MDOT

Lawrence Tech

LRFD Design Example for:

CFCC Prestressed Precast Concrete Box-Beam with Cast-In-Place Concrete Slab

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Disclaimer

"This material is based upon work supported by the Federal Highway Administration under SPR-1690. Any opinions, findings and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the Federal Highway Administration."

About this Design Example

Description

This document provides guidance for the design of CFCC prestressed precast concrete beams according to AASHTO LRFD Bridge Design Specifications with the neccessary ammendmets where applicable, based on available literature and experimental data from tests conducted by Grace et. al at Lawrence Technological University. The cross-section of the bridge is **Type B** as described by **AASHTO Table 4.6.2.2.1-1**.

Standards

The following design standards were utilized in this example:

- AASHTO LRFD Bridge Design Specification, 7th Edition, 2014
- Michigan Department of Transportation Bridge Design Manual, Volume 5
- Michigan Department of Transportation Bridge Design Guide
- ACI 440.4R-04, Prestressing Concrete Structures with FRP Tendons

Code & AASHTO LRFD UPDATES

This Mathcad sheet is developed based on available design guidelines and available AASHTO LRFD edition at the time of writing the sheet. Designer shall check and update design equations according to the latest edition of AASHTO LRFD

General notes

The following notes were considered in this design example:

1- Guarnateed strength of CFRP is reduced to account for environmental effect. The design guarnateec strength is taken as 0.9 x guarnateed strength recommended by manufacturer

2- Initial prestressing stress is limited to 65% of the design (reduced) guaranteed strength according to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations for initial/jacking stress in CFRP strands

3- CFCC strength immediately following transfer is limited to 60% of the design (reduced) guaranteed strength according ccording to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations

4- The depth of the haunch is ignored in calculating section properties or flexural capacity, while is included in calculating the dead loads

5- In strength limit state flanged section design, the concrete strength of the beam portion participating i the stress block was conservatively assumed equal to the concrete strength of the deck (AASHTO LRF C5.7.2.2)

6- Barrier weight was taken as 475 lb/ft. While, weight of midspan diaphragm was 500 lb/beam

7- In the Mathcad sheet, the option of debonding as well as top prestressing strands are offered as means of reducing the end tensile stresses of the beams

8-The example provided herein is a box beam with varying web thickness from a maximum of 12 in. at the beam ends to a minimum of 4.5 in. at midspan. This is the same cs that was used in the construction of M-102 bridge in Southfield, MI. Shear requirements necessitated the increase in the web thickenss near the ends of the span

9-The box beam in this example is also provided with end diaphragms, which affect the stress calculations at beam ends at prestress release

10- In strength limit state check, the design addresses six different failure modes as follows:
Tension controlled rectangular section (depth of stress block is less than or equal the depth of the deck slab and the reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled rectangular section (depth of stress block is less than or equal the depth of the deck slab and the reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Tension controlled flanged section (depth of stress block is larger than the depth of the deck slab but less than the combined depth of the deck slab and beam top flange. The reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled flanged section (depth of stress block is larger than the depth of the deck slab but less than the combined depth of the deck slab and beam top flange. The reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Tension controlled double flanged section (depth of stress block is larger than the combined depth of the deck slab and beam top flange. The reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled double flanged section (depth of stress block is larger than the combined depth of the deck slab and beam top flange. The reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Designer is advised to check the ductility of the beam and the deflection at failure in case of double flanged section because in that case, the N.A. of the section lies within the web of the beam and the ductility of the section may be compromised

11- This design example is developed based on allowable jacking strength and stress immediately after transfer according to the limits presented in the ACI 440.4R-04. The document can be updated using other prestress limits such as those presented in MDOT SPR-1690 research report and guide

$L_{RefToRef} := 75ft$

$D_{RefAtoBearing} := 50.5in$

$D_{RefBtoBearing} := 50.5in$

$L := L_{RefToRef} - D_{RefAtoBearing} - D_{RefBtoBearing} = 66.583\,ft$ Center to center span Length

$brg_{off} := 8in$ Center of bearing offset to end of beam (same vaLue at both ends is assumed)

$L_{beam} := L + 2 \cdot brg_{off} = 67.917 \cdot ft$ TotaL length of beam

$l_{ship} := 12 \cdot in$ Distance from support to the end of the beam after force transfer and during shipping and handling

$L_{ship} := L_{beam} - l_{ship} \cdot 2 = 65.917\,ft$ Distance between supports during handling and shipping

$deck_{width} := 61ft + 8.5in$ Out to out deck width

$clear_{roadway} := 52ft + 0in$ CLear roadway width

$deck_{thick} := 9in$ Deck slab thickness

$t_{wear} := 0 \cdot in$ Wearing surface is included in the structural deck thickness only when designing the deck as per MDOT BDM 7.02.19.A.4. It is not used when designing the beam.

$t_{fws} := 2in$ Future wearing surface is applied as dead laod to accuant for additional deck thickness if a thicker rigid overlay is placed on deck

$walk_{width} := 6ft + 0in$ sidewalk width

$walk_{thick} := 0in$ sidewalk thickness (0" indicates no separate sidewalk pour)

$S := 8ft + 0in$ Center to center beam spacing

$NO_{beams} := 8$ Total number of beams

$haunch := 0in$ Average haunch thickness for section properties and strength calculations

$haunch_d := 2.0in$ Average haunch thickness for Load calculations

$overhang := 2ft + 11.5in$ Deck overhang width (same vaLue on both overhangs is assumed)

$barrier_{width} := 1ft + 2.5in$ Barrier width; include offset from back of barrier to edge of deck

$S_{exterior} := 56ft + 0in$ Hz distance between center of gravity of two exterior girders

$Lanes := floor\left(\frac{clear_{roadway}}{12ft}\right) = 4.00$ The number of design traffic Lanes can be caLcuLated as

$angle_{crossing} := \left(45 + \frac{20}{60} + \frac{51}{3600}\right)deg = 45.35 \cdot deg$ Angle measured from centerline of bridge to the reference line

$\theta_{skew} := 90deg - angle_{crossing} = 44.65 \cdot deg$ Angle measured from a line perpendicular to the centerline of bridge to the reference line

Concrete Material Properties

$f_{c\_deck} := 5ksi$ Deck concrete compressive strength

$f_{c\_beam} := 8ksi$ FinaL beam concrete compressive strength

$f_{ci\_beam} := 0.8f_{c\_beam} = 6.4 \cdot ksi$ Beam concrete compressive strength at reLease

$\omega_{conc} := 0.150\frac{kip}{ft^3}$ Unit weight of reinforced concrete for load calculations

$barrier_{weight} := 0.475\frac{kip}{ft}$ Weight per foot of barrier (aesthetic parapet tube, see MDOT BDG 6.29.10)

Unit weights of concrete used for modulus of eLasticity calculations, AASHTO Table 3.5.1-1

$$\gamma_c(f'_c) := \begin{array}{|l} 0.145\frac{kip}{ft^3} \ \text{ if } f'_c \le 5ksi \\ 0.140\frac{kip}{ft^3} + 0.001 \cdot \left(\frac{f'_c}{ksi}\right)\frac{kip}{ft^3} \ \text{ otherwise} \end{array}$$

$\gamma_{c.deck} := \gamma_c(f_{c\_deck}) = 145 \cdot pcf$

$\gamma_{c.beam} := \gamma_c(f_{c\_beam}) = 148 \cdot pcf$

$\gamma_{ci.beam} := \gamma_c(f_{ci\_beam}) = 146.4 \cdot pcf$

Concrete Modulus of Elasticity

Elastic modulus for concrete is as specified by **AASHTO A 5.4.2.4** with a correction factor of 1.0

$E_{c.beam\_i} := 120000 \cdot \left(\frac{\gamma_{ci.beam}}{\frac{kip}{ft^3}}\right)^{2.0} \cdot \left(\frac{f_{ci\_beam}}{ksi}\right)^{0.33} \cdot ksi = 4745.73 \cdot ksi$ Beam concrete at reLease

$E_{c.beam} := 120000 \cdot \left(\dfrac{\gamma_{c.beam}}{\dfrac{kip}{ft^3}} \right)^{2.0} \cdot \left(\dfrac{f_{c\_beam}}{ksi} \right)^{0.33} \cdot ksi = 5220.65 \cdot ksi$	Beam concrete at 28 days
$E_{c.deck} := 120000 \cdot \left(\dfrac{\gamma_{c.deck}}{\dfrac{kip}{ft^3}} \right)^{2.0} \cdot \left(\dfrac{f_{c\_deck}}{ksi} \right)^{0.33} \cdot ksi = 4291.19 \cdot ksi$	Deck concrete at 28 days

CFCC Material Properties

$d_s := 15.2mm = 0.6 \cdot in$	Prestressing strand diameter
$A_{strand} := 0.179 \cdot in^2$	Effective cross sectionaL area
$E_p := 21000ksi$	Tensile elastic modulus
$T_{guts} := 60.70kip$	Guaranteed ultimate tensile capacity
$f'_{pu} := \dfrac{T_{guts}}{A_{strand}} = 339.11 \cdot ksi$	Calculated ultimate tensile stress
$C_{Ese} := 0.9$ $f_{pu.service} := C_{Ese} \cdot f'_{pu} = 305.2 \cdot ksi$	Environmental reduction factor for prestressed concrete exposed to weather for service limit state calculations
$C_{Est} := 0.9$ $f_{pu} := C_{Est} \cdot f'_{pu} = 305.2 \cdot ksi$	Environmental reduction factor for prestressed concrete exposed to weather for strength limit state calculations

Modular Ratio

$n := \dfrac{E_{c.beam}}{E_{c.deck}} = 1.217$	Modular ratio for beam
$n_p := \dfrac{E_p}{E_{c.deck}} = 4.89$	Modular ratio for Prestressing CFCC

Box-Beam Section Properties:

$b_{ft} := 48in$	Width of top flange
$d_{ft} := 6in$	Thickness of top flange
$b_{fb} := 48in$	Width of bottom flange

$d_{fb} := 6in$ Thickness of bottom flange

$b_{web.min} := 4.5in$ Minimum web thickness

$b_{web.max} := 12in$ Maximum web thickness

$L_{end} := 32 \cdot in$ Length of the solid end block at beam end

$L_{var} := 176 \cdot in$ Length where web is tabered from maximum to minimum width

