

Appendix 4-B

Open Channel Theory

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4.B.1 Open Channel Flow Theory

Design analysis of natural and artificial channels relies on the basic principles of open channel flow (refer to Chow, 1970 and Henderson, 1966). The fluid mechanics principles of continuity, momentum, and energy can be applied to open channel flow, but the position of the free surface is usually an unknown variable. Determining this unknown is one of the challenges of open channel flow analysis because it requires quantification of the flow resistance. Natural channels display a wider range of roughness values than artificial channels.

4.B.2 Concepts

4.B.2.1. Specific Energy

Specific energy, E , is defined as the energy head relative to the channel bottom. If the channel is not too steep (slope less than 10%) and the streamlines are nearly straight and parallel (so that the hydrostatic assumption holds), E becomes the sum of the depth and velocity head:

$$E = y + \alpha \left(\frac{V^2}{2g} \right) \quad (4.B.1)$$

Where:

- y = depth, ft
- α = velocity distribution coefficient (refer to **Equation 4.B.2**)
- V = mean velocity, fps
- g = gravitational acceleration, 32.2 ft/s²

The velocity distribution coefficient assumes a value of 1 for turbulent flow in prismatic channels but may be significantly different than 1 in natural channels (refer to **Equation 4.B.2**). A prismatic channel is a channel of constant cross-section, roughness, and slope in the flow direction.

4.B.2.2 Velocity Distribution Coefficient

Due to the presence of a free surface and friction along the channel boundary, the velocities in a channel are not uniformly distributed in the channel section. As a result, the velocity head of an open channel is greater than the average velocity head computed as $((Q/A_t)^2)/(2g)$. A weighted average value of the velocity head is obtained by multiplying the average velocity head by a velocity distribution coefficient, α , defined as:

$$\alpha = \sum_{i=1}^n \left(\frac{K_i^3}{A_i^2} \right) / \left(\frac{K_t^3}{A_t^2} \right) \quad (4.B.2)$$

