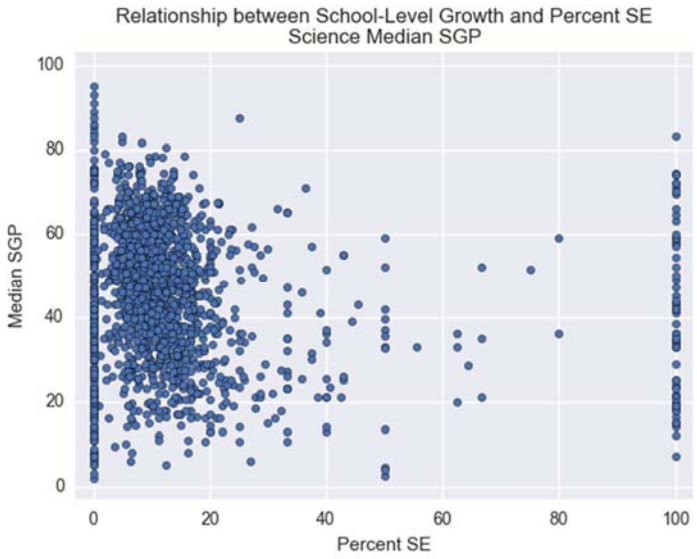
Relationship between School-Level Growth and Percent SE
Science Median SGP
Median SGP
Percent SE
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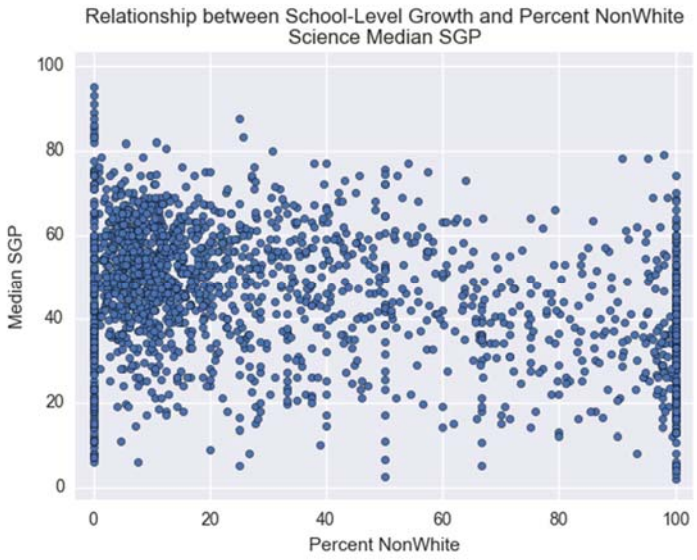
Relationship between School-Level Growth and Percent NonWhite
Science Median SGP
Median SGP
Percent NonWhite
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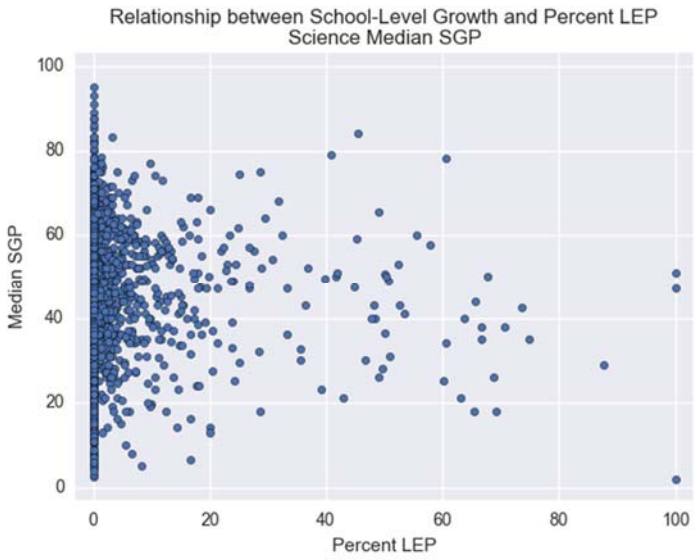
Relationship between School-Level Growth and Percent LEP
Science Median SGP
Median SGP
Percent LEP
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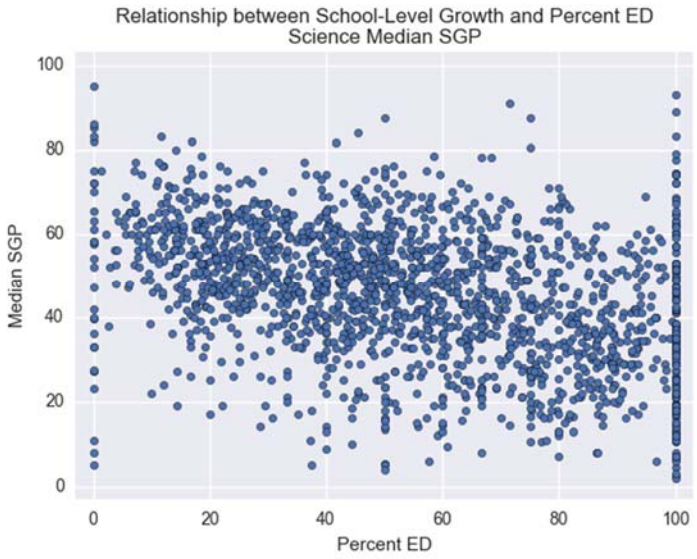