$d := 33in$ Depth of beam

$$b_{webf}(x) := \left| \begin{array}{l} (24 \cdot in) \quad \text{if } 0 \le x < L_{end} \\ \left[b_{web.max} - \left(b_{web.max} - b_{web.min} \right) \cdot \dfrac{x - L_{end}}{L_{var}} \right] \quad \text{if } L_{end} \le x \le L_{end} + L_{var} \\ b_{web.min} \quad \text{if } x > L_{end} + L_{var} \end{array} \right.$$

$$A_{beamf}(x) := \left| \begin{array}{l} b_{ft} \cdot d - 0.5625 \cdot in^2 \quad \text{if } 0 \le x < L_{end} \\ \left[b_{ft} \cdot d - \left(d - d_{ft} - d_{fb} \right) \cdot \left(b_{ft} - 2 \cdot b_{webf}(x) \right) + 17.4375 \cdot in^2 \right] \quad \text{if } L_{end} \le x \end{array} \right.$$

$$I_{beamf}(x) := \left| \begin{array}{l} b_{ft} \cdot \dfrac{d^3}{12} \quad \text{if } 0 \le x < L_{end} \\ b_{ft} \cdot \dfrac{d^3}{12} - \left(b_{ft} - 2 \cdot b_{webf}(x) \right) \cdot \dfrac{\left(d - d_{ft} - d_{fb} \right)^3}{12} + 1485 \cdot in^4 \quad \text{if } L_{end} \le x \end{array} \right.$$

$y_t := 16.5in$ Depth from centroid to top of beam

$y_b := 16.5in$ Depth from centroid to soffit of beam

$S_{Tf}(x) := \dfrac{I_{beamf}(x)}{y_t}$ Section modulus about top flange

$S_{Bf}(x) := \dfrac{I_{beamf}(x)}{y_b}$ Section modulus about bottom flange

Properties of the section at midspan (minimum concrete area)

$A_{beam} := A_{beamf}\left(\frac{L_{beam}}{2}\right) = 782.437 \cdot in^2$

Minimum area of beam section

$b_{web} := b_{webf}\left(\frac{L_{beam}}{2}\right) = 4.5 \cdot in$

width of the web at midspan

$b_v := 2 \cdot b_{webf}\left(\frac{L_{beam}}{2}\right) = 9.00 \cdot in$

combined web width at midspan
(two webs per beam)

$\omega_{beam} := A_{beam} \cdot (150pcf) = 815.04 \cdot plf$

Beam weight per foot

$I_{beam} := I_{beamf}\left(\frac{L_{beam}}{2}\right) = 1.151 \times 10^5 \cdot in^4$

Minimum moment of inertia

$y_t := 16.5in$

Depth from centroid to top of beam

$y_b := 16.5in$

Depth from centroid to soffit of beam

$S_T := \frac{I_{beam}}{y_t} = 6.978 \times 10^3 \cdot in^3$

Minimum section modulus about top flange

$S_B := \frac{I_{beam}}{y_b} = 6977.86 \cdot in^3$

Minimum section modulus about bottom flange

Effective Flange Width of Concrete Deck Slab, AASHTO A 4.6.2.6

Beam_Design := "Interior"

Choose the design of the beam either "Interior" or "Exterior"

$b_{eff.int} := S = 8.00\, ft$

Effective flange width of deck slab for interior beams

$b_{eff.ext} := \frac{1}{2} \cdot S + overhang = 6.96\, ft$

Effective flange width of deck slab for exterior beams

$$b_{eff} := \left| \begin{array}{ll} b_{eff.int} & \text{if Beam\_Design} = \text{"Interior"} \\ b_{eff.ext} & \text{if Beam\_Design} = \text{"Exterior"} \end{array} \right. = 2.438$$

$d_{total} := deck_{thick} + d = 42 \cdot in$

Total depth of section including deck

Dynamic load Allowance

Dynamic load allowance from **AASHTO Table 3.6.2.1-1** is applied as an increment to the static wheel loads to account for wheel load impacts from moving vehicles.

$IM := 1 + 33\% = 1.33$

Design Factors

These factors are related to the ductility, redundancy and operational importance of the bridge structure components and are applied to the strength limit state.

Ductility

For Strength limit State, a factor of 1.05 is used for nonductile components and connections, 1.00 for conventional designs and details complying with these specifications, and 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by these specifications, **AASHTO A 1.3.3.**

$\eta_D := 1.00$

Redundancy

For Strength limit State, a factor of 1.05 is used for nonredundant members, 1.00 for conventional levels of redundancy, foundation elements where ϕ already accounts for redundancy as specified in **AASHTO A 10.5**, and 0.95 for exceptional levels of redundancy beyond girder continuity and a torsionally-closed cross-section, **AASHTO A 1.3.4.**

$\eta_R := 1.00$

Operational Importance

For the Strength limit State, a factor of 1.05 is used for critical or essential bridges, 1.00 for typical bridges, and 0.95 for relatively less important bridges, **AASHTO A 1.3.5.**

$\eta_I := 1.00$

Ductility, redundancy, and operational classification considered in the load modifier, **AASHTO Eqn. 1.3.2.1-2.**

$\eta_i := \eta_D \cdot \eta_R \cdot \eta_I = 1.00$

Composite Section Properties

This is the moment of inertia resisting superimposed dead loads.

Elastic Section Properties - Composite Section: k=2

$k_{sdl} := 2$

$$A_{haunchkn} := \frac{b_{ft}}{k_{sdl}\, n} \cdot haunch = 0 \cdot in^2$$

effective area of haunch

$$d_{haunchkn} := d + \frac{haunch}{2} = 33 \cdot in$$

Depth of centroid of haunch to bottom of beam

$$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$$

$b_{effkn} := \dfrac{b_{eff}}{k_{sdl}\,n} = 39.45 \cdot in$

Transformed deck width

$d_{slabkn} := d + haunch + \dfrac{deck_{thick} - t_{wear}}{2} = 37.5 \cdot in$

Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 355.09 \cdot in^2$

Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 13315.82 \cdot in^3$

Static moment of inertia of transformed section about soffit of beam

$d_k := \dfrac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 23.06 \cdot in$

Depth of CG of composite section from beam soffit

$I_{oslabkn} := \dfrac{b_{effkn} \cdot deck_{thick}^{\ 3}}{12} = 2396.85 \cdot in^4$

Moment of inertia of transformed deck about centroid

$I_{haunchkn} := \dfrac{\dfrac{b_{ft}}{k_{sdl} \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$

Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_{3n} := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \cdots + A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2 = 225243.5 \cdot in^4$$

$y_{b3n} := d_k = 23.055 \cdot in$

Depth of CG of composite section from beam soffit

$S_{b3n} := \dfrac{I_{3n}}{y_{b3n}} = 9769.69 \cdot in^3$

Section modulus about bottom of beam

$y_{t.bm.3n} := d - y_{b3n} = 9.94 \cdot in$

Depth of CG of composite section from top of beam

$S_{t.bm.3n} := \dfrac{I_{3n}}{y_{t.bm.3n}} = 22649.67 \cdot in^3$

Section modulus about top of beam

$y_{t3n} := d + haunch + deck_{thick} - t_{wear} - y_{b3n} = 18.94 \cdot in$

Depth of CG of composite section from top of deck

$S_{t3n} := \dfrac{I_{3n}}{y_{t3n}} = 11889.54 \cdot in^3$

Section modulus about top of deck

<u>Elastic Section Properties - Composite Section: k=1</u>

These properties are used to evaluate the moment of inertia for resisting live loads

Assumed wearing surface not included in the structural design deck thickness, per **MDOT BDM 7.02.19.A.4............**

$k := 1$

$A_{haunchkn} := \frac{b_{ft}}{k\,n} \cdot haunch = 0 \cdot in^2$

effective area of haunch

$d_{haunchkn} := d + \frac{haunch}{2} = 33 \cdot in$

Depth of centroid of haunch to bottom of beam

$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$

$b_{effkn} := \frac{b_{eff}}{k\,n} = 78.91 \cdot in$

Transformed deck width

$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 37.5 \cdot in$

Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 710.18 \cdot in^2$

Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 26631.64 \cdot in^3$

Static moment of inertia of transformed section about soffit of beam

$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 26.49 \cdot in$

Depth of CG of composite section from beam soffit

$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\,3}}{12} = 4793.7 \cdot in^4$

Moment of inertia of transformed deck about centroid

$I_{haunchkn} := \frac{\frac{b_{ft}}{k \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$

Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_n := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \ldots = 284103.5 \cdot in^4$$
$$+ A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2$$

$y_{bn} := d_k = 26.492 \cdot in$

Depth of CG of composite section from beam soffit

$S_{bn} := \frac{I_n}{y_{bn}} = 10724.26 \cdot in^3$	Section modulus about bottom of beam
$y_{t.bm.n} := d - y_{bn} = 6.51 \cdot in$	Depth of CG of composite section from top of beam
$S_{t.bm.n} := \frac{I_n}{y_{t.bm.n}} = 43652.33 \cdot in^3$	Section modulus about top of beam
$y_{tn} := d + haunch + deck_{thick} - t_{wear} - y_{bn} = 15.51 \cdot in$	Depth of CG of composite section from top of deck
$S_{tn} := \frac{I_n}{y_{tn}} = 18319.42 \cdot in^3$	Section modulus about top of deck

live load lateral Distribution Factors

Cross-section classification.. **Type B**

Distribution of live loads from the deck to the beams is evaluated based on the **AASHTO** specified distribution factors. These factors can only be used if generally, the following conditions are met;

- Width of deck is constant.
- Unless otherwise specified, the number of beams is not less than three
- Beams are parallel and have approximately the same stiffness.
- Curvature in plan is less than the limit specified in **AASHTO A 4.6.1.2.4.**
- Unless otherwise specified, the roadway part of the overhang does not exceed 3.0 ft.
- Cross-section is consistent with one of the cross-sections shown in AASHTO **Table 4.6.2.2.1-1.**

Distribution of live loads for Moment in Interior Beams, AASHTO Table 4.6.2.2.2b-1

Range of Applicability.................................