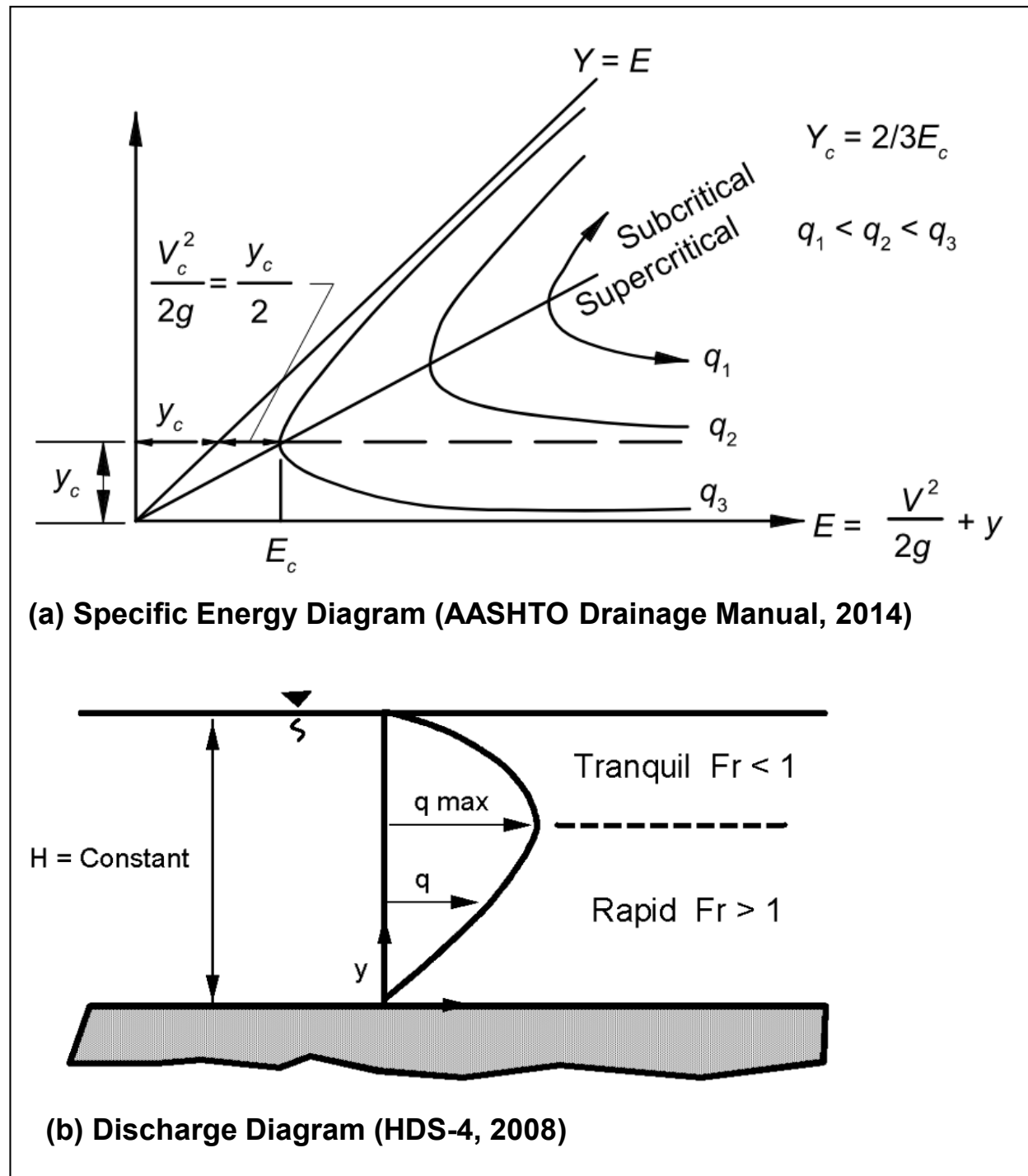
Where:

- K_i = conveyance in subsection (refer to **Equation 4.B.15**)
- K_t = total conveyance in section (refer to **Equation 4.B.15**)
- A_i = cross-sectional area of subsection, sft
- A_t = total cross-sectional area of section, sft
- n = number of subsections

4.B.2.3 Total Energy Head

The total energy head is the specific energy head plus the elevation of the channel bottom with respect to some datum. The difference in elevation of the energy head from one cross-section to the next cross section defines the energy grade line. Refer to **Figure 4-B-1** for a plot of the specific energy diagram.

Figure 4-B-1. Specific Energy and Discharge Diagram for Rectangular Channels



4.B.2.4 Steady and Unsteady Flow

Steady flow occurs when the discharge passing a given cross-section is constant with respect to time. Steady flow in any reach requires that the rates of inflow and outflow be constant and equal. When the discharge varies with time, the flow is unsteady.

4.B.2.5 Nonuniform Flow and Uniform Flow

Nonuniform flow occurs when the velocity and depth vary in the direction of motion, while uniform flow remains constant. Uniform flow can only occur in a prismatic channel. Nonuniform flow can occur either in a prismatic channel or in a natural channel with variable properties.

4.B.2.6 Gradually Varied and Rapidly Varied

A nonuniform flow, in which the depth and velocity change gradually enough in the flow direction that vertical accelerations can be neglected, is referred to as a gradually varied flow; otherwise, it is considered to be rapidly varied flow.

4.B.2.7 Froude Number

The Froude number, F_r , is an important dimensionless parameter in open channel flow. It represents the ratio of inertial forces to gravitational forces and is defined by:

$$F_r = \frac{V}{\left(\frac{g * d}{\alpha}\right)^{0.5}} \quad (4.B.3)$$

Where:

- α = velocity distribution coefficient
- V = mean velocity = Q/A , fps
- g = acceleration of gravity, 32.2 ft/s²
- d = hydraulic depth = A/T , ft
- A = cross-sectional area of flow, sft
- T = channel top width at the water surface, ft
- Q = total discharge, cfs

This expression for Froude number applies to any normal depth channel. For rectangular channels, the hydraulic depth is equal to the flow depth.