Relationship between School-Level Growth and Percent ED
Science Median SGP
Median SGP
Percent ED
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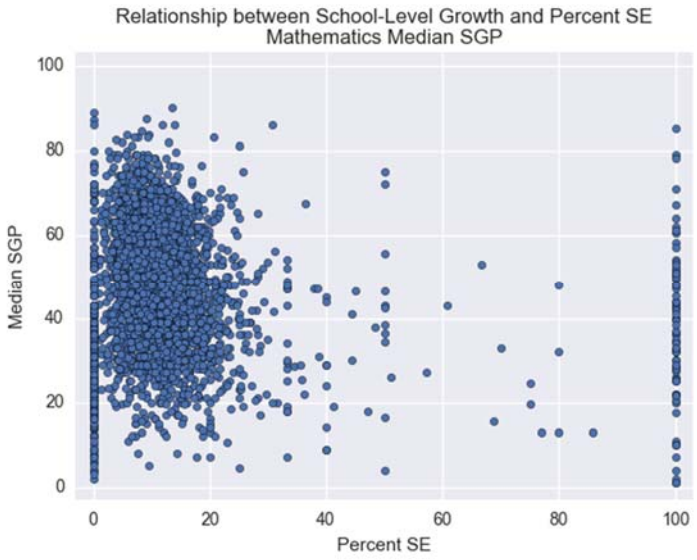
Relationship between School-Level Growth and Percent SE
Mathematics Median SGP
Median SGP
Percent SE
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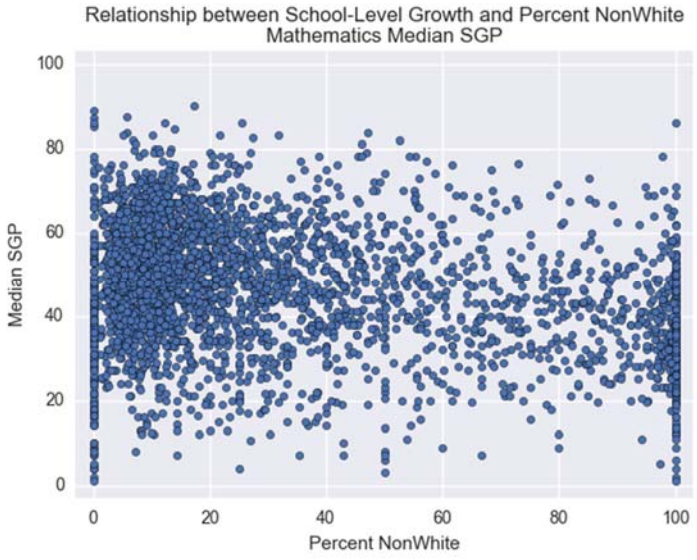
Relationship between School-Level Growth and Percent NonWhite
Mathematics Median SGP
Median SGP
Percent NonWhite
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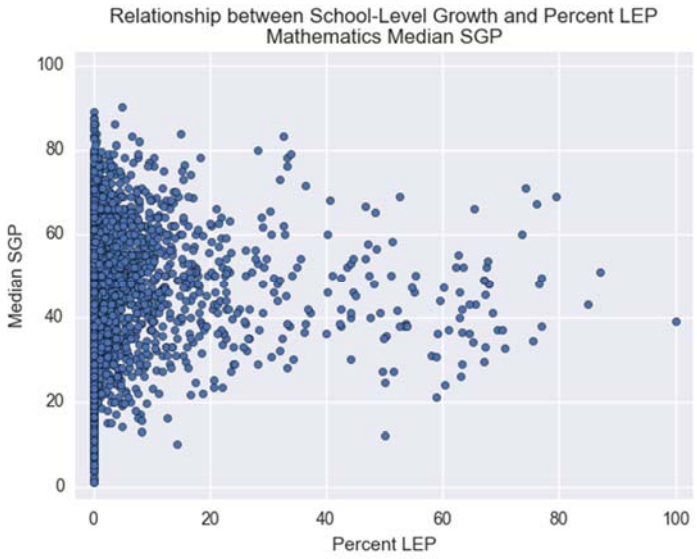