$\mathrm{if}(6ft < S \le 18ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\mathrm{if}(18in < d \le 65in, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\mathrm{if}(20ft < L \le 140ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\mathrm{if}\left(NO_{beams} \ge 3, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

One lane loaded

$$M_{lane1\_int} := \left(\frac{S}{3.0ft}\right)^{0.35} \cdot \left(\frac{S \cdot d}{12.0L^2} \cdot \frac{ft}{in}\right)^{0.25} = 0.374$$

Two or more lanes loaded

$$M_{lane2\_int} := \left(\frac{S}{6.3ft}\right)^{0.6} \cdot \left(\frac{S \cdot d}{12.0L^2} \cdot \frac{ft}{in}\right)^{0.125} = 0.595$$

live load moment disribution factor for interior beam

$$M_{lane\_int} := \max\left(M_{lane1\_int}, M_{lane2\_int}\right) = 0.595$$

Distribution of live loads for Moment in Exterior Beams, AASHTO Table 4.6.2.2.2d-1

One lane loaded (using the lever rule)

The lever rule involves the use of statics to determine the lateral distribution to the exterior girder by summing moments about the adjacent interior girder to determine the wheel-load reaction at the exterio girder assuming the concrete deck is hinged at the interior girder. A wheel cannot be closer than 2'-0" to the toe of barrier, and the standard wheel spacing is 6'-0". The evaluated factor is multiplied by the multiple presence factor, **AASHTO Table 3.6.1.1.2-1**.

Summing moments about the center of the interior beam

$$R := \frac{\left(S + overhang - barrier_{width} - 2 \cdot ft - \frac{6 \cdot ft}{2}\right)}{S} = 0.594$$

This factor is based on the lever arm rule considring the wheel load and not the resultant of both wheel

Moment distribution factor for exterior beam, one load loaded. The 1.2 accounts for the multiple presence factor, *m* from **AASHTO Table 3.6.1.1.2-1** for one lane loaded

$$M_{lane1\_ext} := R \cdot 1.2 = 0.713$$

Two or more lanes loaded

Horizontal distance from the centerline of the exterior web of exterior beam at deck level to the interior web edge of curb or traffic barrier must be greater than 0'-0"

$$d_e := \max\left[overhang - barrier_{width} - \left(0.5b_{fb} - 0.5b_{web}\right), 0ft\right] = 0.00\,ft$$

Range of Applicability

$$\text{if}\left(-1ft \le d_e \le 5.5ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

lane fraction

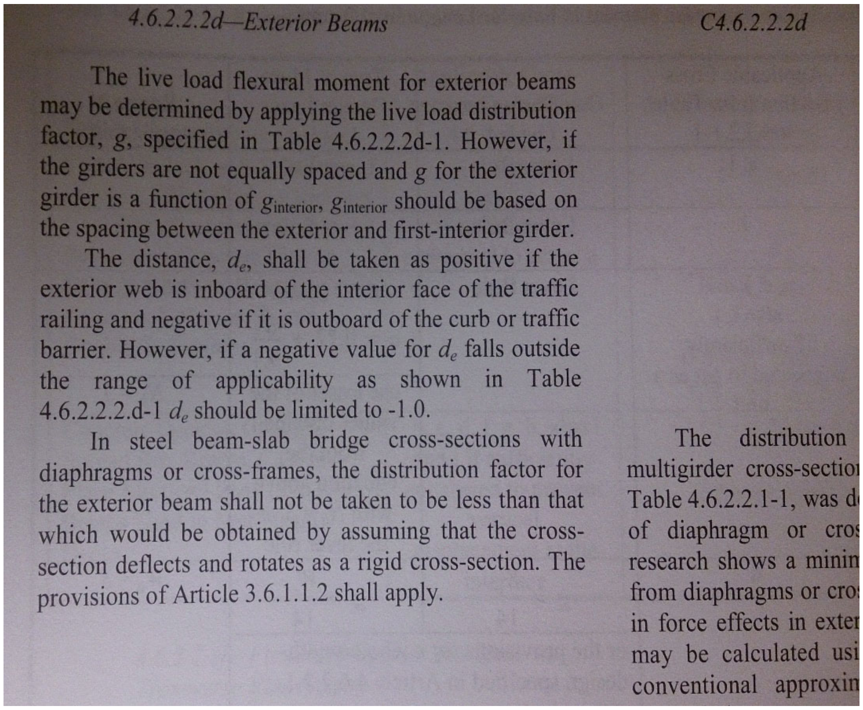
$$e := 0.97 + \frac{d_e}{28.5ft} = 0.97$$

Moment distribution factor for exterior beam, two or more lanes loaded

$$M_{lane2\_ext} := M_{lane\_int} \cdot e = 0.577$$

Distribution of live loads for Moment in Exterior Beams, AASHTO C4.6.2.2.2d

AASHTO LRFD 2014 recommends the rigid plate analysis only for **steel beam-slab bridges**. This was a change from ealier versions of AASHTO. It is up to the designed to ignore the rigid plate analysis or take it into consideration when calculating the DF for exterior beam



4.6.2.2.2d—Exterior Beams

C4.6.2.2.2d

The live load flexural moment for exterior beams may be determined by applying the live load distribution factor, *g*, specified in Table 4.6.2.2.2d-1. However, if the girders are not equally spaced and *g* for the exterior girder is a function of $g_{interior}$, $g_{interior}$ should be based on the spacing between the exterior and first-interior girder.

The distance, d_e, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier. However, if a negative value for d_e falls outside the range of applicability as shown in Table 4.6.2.2.2.d-1 d_e should be limited to -1.0.

In steel beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

The distribution
multigirder cross-sectio
Table 4.6.2.2.1-1, was d
of diaphragm or cros
research shows a minin
from diaphragms or cros
in force effects in exter
may be calculated usi
conventional approxin

Additional special analysis investigation is required because the distribution factor for multigirder in cross section was determined without consideration of diaphragm or cross frames. The multiple presence factors are used per **AASHTO Table 3.6.1.1.2-1.** This analysis should be done by sketching the cross section to determine the variables required for this example, the defined deck geometry is used. For any other geometry, these variables should be hand computed and input:

Horizontal distance from center of gravity of the pattern of girders to the exterior girder

$$X_{ext} := \frac{S_{exterior}}{2} = 28.00\,ft$$

Eccentricity of the center line of the standard wheel from the center of gravity of the pattern of girders

$$e_1 := X_{ext} + overhang - barrier_{width} - 2ft - \frac{6ft}{2} = 24.75\,ft$$

$$e_2 := e_1 - 12ft = 12.75\,ft$$

$$e_3 := e_2 - 12ft = 0.75\,ft$$

$$e_4 := e_3 - 12ft = -11.25\,ft$$

Summation of eccentricities for number of lanes considered:

$e_{NL1} := e_1 = 24.75\,ft$ One lane loaded

$e_{NL2} := e_1 + e_2 = 37.5\,ft$ Two lanes loaded

$e_{NL3} := e_{NL2} + e_3 = 38.25\,ft$ Three lanes loaded

$e_{NL4} := e_{NL3} + e_4 = 27\,ft$ Four lanes loaded

Horizontal distances from the center of gravity of the pattern of girders to each girder

$$X_{beams} := \left| \begin{array}{l} \text{for } i \in 0 .. NO_{beams} - 1 \\ \quad X_i \leftarrow X_{ext} - (i \cdot S) \\ X \end{array} \right. = \begin{pmatrix} 28.00 \\ 20.00 \\ 12.00 \\ 4.00 \\ -4.00 \\ -12.00 \\ -20.00 \\ -28.00 \end{pmatrix} ft$$

Summation of horizontal distances from the center of gravity of the pattern of girders to each girder

$$X_{NB} := \sum X_{beams}{}^2 = 2688.00 \cdot ft^2$$

$$m_{1R} := 1.2 \cdot \left(\frac{1}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL1}}{X_{NB}} \right) = 0.459$$

Reaction on exterior beam when one lane is loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{2R} := 1.0 \cdot \left(\frac{2}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL2}}{X_{NB}} \right) = 0.641$$

Reaction on exterior beam when two lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$m_{3R} := 0.85 \cdot \left(\frac{3}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL3}}{X_{NB}} \right) = 0.657$

Reaction on exterior beam when three lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$m_{4R} := 0.65 \cdot \left(\frac{4}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL4}}{X_{NB}} \right) = 0.508$

Reaction on exterior beam when four lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

live load moment disribution factor for exterior beam

$M_{lane\_ext} := \max\left(M_{lane1\_ext}, M_{lane2\_ext}, m_{1R}, m_{2R}, m_{3R}, m_{4R}\right) = 0.713$

Reduction of load Distribution Factors for Moment in longitudinal Beams on Skewed Supports

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10 degrees, the bending moments and shear forces are reduced in accordance with **AASHTO Table 4.6.2.2.2e-1 and 4.6.2.2.3c-1** respectively.

Moment

Range of Applicability

$$\theta_{skew} := \begin{vmatrix} \theta_{skew} & \text{if } \theta_{skew} \le 60 \cdot deg \\ 60 \cdot deg & \text{if } \theta_{skew} > 60 \cdot deg \end{matrix} = 44.652 \cdot deg$$

$Mcorr_{factor} := \min\left(1.05 - 0.25 \cdot \tan\left(\theta_{skew}\right), 1.0\right) = 0.803$

Correction factor for moment

Reduced distribution factors at strength limit state for interior girders due to skew

$DF_{strength\_moment\_int} := M_{lane\_int} \cdot Mcorr_{factor} = 0.477$

Moment

Reduced distribution factors at strength limit state for exterior girders due to skew

$DF_{strength\_moment\_ext} := M_{lane\_ext} \cdot Mcorr_{factor} = 0.572$

Moment

Design distribution factors for service and strength limit states

Distribution factor for moment at strength limit state

$$DF_{strength\_moment} := \begin{vmatrix} DF_{strength\_moment\_int} & \text{if } Beam\_Design = \text{"Interior"} \\ DF_{strength\_moment\_ext} & \text{if } Beam\_Design = \text{"Exterior"} \end{matrix} = 0.477$$

live load Analysis

Flexure

As per **AASHTO A 3.6.1.2.1**, vehicular live loading designated by the standard HI-93 truck shall be a combination of the design truck or design tandem, and the design lane load. To produce extreme force effects, the spacing between the two 32-kip axles are taken as 14 ft.

Calculate the maximum moment due to the truck load. Maximum truck load moment occurs when the middle axle is positioned at distance 2.33 ft from the midspan. Maximum momment occurs under the middle axle load. Moment due to distributed load occurs at midspan.