4.B.2.8 Critical Flow

Critical flow occurs when the specific energy is at a minimum. The variation of specific energy with depth at a constant discharge shows a minimum in the specific energy at a critical depth in which the Froude number has a value of 1. Critical depth is also the depth of maximum discharge when the specific energy is held constant. These

relationships are illustrated in **Figure 4-B-1**. During critical flow, the velocity head is equal to half the hydraulic depth. The general expression for flow at critical depth is:

$$\alpha \frac{Q^2}{g} = \frac{A^3}{T} \quad (4.B.4)$$

Where:

- α = velocity distribution coefficient
- Q = total discharge, cfs
- g = gravitational acceleration, 32.2 ft/s²
- A = cross-sectional area of flow, sft
- T = channel top width at the water surface, ft

When flow is at critical depth, **Equation 4.B.4** must be satisfied, no matter what the shape of the channel.

4.B.2.9 Subcritical Flow

Depths greater than critical depth occur in subcritical flow and the Froude number is less than 1. In this state of flow, small water surface disturbances can travel both upstream and downstream, and the hydraulic control is located downstream.

4.B.2.10 Supercritical Flow

Depths less than critical depth occur in supercritical flow and the Froude number is greater than 1. Small water surface disturbances travel downstream in supercritical flow and the location of the hydraulic control is upstream.

4.B.2.11 Hydraulic Jump

A hydraulic jump occurs as an abrupt transition from supercritical to subcritical flow. There are significant changes in the depth and velocity at the hydraulic jump as energy is dissipated. For this reason, the hydraulic jump is often employed under controlled situations and facilities to dissipate energy and control erosion at highway drainage structures.

A hydraulic jump will not occur until the ratio of the flow depth, y_1 , in the approach channel to the flow depth y_2 in the downstream channel reaches a specific value which depends on the channel geometry. The depth before the jump is called the initial depth, y_1 , and the depth after the jump is the conjugate depth, y_2 . When a hydraulic jump is used as an energy dissipater, controls to create sufficient tailwater depth are often necessary to control the location of the jump and to ensure that a jump will occur during the desired range of discharges. Sills can be used to control a hydraulic jump if the tailwater depth is less than the conjugate depth. A sharp crested weir, broad crested weir, or an abrupt rise and drop in channel floor are examples of a sill. If the tailwater depth is higher than the conjugate depth, a drop in the channel floor is used to ensure a jump.

4.B.2.12 Velocity Distribution

In stream channels, the transverse variation of velocity in any cross-section is a function of subsection geometry and roughness and may vary considerably from one stage and discharge to another. It is important to know this variation for purposes of designing erosion control measures and locating relief openings in highway fills for example. Current meter measurements are the best method to determine transverse velocity variations. If this is not possible, the normal depth method can be used by dividing the cross-section into subsections of relatively uniform roughness and geometry. It is assumed that the energy grade line slope is the same across the cross-section so that the total conveyance K_t of the cross-section is the sum of the subsection conveyances. The total discharge is $K_t S^{1/2}$ and the discharge in each subsection is proportional to its conveyance. The velocity in each subsection is obtained from the continuity equation, $V = Q/A$.