Relationship between School-Level Growth and Percent LEP
Mathematics Median SGP
Median SGP
Percent LEP
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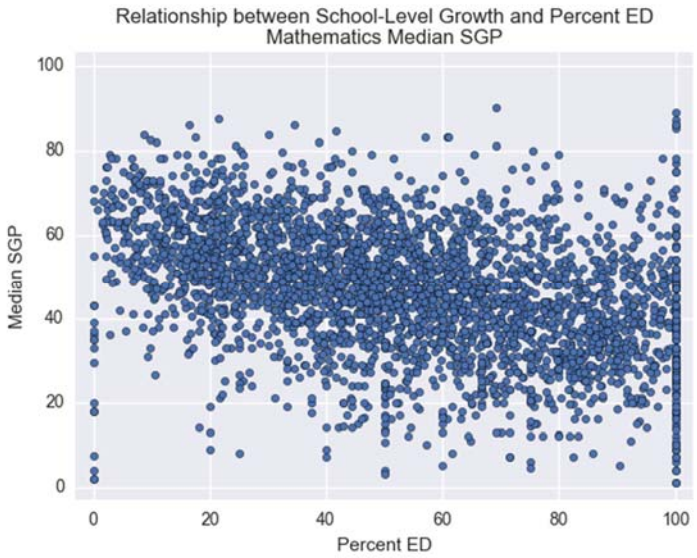
Relationship between School-Level Growth and Percent ED
Mathematics Median SGP
Median SGP
Percent ED
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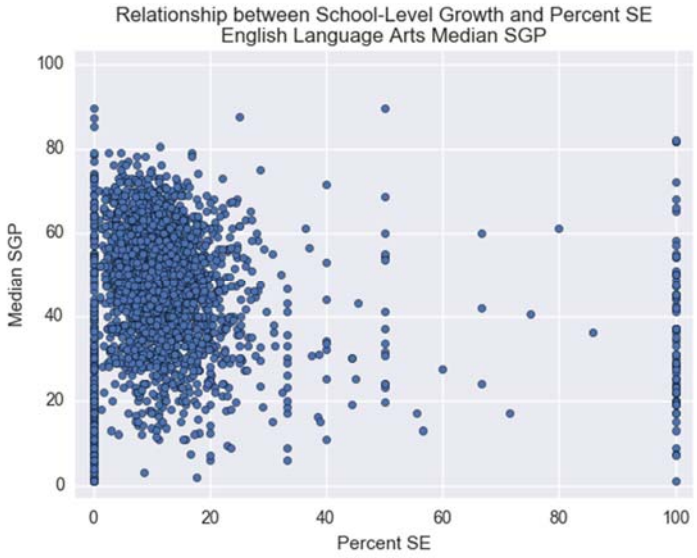
Relationship between School-Level Growth and Percent SE
English Language Arts Median SGP
Median SGP
Percent SE
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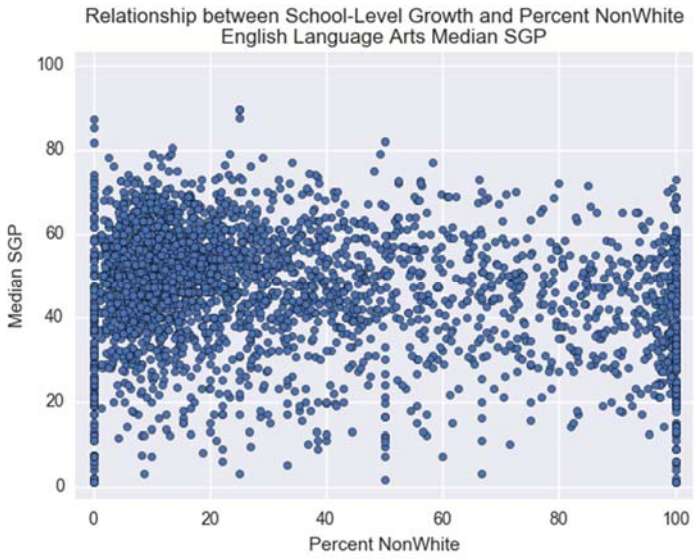