Unless more detailed analysis is performed to determine the location and value for the maximum moment under combined truck and distributed loads at both service and strength limit state, the maximum moment from the truck load at distance 2.33 ft from midspan can be assumed to occur at the midspan and combined with the maximum moment from other dead and live distributed loads

Calculate the reaction at the end of the span

$$R := \frac{8\text{kip}\cdot\left(\frac{L}{2} - 16.33\text{ft}\right) + 32\text{kip}\cdot\left(\frac{L}{2} - 2.33\text{ft}\right) + 32\text{kip}\cdot\left(\frac{L}{2} + 11.67\text{ft}\right)}{L} = 38.527\cdot\text{kip}$$

Calculate the maximum moment

$$M_{\text{truck}} := R\cdot\left(\frac{L}{2} + 2.33\text{ft}\right) - 32\cdot\text{kip}\cdot14\cdot\text{ft} = 924.387\cdot\text{kip}\cdot\text{ft}$$

Maximum moment due to design lane load, **AASHTO A 3.6.1.2.4**

$$X := \frac{L}{2} = 33.292\,\text{ft}$$

$$M_{\text{lane}} := \frac{0.64\text{klf}\cdot L\cdot X}{2} - 0.64\text{klf}\cdot\frac{X^2}{2} = 354.67\cdot\text{kip}\cdot\text{ft}$$

Maximum moment due to design tandem, **MDOT BDM 7.01.04.A**

$$M_{\text{tandem}} := \frac{60\text{kip}\cdot L}{4} = 998.75\cdot\text{kip}\cdot\text{ft}$$

Maximum moment due to vehicular live loading by the modified HI-93 design truck and tandem per **MDOT BDM 7.01.04.A**. Modification is by multiplying the load effects by a factor of 1.20. Dynamic load allowance is considered only for the design truck and tandem, **AASHTO A 3.6.1.2.2, 3.6.1.2.3 & 3.6.1.2.4.**

$$M_{\text{LLI}} := \left[1.20M_{\text{lane}} + \text{IM}\cdot\left(1.20\cdot\max\left(M_{\text{truck}}, M_{\text{tandem}}\right)\right)\right]\cdot\text{DF}_{\text{strength\_moment}} = 964.28\cdot\text{kip}\cdot\text{ft}$$

Dead load Analysis

Noncomposite Dead load (DC_1)

$$w_{webf}(x) := \left(A_{beamf}(x) - A_{beamf}\left(\frac{L_{beam}}{2}\right)\right)\cdot\omega_{conc}$$

$w_1 := w_{webf}(0\cdot in) = 0.834\cdot\frac{kip}{ft}$ acting on $L_{end} = 2.667\,ft$ as uniform load

$w_2 := w_{webf}\left(L_{end}\right) = 0.328\cdot\frac{kip}{ft}$ acting on $L_{var} = 14.667\,ft$ as triangular load

Additional moment at mid-span due to weight of varying web width

$$M_{sw.web} := w_1\cdot\frac{L_{end}^{\;2}}{2} + 0.5\cdot w_2\cdot L_{var}\cdot\left(L_{end} + \frac{L_{var}}{3}\right) = 21.15\cdot kip\cdot ft$$

$M_{beam} := \frac{\omega_{beam}\cdot L^2}{8} = 451.67\cdot kip\cdot ft$

Total moment due to selfweight of beam without the varying width

$M_{swbeam} := M_{sw.web} + M_{beam} = 472.82\cdot kip\cdot ft$

Total moment due to selfweight of beam

$deck := \left(deck_{thick}\cdot b_{eff} + haunch_d\cdot b_{ft}\right)\cdot 0.15\frac{kip}{ft^3} = 1.00\cdot klf$

Selfweight of deck and haunch on beam

$M_{deck} := \frac{deck\cdot L^2}{8} = 554.17\cdot kip\cdot ft$

Moment due to selfweight of deck and haunch

$sip := 15psf\cdot\left(b_{eff} - b_{ft}\right) = 0.06\cdot klf$

15 psf weight included for stay-in-place forms per MDOT BDM 7.01.04.I

$M_{sip} := \frac{sip \cdot L^2}{8} = 33.25 \cdot kip \cdot ft$

Moment due to stay-in-place forms

$dia_{int} := 0 \cdot kip$

Weight of diaphragm at mid-span per each interior beam. Zero if no diaphragm is used

$dia_{ext} := 0 \cdot kip$

Weight of diaphragm at mid-span per each exterior beam. Zero if no diaphragm is used

$$diaphragm := \left| \begin{array}{l} dia_{int} \ \text{ if } \ Beam\_Design = \text{"Interior"} \\ dia_{ext} \ \text{ if } \ Beam\_Design = \text{"Exterior"} \end{array} \right. = 0 \cdot kip$$

$spa_{dia} := 2\left(S - b_{fb}\right) \cdot \tan\left(\theta_{skew}\right) = 7.904\,ft$

One row of diaphragms at midspan are used.

$M_{dia} := diaphragm \cdot \frac{L}{4} = 0 \cdot kip \cdot ft$

$DC_1 := \omega_{beam} + deck + sip = 1.875 \cdot klf$

Dead load (wt of beam+ deck+ SIP forms) acting on non-composite section

$M_{DC1} := M_{swbeam} + M_{deck} + M_{sip} + M_{dia} = 1060.23 \cdot kip \cdot ft$

Total midspan moment acting on the non-composite section

Composite Dead load (DC_2)

$util := \frac{1}{2} \cdot (0plf) = 0 \cdot klf$

No utilities are supported by the superstructure

$barrier1_{weight} := 0.475 \frac{kip}{ft}$

Weight per foot of first barrier (aesthetics parapet tube, **MDOT BDG 6.29.10**)

$barrier2_{weight} := 0.475 \frac{kip}{ft}$

Weight per foot of first barrier (aesthetics parapet tube, **MDOT BDG 6.29.10**)

$sidewalk := \frac{2 \cdot walk_{width} \cdot walk_{thick} \cdot \omega_{conc}}{NO_{beams}} = 0.00 \cdot klf$

Weight to due extra thickness of sidewalk per beam

$barrier := \frac{barrier1_{weight} + barrier2_{weight}}{NO_{beams}} = 0.12 \cdot klf$

Total barrier weight per beam

$soundwall_{weight} := 0.0 \cdot \frac{kip}{ft}$

Weight of the sound wall, if there is a sound wall

Weight of the sound wall **for exterior beam** design assuming lever arm and an inetremiate hinge on the first interior beam

$$soundwall := \begin{vmatrix} 0\cdot\frac{kip}{ft} \ \text{ if } Beam\_Design = \text{"Interior"} \\ \left[soundwall_{weight}\cdot\frac{(S + overhang)}{S}\right] \ \text{ if } Beam\_Design = \text{"Exterior"} \end{vmatrix} = 0\cdot\frac{kip}{ft}$$

$$DC_2 := sidewalk + barrier + util + soundwall = 0.119\cdot klf$$

Total dead load acting on the composite section

$$M_{DC2} := \frac{DC_2\cdot L^2}{8} = 65.81\cdot kip\cdot ft$$

Total midspan moment acting on the composite section

(DW) Wearing Surface load

$$DW := \left(b_{eff}\right)\cdot 0.025\frac{kip}{ft^2} = 0.2\cdot klf$$

Self weight of future wearing surface

Maximum unfactored dead load moments

$$M_{DC} := M_{DC1} + M_{DC2} = 1126.04\cdot kip\cdot ft$$

Total midspan moment due to loads acting on the composite and non-composite section

$$M_{DW} := \frac{DW\cdot L^2}{8} = 110.83\cdot kip\cdot ft$$

Midspan moment due to weight of future wearing surface

Wind load on the sound wall

If a tall sound wall is provided, wind effect shall be calculated and considered in the design. Assuming lever arm rule and an intermediate hinge at the first interior beam after the exterior beam, the wind load will affect the loads on the exterior beam and the first interior beam. In the following set of calculations, the wind effect was calculated as a concentrated moment at the end of the overhang of the bridge.

$$M_{wind} := 0.0\cdot ft\cdot\frac{kip}{ft}$$

Moment due to wind acting at the sound wall

$$W := \frac{M_{wind}}{S} = 0 \cdot \frac{kip}{ft}$$

Extra load on the interior/exterior beam due to wind load assuming lever arm analysis and an intermediate hinge at the first interior beam

$$M_{WS} := \frac{W \cdot L^2}{8} = 0 \cdot kip \cdot ft$$

Interior beam moment due to wind acting at the sound wall

load Combinations

Load Combinations: Strength, Extreme Event, Service and Fatigue load combinations are defined per AASHTO 3.4.1. Verify which combination are appropriate. For this concrete box beam design, wind load is not evaluated, and no permit vehicle is specified. However, the design live loading is MDOT HL-93 Modified which accounts for Michigan's inventory of legal and permit vehicles.

Strength I, III, IV and Strength V limit states are considered for the design of this beam. Load combinations factors according to **AASHTO LRFD 2016 Interim revision are used (Check for latest AASHTO LRFD edition)**

$$M\_Strength_I := \eta_i \cdot \left(1.25M_{DC} + 1.50M_{DW} + 1.75M_{LLI}\right) = 3261.28 \cdot kip \cdot ft$$

$$M\_Strength_{III} := \eta_i \cdot \left(1.25M_{DC} + 1.50M_{DW} + 1.0M_{WS}\right) = 1573.80 \cdot kip \cdot ft$$

$$M\_Strength_{IV} := \eta_i \cdot \left[1.50 \cdot \left(M_{DC} + M_{DW}\right)\right] = 1855.31 \cdot kip \cdot ft$$

$$M\_Strength_V := \eta_i \cdot \left(1.25M_{DC} + 1.50M_{DW} + 1.35M_{LLI} + 1.0 \cdot M_{WS}\right) = 2875.57 \cdot kip \cdot ft$$

$$M_{u\_strength} := \max\left(M\_Strength_I, M\_Strength_{III}, M\_Strength_{IV}, M\_Strength_V\right) = 3261.28 \cdot kip \cdot ft$$

Number of Prestressing Strands

The theoretical number of strands required is calculated using the **Service III** limit state

$$f_b := \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = 2.9 \cdot ksi$$

Tensile stress in bottom flange due to applied loads

Allowable stress limits for concrete

$$f_{ti} := 0.24 \cdot \sqrt{f_{ci\_beam} \cdot ksi} = 0.61 \cdot ksi$$

Initial allowable tensile stress

$$f_{ci} := -0.65 \cdot f_{ci\_beam} = -4.16 \cdot ksi$$

Initial allowable compressive stress (according to AASHTO LRFD 2016 interim revision)

$$f_{tf} := 0 \cdot \sqrt{f_{c\_beam} \cdot ksi} = 0.00 \cdot ksi$$

Final allowable tensile stress (allowing no tension)

No tension is allowed under service III limit state to avoid potential cracks and shear action on the strands

$f_{cfp} := -0.45 \cdot f_{c\_beam} = -3.60 \cdot ksi$

Final allowable compressive stress in the beam due to sum of effective prestress and permanent loads

$f_{cf.deckp} := -0.45 \cdot f_{c\_deck} = -2.25 \cdot ksi$

Final allowable compressive stress in the slab due to permanent loads

$f_{cf} := -0.6 \cdot f_{c\_beam} = -4.80 \cdot ksi$

Final allowable compressive stress in the beam due to sum of effective prestress, permanent loads, & transient loads

$f_{cf.deck} := -0.6 \cdot f_{c\_deck} = -3.00 \cdot ksi$

Final allowable compressive stress in the beam due to sum of permanent loads and transient loads

$f_p := f_b - f_{tf} = 2.9 \cdot ksi$

Excess tension in the bottom flange due to applied loads

Assuming strand pattern center of gravity is midway between the bottom two rows of strands, i.e. the same number of strands are used in the top and bottom rows of the bottom flange.