4.B.2.13 Standard Step Method

The following discussion explains the theory behind the standard step method, beginning with the energy equation. The energy equation is:

$$h_1 + \alpha_1 \left(\frac{V_1^2}{2g} \right) = h_2 + \alpha_2 \left(\frac{V_2^2}{2g} \right) + h_L \quad (4.B.5)$$

Where:

- h_1 and h_2 are the upstream and downstream stages, respectively, ft
- α = velocity distribution coefficient
- V = mean velocity, fps
- g = acceleration due to gravity, ft/s²
- h_L = head loss due to local cross-sectional changes (minor loss) and boundary resistance, ft

The stage, h , is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y , i.e., $h = z + y$. The energy equation is solved between successive stream reaches with nearly uniform roughness, friction slope, and cross-sectional properties.

The total head loss is calculated from:

$$h_L = K_m \left[\left(\frac{\alpha_1 V_1^2}{2g} \right) - \left(\frac{\alpha_2 V_2^2}{2g} \right) \right] + S_{f(avg)} L \quad (4.B.6)$$

Where:

- K_m = expansion or contraction loss coefficient
- $S_{f(avg)}$ = average friction slope, ft/ft
- L = discharge-weighted or conveyance-weighted reach length, ft

The standard step method solves these equations from cross-section to cross-section.

Guidance on the selection of expansion and contraction loss coefficients for bridges is included in the USACE HEC-RAS Hydraulic Reference Manual. Guidance on the design of transitions for culverts and the associated expansion and contraction loss coefficients is included in FHWA HEC-14 (2006).

The default values of the minor loss coefficient, K_m , are 0.1 for contractions and 0.3 for expansions in HEC-RAS. The range of these coefficients, from ideal transitions to abrupt changes, are 0.1 to 0.5 for contractions and 0.3 to 1.0 for expansions, respectively.

HEC-RAS calculates a discharge-weighted reach length, L , as:

$$L = \frac{L_{lob}Q_{lob(avg)} + L_{ch}Q_{ch(avg)} + L_{rob}Q_{rob(avg)}}{Q_{lob(avg)} + Q_{ch(avg)} + Q_{rob(avg)}} \quad (4.B.7)$$

Where:

L_{lob} , L_{ch} , L_{rob} = flow distance between cross-sections in the left overbank, main channel and right overbank, respectively, ft

$Q_{lob(avg)}$, $Q_{ch(avg)}$, $Q_{rob(avg)}$ = arithmetic average of flows between cross-section for the left overbank, main channel and right overbank, respectively, cfs

HEC-RAS allows the user the following options for determining the average friction slope, $S_{f(avg)}$:

- Average conveyance equation

$$S_{f(avg)} = \left(\frac{Q_u + Q_d}{K_u + K_d} \right)^2 \quad (4.B.8)$$

- Average friction slope equation

$$S_{f(avg)} = \frac{S_{fu} + S_{fd}}{2} \quad (4.B.9)$$

- Geometric mean friction slope equation

$$S_{f(avg)} = \sqrt{S_{fu}S_{fd}} \quad (4.B.10)$$

- Harmonic mean friction slope equation

$$S_{f(avg)} = \frac{2S_{fu}S_{fd}}{S_{fu} + S_{fd}} \quad (4.B.11)$$

Where:

- Q_u, Q_d = discharge at the upstream and downstream cross-sections, respectively, cfs
 K_u, K_d = conveyance at the upstream and downstream cross-sections, respectively
 S_{fu}, S_{fd} = friction slope at the upstream and downstream cross-sections, respectively, ft/ft

The default option is the average conveyance equation in HEC-RAS.

4.B.2.14 Standard Step Computation Worksheet

A sample standard step computation worksheet is used to calculate water surface profiles (taken from *"Hydrologic Engineering Methods for Water Resources Development - Volume 6, Water Surface Profiles,"* The Hydrologic Engineering Center, USACE, Davis, CA). The worksheet, as shown in **Figure 4-B-2**, follows the concepts presented in **Section 4.4.3.7**. Because of the large amount of calculation required, most designers will choose to use the HEC-RAS software.