Relationship between School-Level Growth and Percent NonWhite
English Language Arts Median SGP
Median SGP
Percent NonWhite
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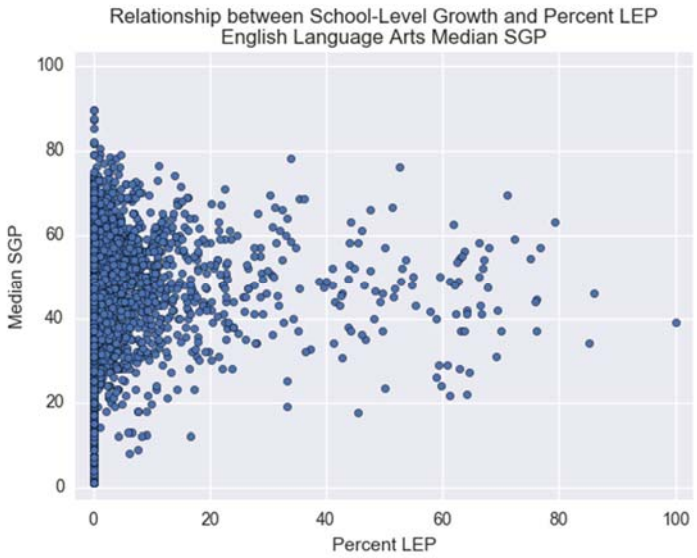
Relationship between School-Level Growth and Percent LEP
English Language Arts Median SGP
Median SGP
Percent LEP
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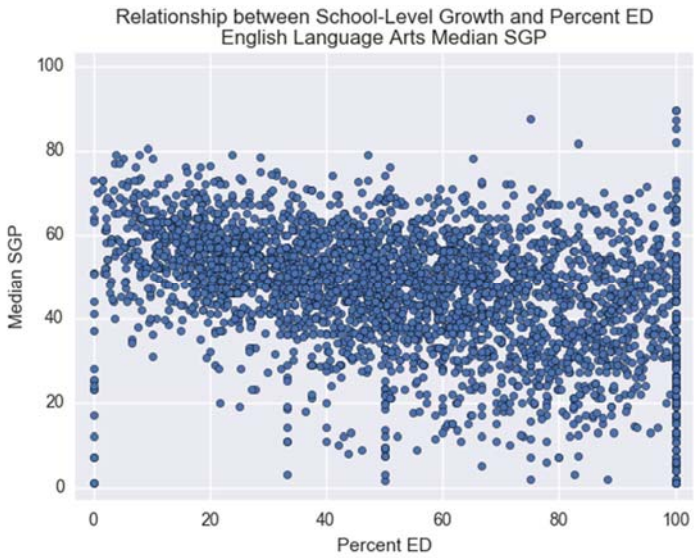
80
100


Relationship between School-Level Growth and Percent ED
English Language Arts Median SGP
Median SGP
Percent ED
0
20
40
60
80
100