$y_{bs} := 3in$

Distance from soffit of beam to center of gravity of strands

$e_{st} := y_b - y_{bs} = 13.50 \cdot in$

Eccentricity of strands from the centroid of beam

Final prestressing force required to counteract excess tension in the bottom flange. Set allowable stress equal to the excess tension, solve for P_e.

$$P_{et} := \frac{f_p}{\left(\frac{1}{A_{beam}} + \frac{e_{st}}{S_B}\right)} = 903.732 \cdot kip$$

$f_{j.max} := 0.65 \cdot f_{pu.service} = 198.377 \cdot ksi$

Maximum allowable Jacking stress, **ACI 440.4R Table 3.3**

$P_j := A_{strand} \cdot f_{j.max} = 35.51 \cdot kip$

Maximum Jacking prestressing force per strand

$f_t := 0.64 f_{pu.service} = 195.33 \cdot ksi$

Initial prestressing stress immediately **prior** to transfer. shall be less than or equal to the maximum jacking strength, and shall be adjusted accordingly to make sure the stress immedietely following transfer is not exceeding 0.6 times guaranteed strength as shown on the following page

$P_{in} := A_{strand} \cdot f_t = 34.96 \cdot kip$

Initial prestressing force per strand prior to transfer

$P_{pet} := A_{strand} \cdot f_t \cdot 0.75 = 26.22 \cdot kip$

Effective prestressing force assuming 25% final prestress losses per 0.6" diameter strand

$NO_{strands\_i} := ceil\left(\frac{P_{et}}{P_{pet}}\right) = 35$

Minimum number of strands required

Strand distribution per row. Row 0 is the bottom most row in the beam. Start adding strands from the bottom row going up until the number of strands is reached. do not skip rows inbetween. Extra rows with zero strands will be eliminated in the analysis.

$row_0 := 15$ $row_1 := 18$ $row_2 := 4$ $row_3 := 2$ $row_4 := 0$ $row_5 := 0$

$row_6 := 0$ $row_7 := 0$ $row_8 := 0$ $row_9 := 0$

$$row = \begin{pmatrix} 15 \\ 18 \\ 4 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
Row := | $a \leftarrow 0$
       | for $i \in 0 .. length(row) - 1$
       |     $a \leftarrow a + 1$  if $row_i > 0$
       |     $a \leftarrow a$  otherwise
       | for $j \in 0 .. a - 1$
       |     $D_j \leftarrow row_j$
       | $D$
```

$$Row = \begin{pmatrix} 15 \\ 18 \\ 4 \\ 2 \end{pmatrix}$$

$NO_{strands} := \sum Row = 39.00$

Total number of prestressing strands

$$d_{strand} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}(Row)-1 \\ \quad d_{s_i} \leftarrow d-(2in)-(2in)i \\ d_s \end{array}\right. = \begin{pmatrix}31.00\\29.00\\27.00\\25.00\end{pmatrix}\cdot in$$

Depth of CFCC strands in each layer from the top of the beam section. This calculation assumes a 2" vertical spacing of the strand rows

$$CG := \frac{\left[Row\cdot\left(d-d_{strand}\right)\right]}{\sum Row} = 3.64\cdot in$$

Center of gravity of the strand group measured from the soffit of the beam section

$$d_f := (d-CG)+haunch+deck_{thick} = 38.36\cdot in$$

Depth from extreme compression fiber to centroid of CFCC tension reinforcement

$$e_s := y_b - CG = 12.86\cdot in$$

Eccentricity of strands from centroid of beam

$$A_{ps} := A_{strand}\cdot NO_{strands} = 6.98\cdot in^2$$

Total area of prestressing CFCC strands

Prestress losses

loss due to Elastic Shortening, AASHTO Eqn. C5.9.5.2.3a-1

$$\Delta f_{PES} := \frac{A_{ps}\cdot f_t\cdot\left(I_{beam}+e_s^{\,2}\cdot A_{beam}\right)-e_s\cdot M_{swbeam}\cdot A_{beam}}{A_{ps}\cdot\left(I_{beam}+e_s^{\,2}\cdot A_{beam}\right)+\dfrac{A_{beam}\cdot I_{beam}\cdot E_{c.beam\_i}}{E_p}} = 12.52\cdot ksi$$

$$F_{pt} := f_t - \Delta f_{PES} = 182.80\cdot ksi$$

Prestressing stress immediately following transfer

$$P_t := A_{ps}\cdot F_{pt} = 1276.141\cdot kip$$

According to ACI 440.4R, Table 3.3, the allowable stress immediately after transfer shall not exceed 0.6 fpu

$$0.6\cdot f_{pu.service} = 183.117\cdot ksi$$

$$\text{if}\left(F_{pt} \le 0.6\cdot f_{pu.service}, \text{"Ok"}, \text{"Not Ok"}\right) = \text{"Ok"}$$

Approximate Estimate of Time dependent losses, AASHTO A 5.9.5.3

$H := 75$	Average annual ambient relative humidity
$\gamma_h := 1.7 - 0.01 \cdot H = 0.95$	Correction factor for relative humidity of ambient air
$\gamma_{st} := \dfrac{5}{1 + \dfrac{f_{ci\_beam}}{ksi}} = 0.68$	Correction factor for specified concrete strength at time of prestress transfer to the concrete member
$\Delta f_{pR} := f_t \cdot 1.75\% = 3.42 \cdot ksi$	Relaxation loss taken as 1.75% of the initial pull per experimental results from Grace et. al based on 1,000,000 hours (114 years)
$\Delta f_{pLT} := 10 \cdot \dfrac{f_t \cdot A_{ps}}{A_{beam}} \cdot \gamma_h \cdot \gamma_{st} + 12ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} = 22.31 \cdot ksi$	long term prestress loss

Difference in thermal coefficient expansion between concrete and CFCC

$\alpha := 6 \cdot 10^{-6} \cdot \dfrac{1}{F}$	Difference in coefficient of thermal expansion between concrete and CFCC
$t_{amb} := 68F$	Ambient temperature
$t_{low} := -10F$	lowest temperature in Michigan according to AASHTO IRFD 3.12.2
$\Delta t := t_{amb} - t_{low} = 78\,F$	Change in the temperature
$\Delta f_{pt} := \alpha \cdot \Delta t \cdot E_p = 9.83 \cdot ksi$	Prestress losses due to temp. effect
$f_{pe} := f_t - \Delta f_{pLT} - \Delta f_{PES} - \Delta f_{pt} = 150.67 \cdot ksi$	Effective prestressing stress after all losses
$P_e := A_{ps} \cdot f_{pe} = 1051.81 \cdot kip$	Effective prestressing force after all losses
$f_t = 195.33 \cdot ksi$	Initial prestress prior to transfer, not including anchorage losses
$f_{pe} = 150.67 \cdot ksi$	Prestress level after all losses
$loss := \dfrac{f_t - f_{pe}}{f_t} = 22.86 \cdot \%$	Total prestress loss

Debonding Criteria

Estimate the location from each beam end where top prestressing or debonding is no longer needed The vectors are developed for possible two different deboning lengths per row. Enter the number of debonded strands and the estimated debonding length in the vectors below per each row location

Location: number of strands: debonding length:

$$Row_{db} := \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \\ 5 \\ 5 \\ 6 \\ 6 \end{pmatrix} \qquad n_{db} := \begin{pmatrix} 3 \\ 2 \\ 2 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad l_{db} := \begin{pmatrix} 15 \\ 5 \\ 10 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot ft$$

For debonding pattern, follow staggering guidelines in MDOT BDM 7.02.18.A.2

```
row_db := | for  i ∈ 0 .. 2 length(Row) − 1
          |    D_i ← Row_db_i
          | D
```

```
N_db := | for  i ∈ 0 .. length(row_db) − 1
        |    D_i ← n_db_i
        | D
```

```
L_db := | for  i ∈ 0 .. length(row_db) − 1
        |    D_i ← l_db_i
        | D
```

$$row_{db} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \end{pmatrix} \qquad N_{db} = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad L_{db} = \begin{pmatrix} 15 \\ 5 \\ 10 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} ft$$

$$\sum N_{db} = 11$$

$$Debond_{tot} := \frac{\sum N_{db}}{NO_{strands}} = 28.21 \cdot \%$$

Portion of partially debonded strands in beam section

$$\text{if}\left(Debond_{tot} \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

Total number of debonded strands in rows

```
N_db.row :=
    for $i \in 0..\text{length}(Row) - 1$
        $a_i \leftarrow 0$
        for $j \in 0..\text{length}(N_{db}) - 1$
            $a_i \leftarrow a_i + N_{db_j}$  if $row_{db_j} = i + 1$
    a
```

$$= \begin{pmatrix} 5.00 \\ 6.00 \\ 0.00 \\ 0.00 \end{pmatrix}$$