In the worksheet, Columns 2 and 4 through 12 solve Manning's equation to obtain the energy loss due to friction. Columns 13 and 14 calculate the velocity distribution across the section. Columns 15 through 17 calculate the average kinetic energy. Column 18 calculates "other losses" (expansion and contraction losses). Column 19 calculates the change in water surface elevation. Proceeding from cross-section to cross-section down the computation form accounts for the conservation of energy.

Column 1 CROSS-SECTION NO. is the cross-section identification number.

Column 2 ASSUMED is the water surface elevation which must agree with the resulting computed water surface elevation within ± 0.5 inch, or some allowable tolerance, for trial calculations to be successful.

Column 3 COMPUTED is the rating curve value for the first section, but thereafter is the value calculated by adding WS to the computed water surface elevation for the previous cross-section.

Column 4 A is the cross-section area. If the section is complex and has been subdivided into several parts (e.g., left overbank, channel, and right overbank) use one line of the form for each sub-section and sum to get A_t , the total area of cross-section.

- Column 5** R is the hydraulic radius. Use the same procedure as for Column 4 if the section is complex, but do not sum subsection values.
- Column 6** $R^{2/3}$ is 2/3 power of hydraulic radius.
- Column 7** n is Manning's roughness coefficient.
- Column 8** K is conveyance and defined as $(CAR^{2/3} / n)$ where C is 1.49 for English units (C is 1.0 for Metric units). If the cross-section is complex, sum subsection K values to get K_t .
- Column 9** $K_{t(avg)}$ is average conveyance for the reach and calculated by $0.5 * (K_{td} + K_{tu})$ where subscripts D and U refer to downstream and upstream ends of the reach, respectively.
- Column 10** $S_{f(avg)}$ is the average slope through the reach determined by $(Q / K_{t(avg)})^2$.
- Column 11** L is the discharge-weighted or conveyance-weighted reach length.
- Column 12** h_f is energy loss due to friction through the reach and calculated by $h_f = (Q / K_{t(avg)})^2 * L = S_{f(avg)}L$.
- Column 13** (K^3 / A^2) is part of the expression relating distributed flow velocity to an average value. If the section is complex, calculate separate values for each subsection and sum all subsection values to get a total. If one subsection is used, Column 13 is not needed and Column 14 equals one.
- Column 14** α is the velocity distribution coefficient and calculated by $(K^3 / A^2) / (K_t^3 / A_t^2)$ where the numerator is the sum of values in Column 13 and the denominator is calculated from K_t and A_t .
- Column 15** V is the average velocity and calculated by Q / A_t .
- Column 16** $\alpha V^2 / 2g$ is the average velocity head corrected for flow distribution.
- Column 17** $\Delta * (\alpha V^2 / 2g)$ is the difference between velocity heads at the downstream and upstream sections. A positive value indicates velocity is increasing, therefore, use a contraction coefficient for "other losses." A negative value indicates the expansion coefficient should be used in calculating "other losses."
- Column 18** h_o is "other losses," and calculated by multiplying either the expansion or contraction coefficient, K_m , times the absolute value of Column 17.
- Column 19** $\Delta W S$ is the change in water surface elevation from the previous cross-section. It is the algebraic sum of Columns 12, 17, and 18.

Figure 4-B-2. Standard Step Computation Worksheet

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4.B.3 Flow Classification

The classification of open channel flow can be summarized as follows:

Steady Flow:

- A. Uniform Flow
- B. Nonuniform Flow
 - a. Gradually Varied Flow
 - b. Rapidly Varied Flow

Unsteady Flow:

- A. Unsteady Uniform Flow (rare)
- B. Unsteady Nonuniform Flow
 - a. Gradually Varied Unsteady Flow
 - b. Rapidly Varied Unsteady Flow

The steady, uniform flow case and the steady, nonuniform flow case are the fundamental types of flow treated in highway engineering hydraulics. The following procedures address steady flow.

4.B.4 Equations

The following equations are used to analyze open channel flow. Use of these equations in analyzing open channel hydraulics is discussed in **Sections 4.B.1** and **4.B.2**.