```
Debond_row :=
    for $i \in 0..\text{length}(Row) - 1$
        $a_i \leftarrow 0$
        $a_i \leftarrow \frac{N_{db.row_i}}{Row_i}$  if $Row_i > 0$
        0  otherwise
    a
```

$$= \begin{pmatrix} 33.33 \\ 33.33 \\ 0.00 \\ 0.00 \end{pmatrix} \cdot \%$$

$$\text{if}\left(\max\left(Debond_{row}\right) \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

The limit of 40% is taken according to MDOT BDM 7.02.18. A2

Optional: only needed if debonding scheme is not sufficient to eliminate the tensile stresses at beam ends either at transfer or due to handling and shipping

CFCC strand transfer length, **ACI 440.4R Table 6.1**

$$L_t := 50d_s = 2.49\,\text{ft}$$

Number of top prestressing strands in the top flange

$Row_{top} := \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

Depth of the top prestressing strands from the top surface of the beam

$d_{top} := \begin{pmatrix} 3 \\ 0 \end{pmatrix} \cdot in$

Initial prestressing stress/force at the top prestressing strands

$F_{p\_top} := 50 \cdot ksi$

Distance from the end of the beam to the point where the top prestressing is no longer needed

$x_{p\_top} := 15 \cdot ft$

Top prestressing strands shall not extend the the middle third of the beam. Otherwise, it could affect the stresses at service limit state

$$Check\_Top\_prestressing\_Length := \left| \begin{array}{l} \text{"Okay"} \quad \text{if } x_{p\_top} \le \frac{L_{beam}}{3} \\ \text{"Check service stress @ x.p\_top"} \quad \text{if } x_{p\_top} > \frac{L_{beam}}{3} \end{array} \right. = \text{"Okay"}$$

Distance from the end of the beam to the pocket where top prestressing strand is cut after concrete pouring. The middle region between the cut pockets shall be dobonded to avoid force transfer to the middle region

$x_{pocket} := x_{p\_top} + L_t = 17.493\,ft$

Serviceability Checks

Stress check locations along the beam

Stress locations after the transfer length for bonded and de-bonded strands

Since this beam is provided with and end block, it is important to check the stresses at the end of the end block. Therefore, L.end was added to the x.release

$$X_{release} := \mathrm{sort}\left[\mathrm{stack}\left[L_t, L_{end}, \left(L_{db} + L_t\right), x_{p\_top}, x_{pocket}\right]\right] = \begin{pmatrix} 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.667 \\ 7.493 \\ 7.493 \\ 12.493 \\ 15 \\ 17.493 \\ 17.493 \end{pmatrix} \mathrm{ft}$$

Extracting repreated X from the vector

```
$x_{release} :=$ | $k \leftarrow 0$
                 | $x_0 \leftarrow L_t$
                 | for  $i \in 1 .. \mathrm{length}(X_{release}) - 1$
                 |     | $k \leftarrow k + 1$  if  $(X_{release_i} \neq X_{release_{i-1}})$
                 |     | $x_k \leftarrow X_{release_i}$
                 | $x$
```

$$x_{release} = \begin{pmatrix} 2.493 \\ 2.667 \\ 7.493 \\ 12.493 \\ 15 \\ 17.493 \end{pmatrix} \cdot \mathrm{ft}$$

Area of strands in each row at each stress check location

```
$A_{db} :=$
  for $i \in 0..\text{length}(x_{release}) - 1$
    for $z \in 0..\text{length}(Row) - 1$
      $A_{i,z} \leftarrow Row_z \cdot A_{strand}$
    for $j \in 0..\text{length}(N_{db}) - 1$
      $n \leftarrow N_{db_j}$
      $row \leftarrow row_{db_j}$
      $L \leftarrow L_{db_j}$
      $A_{i,row-1} \leftarrow (A_{i,row-1} - n \cdot A_{strand}) \cdot \frac{x_{release_i}}{L_t}$  if $x_{release_i} < L_t$
      $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand}$  if $L_t \le x_{release_i} \le L$
      $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand} \cdots + n \cdot A_{strand} \cdot \frac{(x_{release_i} - L)}{L_t}$  if $L < x_{release_i} \le L + L_t$
  $A$
```

$$A_{db} = \begin{pmatrix} 1.79 & 2.15 & 0.72 & 0.36 \\ 1.79 & 2.15 & 0.72 & 0.36 \\ 2.15 & 2.86 & 0.72 & 0.36 \\ 2.15 & 3.22 & 0.72 & 0.36 \\ 2.15 & 3.22 & 0.72 & 0.36 \\ 2.69 & 3.22 & 0.72 & 0.36 \end{pmatrix} \cdot in^2$$

Beam stresses at release due to prestressing only

Sign convention; negative and positive stresses/forces for compression and tension respectively

$$P_{ps} := -F_{pt} \cdot A_{db} = \begin{pmatrix} -327.22 & -392.66 & -130.89 & -65.44 \\ -327.22 & -392.66 & -130.89 & -65.44 \\ -392.66 & -523.55 & -130.89 & -65.44 \\ -392.66 & -588.99 & -130.89 & -65.44 \\ -392.66 & -588.99 & -130.89 & -65.44 \\ -490.82 & -588.99 & -130.89 & -65.44 \end{pmatrix} \cdot kip$$

Midspan moment due to prestressing at release

$$M_{ps} := P_{ps} \cdot \left(d_{strand} - y_t\right) = \begin{pmatrix} -965.286 \\ -965.286 \\ -1180.703 \\ -1248.873 \\ -1248.873 \\ -1367.489 \end{pmatrix} \cdot kip \cdot ft$$

Top and bottom concrete stresses at check locations due to prestressing ONLY

```
f_ps := | for $i \in 0 .. \text{length}(x_{release}) - 1$
        |     | $M \leftarrow M_{ps_i}$
        |     | $P \leftarrow \sum_{j=0}^{\text{cols}(P_{ps})-1} P_{ps_{i,j}}$
        |     | $A \leftarrow A_{beamf}(x_{release_i})$
        |     | $S_{top} \leftarrow S_{Tf}(x_{release_i})$
        |     | $S_{bott} \leftarrow S_{Bf}(x_{release_i})$
        |     | $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
        |     | $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
        | f
```

$$f_{ps} = \begin{array}{cc} \text{top} & \text{bottom} \\ \hline 751 & -1908 \\ 674 & -2343 \\ 783 & -3022 \\ 750 & -3408 \\ 699 & -3529 \\ 721 & -3983 \end{array} \cdot psi$$

Beam stresses at release due to selfweight

For the selfweight moment due to the varying web thickness, assume a linear varying moment from the end of the beam to the point of constant web thickness. This will result in a slight under-estimation of the self weight in the area of the varying web thickness.

Moment due to self weight of beam at check locations

$$M_{sw}(x) := \left| \begin{array}{l} \left(\omega_{beam} \cdot \frac{L_{beam}}{2} + w_1 \cdot L_{end} + 0.5 \cdot w_2 \cdot L_{var}\right) \cdot x - \left(\omega_{beam} + w_1\right) \cdot \frac{x^2}{2} \quad \text{if } 0 \le x < L_{end} \\ \left(\omega_{beam} \cdot \frac{L_{beam}}{2} + w_1 \cdot L_{end} + 0.5 \cdot w_2 \cdot L_{var}\right) \cdot x - \left[\omega_{beam} \cdot \frac{x^2}{2} + w_1 \cdot L_{end} \cdot \left(x - \frac{L_{end}}{2}\right) + 0.5 \cdot w_2 \cdot \right. \\ \left(\omega_{beam} \cdot \frac{L_{beam}}{2} + w_1 \cdot L_{end} + 0.5 \cdot w_2 \cdot L_{var}\right) \cdot x - \left[\omega_{beam} \cdot \frac{x^2}{2} + w_1 \cdot L_{end} \cdot \left(x - \frac{L_{end}}{2}\right) + 0.5 \cdot w_2 \cdot L\right. \end{array} \right.$$

Top and bottom concrete stresses at check locations due to beam self weight ONlY

```
$f_{sw} :=$ for $i \in 0 .. \text{length}(x_{release}) - 1$
        $M \leftarrow M_{sw}(x_{release_i})$
        $f_{i,0} \leftarrow \frac{-M}{S_{Tf}(x_{release_i})}$
        $f_{i,1} \leftarrow \frac{M}{S_{Bf}(x_{release_i})}$
    $f$
```

$$f_{sw} = \begin{array}{c} \text{top} \quad \text{bottom} \\ \begin{pmatrix} -104 & 104 \\ -125 & 125 \\ -328 & 328 \\ -511 & 511 \\ -593 & 593 \\ -655 & 655 \end{pmatrix} \end{array} \cdot psi$$

Area of **top prestressing** strands at distance X.release from the end

```
$A_{top} :=$ for $i \in 0 .. \text{length}(x_{release}) - 1$
        for $z \in 0 .. \text{length}(Row_{top}) - 1$
            $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} \cdot \frac{x_{release_i}}{L_t}$  if $x_{release_i} \le L_t$
            $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand}$  if $L_t < x_{release_i} \le x_{p\_top}$
            $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} - \frac{x_{release_i} - x_{p\_top}}{L_t} \cdot (Row_{top_z} \cdot A_{strand})$  if $x_{p\_top} < x_{release_i} \le x$
            $A_{i,z} \leftarrow 0$  if $x_{release_i} > x_{p\_top} + L_t$
    $A$
```

$$A_{top} = \begin{pmatrix} 0.895 & 0 \\ 0.895 & 0 \\ 0.895 & 0 \\ 0.895 & 0 \\ 0.895 & 0 \\ 0 & 0 \end{pmatrix} \cdot in^2 \qquad x_{release} = \begin{pmatrix} 2.493 \\ 2.667 \\ 7.493 \\ 12.493 \\ 15 \\ 17.493 \end{pmatrix} ft$$

$$P_{p\_top} := -F_{p\_top} \cdot A_{top} = \begin{pmatrix} -44.75 & 0.00 \\ -44.75 & 0.00 \\ -44.75 & 0.00 \\ -44.75 & 0.00 \\ -44.75 & 0.00 \\ -0.00 & 0.00 \end{pmatrix} \cdot kip$$

$$M_{p\_top} := P_{p\_top} \cdot \left(d_{top} - y_t\right) = \begin{pmatrix} 50.344 \\ 50.344 \\ 50.344 \\ 50.344 \\ 50.344 \\ 0 \end{pmatrix} \cdot kip \cdot ft$$