4.B.4.1 Continuity Equation

The continuity equation is the statement of conservation of mass in fluid mechanics. For the special case of one dimensional, steady flow of an incompressible fluid, it assumes the simple form:

$$Q = A_1 V_1 = A_2 V_2 \quad (4.B.12)$$

Where:

- Q = discharge, cfs
- A = cross-sectional area of flow, sft
- V = mean cross-sectional velocity, fps (velocity perpendicular to the cross-section)

Subscripts 1 and 2 refer to successive cross-sections along the flow path.

4.B.4.2 Manning's Equation

For a given depth of flow in an open channel with a steady, uniform flow, the mean velocity, V, can be computed with Manning's equation:

$$V = \left(\frac{1.49}{n} \right) R^{2/3} \sqrt{S} \quad (4.B.13)$$

Where:

- V = velocity, fps
- n = Manning's roughness coefficient
- R = hydraulic radius = A / P, ft
- A = cross-sectional area of flow, sft
- P = wetted perimeter, ft
- S = hydraulic slope, ft/ft
(Note: For steady uniform flow, S = channel slope, ft/ft)

Selection of Manning's n is generally based on observation; however, considerable experience is essential in selecting appropriate n values. Selection of Manning's n is discussed in **Section 4.4.3.3**. **Table 4-3** provides the range of n values for various types of channels and floodplains.

The continuity equation can be combined with Manning's equation to obtain the steady, uniform flow discharge as:

$$Q = \left(\frac{1.49}{n} \right) A R^{2/3} \sqrt{S} \quad (4.B.14)$$

For a given channel geometry, slope, and roughness, and a specified value of discharge Q , a unique value of depth occurs in steady, uniform flow. It is called normal depth and computed from **Equation 4.B.14** by expressing the area and hydraulic radius in terms of depth. The resulting equation may require a trial and error solution. Refer to **Section 4.4.3.5** for a more detailed discussion of the computation of normal depth.

If the normal depth is greater than the critical depth, the slope is classified as a mild slope, while on a steep slope, the normal depth is less than critical depth. Thus, uniform flow is subcritical on a mild slope and supercritical on a steep slope.

4.B.4.2.1 Conveyance

In channel analysis, it is often convenient to group the channel properties in a single term called the channel conveyance K :

$$K = \left(\frac{1.49}{n} \right) A R^{2/3} \quad (4.B.15)$$

and then **Equation 4.B.14** can be written as:

$$Q = K\sqrt{S} \quad (4.B.16)$$

The conveyance represents the carrying capacity of a stream cross-section based upon its geometry and roughness characteristics alone and is independent of the streambed slope.

The concept of channel conveyance is useful when computing the distribution of overbank flood flows in the stream cross-section and the flow distribution through the opening in a proposed stream crossing. Channel conveyance is used to determine the velocity distribution coefficient, α (refer to **Equation 4.B.2**).

4.B.4.3 Energy Equation

The energy equation expresses conservation of energy in open channel flow. The energy is expressed as energy per unit weight of fluid, which has dimensions of length, and is called energy head. The energy head is composed of potential energy head (elevation head), pressure head, and kinetic energy head (velocity head). These energy heads are scalar quantities which give the total energy head at any cross-section when added. Written between an upstream open channel cross-section designated 1 and a downstream cross-section designated 2, the energy equation is:

$$h_1 + \alpha_1 \left(\frac{V_1^2}{2g} \right) = h_2 + \alpha_2 \left(\frac{V_2^2}{2g} \right) + h_L \quad (4.B.17)$$

Where:

h_1 and h_2 are the upstream and downstream stages, respectively, ft

α = velocity distribution coefficient

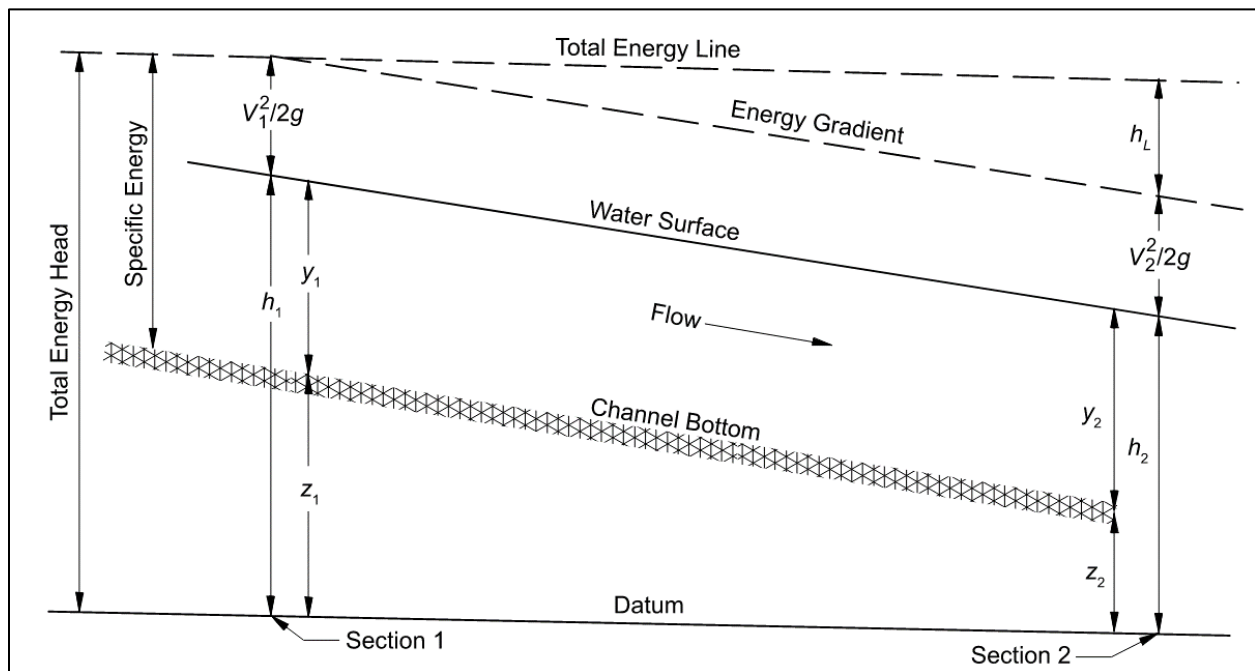
V = mean velocity, fps

g = acceleration due to gravity, ft/s²

h_L = head loss due to local cross-sectional changes (minor loss) and boundary resistance, ft

The stage h is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y , i.e., $h = z + y$. The terms in the energy equation are illustrated graphically in **Figure 4-B-3**. The energy equation states that the total energy head at an upstream cross-section is equal to the energy head at a downstream section plus the intervening energy head loss. The energy equation can only be applied between two cross-sections at which the streamlines are nearly straight and parallel so that vertical accelerations can be neglected (slopes approximately less than 10%).

Figure 4-B-3. Terms in the Energy Equation



4.B.4.4 Shear Stress Equation

Shear stress is the longitudinal stress on a channel boundary produced by flowing water. Shear stress is a primary cause of channel bank erosion.

The equation for shear stress is:

$$\tau_o = \gamma R S_f \quad (4.B.18)$$

Where:

- τ_o = shear stress, lb/ft²
- γ = unit weight of water, 62.4 lb/ft³
- R = hydraulic radius = A / P, ft
- A = cross-sectional area of flow, sft
- P = wetted perimeter, ft
- S_f = friction slope, ft/ft

In wide channels, the hydraulic radius is approximately equivalent to water depth. For uniform flow, the channel slope equals the slope of the energy grade line.

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