```
$f_{p\_top} :=$
    for $i \in 0 .. \mathrm{length}(x_{release}) - 1$
        $M \leftarrow M_{p\_top_i}$
        $P \leftarrow \sum_{j=0}^{\mathrm{cols}(P_{p\_top})-1} P_{p\_top_{i,j}}$
        $A \leftarrow A_{beamf}(x_{release_i})$
        $S_{top} \leftarrow S_{Tf}(x_{release_i})$
        $S_{bott} \leftarrow S_{Bf}(x_{release_i})$
        $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
        $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
    $f$
```

Stresses in the beam due to the top prestressing strands only

$$f_{p\_top} = \begin{pmatrix} -97.605 & 41.083 \\ -119.445 & 37.891 \\ -126.137 & 36.076 \\ -134.283 & 33.311 \\ -138.965 & 31.464 \\ -5.399 \times 10^{-14} & 1.103 \times 10^{-14} \end{pmatrix} psi$$

Check for beam stresses at release against allowable stresses

Beam stresses at release

$$f_{c.release} := f_{ps} + f_{sw} + f_{p\_top} = \begin{array}{c} \text{top} \quad\quad \text{bottom} \\ \begin{pmatrix} 549.472 & -1763.229 \\ 428.6 & -2179.869 \\ 328.515 & -2657.588 \\ 104.021 & -2862.927 \\ -32.475 & -2904.826 \\ 66.177 & -3328.14 \end{pmatrix} \end{array} \cdot psi \qquad x_{release} = \begin{pmatrix} 2.49 \\ 2.67 \\ 7.49 \\ 12.49 \\ 15.00 \\ 17.49 \end{pmatrix} ft$$

$f_{ti.release} := \max(f_{c.release}) = 549\,psi$ Maximum tensile stress at release

$f_{ci.release} := \min(f_{c.release}) = -3328\,psi$ Maximum compressive stress at release

$\text{if}(f_{ti} \geq f_{ti.release}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$ Allowable tension check $f_{ti} = 607\,psi$

$\text{if}(-f_{ci} \geq -f_{ci.release}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$ Allowable compression check $f_{ci} = -4160\,psi$

Camber immediately after transfer

Camber calculations ignores the variable cross section

Camber due to prestressing assuming constant maximum force (ignore debonding)

$$\frac{-\min(M_{ps}) \cdot L_{beam}^{2}}{8 \cdot E_{c.beam\_i} \cdot I_{beam}} = 2.494 \cdot in$$

Deflection due to top prestressing assuming constant maximum force (including debonding transfer length)

$$\delta_{p\_top} := \frac{M_{p\_top_0} \cdot x_{p\_top}^{2}}{2 \cdot (E_{c.beam\_i} \cdot I_{beam})} = 0.018 \cdot in$$

Deflection due to selfweight of the beam

$$\frac{-5 \cdot \omega_{beam} \cdot L_{beam}^{4}}{384 \cdot E_{c.beam\_i} \cdot I_{beam}} = -0.714 \cdot in$$

$$d_{strand.db} := \left| \begin{array}{l} \text{for } i \in 0 .. \text{length}\left(row_{db}\right) - 1 \\ \quad d_{s_i} \leftarrow d - (2in)row_{db_i} \\ d_s \end{array} \right. = \begin{pmatrix} 31.00 \\ 31.00 \\ 29.00 \\ 29.00 \\ 27.00 \\ 27.00 \\ 25.00 \\ 25.00 \end{pmatrix} \cdot in$$

$$\delta_{db} := \frac{\overrightarrow{\left[N_{db} \cdot A_{strand} \cdot F_{pt} \cdot \left(d_{strand.db} - y_t\right) \cdot \left(L_{db} + L_t\right)^2\right]}}{2 \cdot E_{c.beam\_i} \cdot I_{beam}} = \begin{pmatrix} 0.057 \\ 7.021 \times 10^{-3} \\ 0.017 \\ 0.012 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot in$$

$$\sum \delta_{db} = 0.093 \cdot in$$

$$Camber_{tr} := \frac{-\min\left(M_{ps}\right) \cdot L_{beam}^{\ 2}}{8 \cdot E_{c.beam\_i} \cdot I_{beam}} - \frac{5 \cdot \omega_{beam} \cdot L_{beam}^{\ 4}}{384 \cdot E_{c.beam\_i} \cdot I_{beam}} - \sum \delta_{db} - \delta_{p\_top} = 1.668 \cdot in$$

Positive sign indicates camber upwards. Negative sign indeicates deflection

Check the stresses of the beam during shipping and handling, where the supports are not at the ends of the beam (Find the exact location of the supports during shipping and handling)

Moment due to self weight of beam at check locations

$$M_{sw.ship}(x) := \left| \begin{array}{l} -(\omega_{beam}+w_1)\cdot\frac{x^2}{2} \quad \text{if } 0 \le x < L_{end} \wedge 0\cdot in \le x \le l_{ship} \\ -\left[\omega_{beam}\cdot\frac{x^2}{2}+w_1\cdot L_{end}\cdot\left(x-\frac{L_{end}}{2}\right)+0.5\cdot w_2\cdot\frac{(x-L_{end})^2}{2}\right] \quad \text{if } L_{end} \le x < L_{var}+L_{end} \\ -\left[\omega_{beam}\cdot\frac{x^2}{2}+w_1\cdot L_{end}\cdot\left(x-\frac{L_{end}}{2}\right)+0.5\cdot w_2\cdot L_{var}\cdot\left(x-L_{end}-\frac{L_{var}}{3}\right)\right] \quad \text{if } x \ge L_{var}+L_{e} \\ \left(\omega_{beam}\cdot\frac{L_{beam}}{2}+w_1\cdot L_{end}+0.5\cdot w_2\cdot L_{var}\right)\cdot(x-l_{ship})-(\omega_{beam}+w_1)\cdot\frac{x^2}{2} \quad \text{if } 0 \le x < L \\ \left(\omega_{beam}\cdot\frac{L_{beam}}{2}+w_1\cdot L_{end}+0.5\cdot w_2\cdot L_{var}\right)\cdot(x-l_{ship})-\left[\omega_{beam}\cdot\frac{x^2}{2}+w_1\cdot L_{end}\cdot\left(x-\frac{L_{en}}{2}\right.\right. \\ \left(\omega_{beam}\cdot\frac{L_{beam}}{2}+w_1\cdot L_{end}+0.5\cdot w_2\cdot L_{var}\right)\cdot(x-l_{ship})-\left[\omega_{beam}\cdot\frac{x^2}{2}+w_1\cdot L_{end}\cdot\left(x-\frac{L_{en}}{2}\right.\right. \end{array} \right.$$

Top and bottom concrete stresses at check locations due to beam self weight ONIY

$$f_{sw.ship} := \left| \begin{array}{l} \text{for } i \in 0..\,\text{length}(x_{release})-1 \\ \quad \left| \begin{array}{l} M \leftarrow M_{sw.ship}(x_{release_i}) \\ f_{i,0} \leftarrow \frac{-M}{S_{Tf}(x_{release_i})} \\ f_{i,1} \leftarrow \frac{M}{S_{Bf}(x_{release_i})} \end{array} \right. \\ f \end{array} \right.$$

$$f_{sw.ship} = \begin{array}{cc} \text{top} & \text{bottom} \\ \hline -59 & 59 \\ -75 & 75 \\ -276 & 276 \\ -458 & 458 \\ -538 & 538 \\ -599 & 599 \end{array} \cdot psi$$

Check for beam stresses during handling & shipping against allowable stresses

Beam stresses during shipping @ handling

$$f_{c.ship} := f_{ps} + f_{sw.ship} + f_{p\_top} = \begin{array}{cc} \text{top} & \text{bottom} \\ \hline 593.974 & -1807.731 \\ 479.086 & -2230.355 \\ 380.566 & -2709.639 \\ 157.799 & -2916.704 \\ 22.213 & -2959.513 \\ 121.739 & -3383.702 \end{array} \cdot psi$$

$$x_{release} = \begin{pmatrix} 2.49 \\ 2.67 \\ 7.49 \\ 12.49 \\ 15.00 \\ 17.49 \end{pmatrix} ft$$

$f_{ti.ship} := \max(f_{c.ship}) = 594\,psi$ Maximum tensile stress at release

$f_{ci.ship} := \min(f_{c.ship}) = -3384\,psi$ Maximum compressive stress at release

$\text{if}\left(f_{ti} \geq f_{ti.ship}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable tension check $f_{ti} = 607\,\text{psi}$

$\text{if}\left(-f_{ci} \geq -f_{ci.ship}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable compression check $f_{ci} = -4160\,\text{psi}$

Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent loads only

Compressive stress at top of deck due to loads on composite section

$$f_{cf\_actual\_mid} := \frac{-\left(M_{DC2} + M_{DW}\right)}{S_{t3n} \cdot k_{sdl} \cdot n} = -73\,\text{psi}$$

$\text{if}\left(-f_{cf.deckp} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress and permament loads only

Compressive stress at top flange of beam due to prestressing and permanent loads

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} = -1323\,\text{psi}$$

$\text{if}\left(-f_{cfp} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent and transient loads

Compressive stress at top of deck due to loads on composite section **including wind effect according to AASHTO LRFD 2016 Interim revision**

$$f_{cf\_actual\_mid} := \frac{-\left(M_{DC2} + M_{DW}\right)}{S_{t3n} \cdot k_{sdl} \cdot n} - \frac{1.0 M_{LLI}}{S_{tn} \cdot k \cdot n} - \frac{1.0 M_{WS}}{S_{tn} \cdot k \cdot n} = -592\,\text{psi}$$

$\text{if}\left(-f_{cf.deck} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$ Allowable stress check

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress, permanent, and transient loads

Compressive stress at top flange of beam due to prestressing and all loads................

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} - \frac{M_{LLI}}{S_{t.bm.n}} - \frac{1.0 \cdot M_{WS}}{S_{t.bm.n}} = -1588\,\text{psi}$$

$\text{if}\left(-f_{cf} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

Allowable stress check

Service III limit State - Check for tensile stresses at bottom flange of beam at service conditions

Tensile stress at bottom flange of beam due to prestressing and all loads

$$f_{tf\_actual\_mid} := \frac{-P_e}{A_{beam}} - \frac{P_e \cdot e_s}{S_B} + \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = -379 \cdot psi$$

$\text{if}\left(f_{tf} > f_{tf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

Allowable stress check

Calculate bar area required to resist tension in the top flange at release, AASHTO Table 5.9.4.1.2-1:

$f_{ti.ship} = 593.974\,psi$

Maximum top flange tensile stress at release or handling, whichever is larger (usually, handling stresses are larger)

$f_c := \text{vlookup}\left(f_{ti.ship}, f_{c.ship}, 1\right)_0 = -1.808 \times 10^3\,psi$

Bottom flange compressive stress corresponding to the maximum top flange tensile stress at release/shipping

$i_c := \text{match}\left(f_{ti.ship}, f_{c.ship}\right)_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Finding the location of the maximum tensile stresses to calculate the section dimensions

$x_c := x_{release}\left(i_{c_0}\right) = 2.493\,ft$

$$slope_m := \frac{f_{ti.ship} - f_c}{d} = 72.779 \cdot \frac{psi}{in}$$

Slope of the section stress over the depth of the beam

$$x_o := \frac{f_{ti.ship}}{slope_m} = 8.161 \cdot in$$

Distance measured from the top of the beam to the point of zero stress

Calculate the width of the beam where the tensile stresses are acting

```
$b_{ten} :=$ | for $i \in 0..\text{ceil}\left(\frac{x_o}{in}\right)$
             |     $x_i \leftarrow \frac{x_o \cdot i}{\text{ceil}\left(\frac{x_o}{in}\right)}$
             |     $b_i \leftarrow b_{ft}$  if $0 \le x_i \le d_{ft}$
             |     $b_i \leftarrow 2 \cdot b_{webf}\left(x_c\right)$  if $x_i > d_{ft}$
             | $b$
```

Calculate the tensile stress values every inch of depth starting from the top surface of the beam

```
$f :=$ | for $i \in 0..\text{ceil}\left(\frac{x_o}{in}\right)$
       |     $x_i \leftarrow \frac{x_o \cdot i}{\text{ceil}\left(\frac{x_o}{in}\right)}$
       |     $f_i \leftarrow f_{ti.ship} - slope_m \cdot x_i$
       | $f$
```

$$f = \begin{pmatrix} 593.974 \\ 527.977 \\ 461.98 \\ 395.983 \\ 329.986 \\ 263.989 \\ 197.991 \\ 131.994 \\ 65.997 \\ 0 \end{pmatrix} psi \qquad b_{ten} = \begin{pmatrix} 48 \\ 48 \\ 48 \\ 48 \\ 48 \\ 48 \\ 48 \\ 48 \\ 48 \\ 48 \end{pmatrix} \cdot in$$

Calculate the tensile force that shall be resisted by top reinforcement

$$T := \sum_{i=0}^{\text{length}(f)-2} \left[\frac{1}{4} \cdot \left(f_i + f_{i+1}\right) \cdot \left(b_{ten_i} + b_{ten_{i+1}}\right) \cdot \frac{x_o}{\text{ceil}\left(\frac{x_o}{in}\right)} \right] = 116.343 \cdot kip$$

$$A_{s.top} := \frac{T}{30 \cdot ksi} = 3.878 \cdot in^2$$

Calculate area of tensile reinforcement required in the top of the beam. The stress in bars is limited to 30ksi per AASHTO 5.9.4.1.2. See Figure C.5.9.4.1.2-1 which is based upon .5 f.y of steel rebar

$A_{bar.top} := 0.44 \cdot in^2$ Cross sectional area of No. 6 steel rebars

$n_{bar.release} := \mathrm{Ceil}\left(\dfrac{A_{s.top}}{A_{bar.top}}, 1\right) = 9$ number of No. 6 bars provided in the top flange to resist tension at release in the beam ends.

Calculation of minimum length of top tensile reinforcement

AASHTO LRFD Table 5.9.4.1.2-1 specifies a maximum concrete tensile stress of $0.0948 \cdot \sqrt{f_{ci\_beam}} \le 0.2\ ksi$ for tensile zones without bonded reinforcement

$$f_{t.max} := \min\left(0.0948 \cdot \sqrt{\frac{f_{ci\_beam}}{ksi}}, 0.2\right) \cdot ksi = 0.2 \cdot ksi$$

Calculate the minimum required length of top reinforcement based on the stress calculated at distances x.release during release or shipping and handling, whichever is greater. If all the stresses are larger tha f.t.max, estimate the stress after the last point of debonding. The change in the web width is ignored

```
L_topr :=  | $h \leftarrow x_{release}$
           | $f \leftarrow f_{c.ship}^{\langle 0 \rangle}$
           | $i \leftarrow \mathrm{length}(f) - 1$
           | while $f_i < f_{t.max}$
           |     | break if $i = 0$
           |     | $i \leftarrow i - 1$
           | $x \leftarrow 1 \cdot ft$
           | $f_{ps} \leftarrow f_{ps_{\mathrm{rows}(f_{ps})-1,0}}$
           | $S(x) \leftarrow f_{ps} - f_{t.max} - \dfrac{\dfrac{\omega_{beam} \cdot L_{beam} \cdot (x - l_{ship})}{2} - \dfrac{(\omega_{beam} \cdot x^2)}{2}}{S_T}$
           | $g \leftarrow \mathrm{root}(S(x), x)$
           | $g$ if $f_{\mathrm{length}(f)-1} > f_{t.max}$
           | $\dfrac{L_{beam}}{2}$ if $\mathrm{Im}(g) \ne 0 \wedge f_{\mathrm{length}(f)-1} > f_{t.max}$
           | $h_{i+1}$ otherwise
```

$L_{topr} = 12.493\ ft$

$$l_d := 1.4 \cdot \frac{1.25 \cdot \frac{\pi \cdot \left(\frac{6}{8}\right)^2}{4} \cdot 60}{\sqrt{\frac{f_{c\_beam}}{ksi}}} \cdot in = 1.367 \cdot ft$$

Calculate the tension development length required for the tensile reinforcement in the top of the beam. As provided AASHTO 5.11.2.1.1 taking into account 1.4 modification factor per AASHTO 5.11.2.1.2

$$L_{topR} := L_{topr} + l_d = 13.86\,ft$$

Minimum length required for the top reinforcement from each end

Flexural Capacity

Stress block factor, **AASHTO 5.7.2.2.** Assuming depth of neutral axis lies within the deck

$$\beta_1 := \left| \begin{array}{ll} 0.65 \ \text{if} \ f_{c\_deck} \geq 8000psi & \\ 0.85 \ \text{if} \ f_{c\_deck} \leq 4000psi & \\ \left[0.85 - \left(\frac{f_{c\_deck} - 4000psi}{1000psi}\right)0.05\right] & \text{otherwise} \end{array} \right. = 0.8$$

$$\varepsilon_{cu} := 0.003$$

Maximum usable concrete compressive strain

$$\varepsilon_{pu} := \frac{f_{pu}}{E_p} = 0.0145$$

Ultimate tensile strain of CFCC strand

$$\varepsilon_{pe} := \frac{f_{pe}}{E_p} = 0.0072$$

Effective CFCC prestressing strain

$$\varepsilon_0 := \varepsilon_{pu} - \varepsilon_{pe} = 0.0074$$

Reserve strain in CFCC

$$d_i := d_{strand} + haunch + deck_{thick} = \begin{pmatrix} 40.00 \\ 38.00 \\ 36.00 \\ 34.00 \end{pmatrix} \cdot in$$

Depth of prestressing strands from top of concrete deck

$$A_f := A_{strand} \cdot Row = \begin{pmatrix} 2.69 \\ 3.22 \\ 0.72 \\ 0.36 \end{pmatrix} \cdot in^2$$

Area of strands in rows

$$P_{row} := A_f \cdot f_{pe} = \begin{pmatrix} 404.54 \\ 485.45 \\ 107.88 \\ 53.94 \end{pmatrix} \cdot kip$$

Effective prestressing force of strands in rows

$$s_i := \left| \begin{array}{l} \text{for } i \in 0 \,..\, \text{length}(Row) - 1 \\ \quad s_i \leftarrow d_{i_0} - d_{i_i} \\ s \end{array} \right. = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \end{pmatrix} \cdot in$$

Distance from each layer of prestressing strands to the bottom prestressting layer

$$deck_{eff} := deck_{thick} - t_{wear} = 9 \cdot in$$

Effective deck thickness (total thickness minus assumed sacrificial wearing surface thickness)

Balanced reinforcement ratio

$$c_{bal} := \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_0} \cdot d_{i_0} = 11.585 \cdot in$$

Depth of neutral axis at balanced failure

Balanced reinforcement ratio assuming **Rectangular section**

$$\rho_{R\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{eff} \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} = 0.0046$$

Balanced reinforcement ratio assuming **Flanged section**

$$\rho_{Fl\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot deck_{eff} \cdot \left(b_{eff} - b_{ft}\right) + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{ft} \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} = 0.0045$$

Balanced reinforcement ratio assuming **Double Flanged section**

$$\rho_{DFl\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot deck_{eff} \cdot \left(b_{eff} - b_v\right) + 0.85 \cdot f_{c\_deck} \cdot d_{ft} \cdot \left(b_{ft} - b_v\right) + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_v \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} =$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Tension contorlled** section

```
$Fl\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e - 0.85 \cdot f_{c\_deck} \cdot \left(b_{eff} - b_{ft}\right) \cdot deck_{eff}}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{ft}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_T = \begin{pmatrix} 1.5268 \\ 0.0017 \end{pmatrix}$$

$$c_{Fl\_T} := Fl\_T_0 \cdot in = 1.527 \cdot in$$

$$\rho_{Fl\_T} := Fl\_T_1 = 0.0017$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Tension contorlled** section

```
$R\_T :=$
    $c \leftarrow 1.0\cdot in$
    $A_{eq\_s} \leftarrow 1.0\cdot in^{2}$
    $A_{eq\_f} \leftarrow 2.0\cdot in^{2}$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^{2}$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p\cdot \varepsilon_0\cdot A_{eq\_s} + P_e}{0.85\cdot f_{c\_deck}\cdot \beta_1\cdot b_{eff}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$R\_T = \begin{pmatrix} 6.366119 \\ 0.001729 \end{pmatrix}$$

$$c_{R\_T} := R\_T_0\cdot in = 6.366\cdot in$$

$$\rho_{R\_T} := R\_T_1 = 0.0017$$

Depth of the N.A. and reinforcement ratio assuming **Double-Flanged Tension contorlled** section. The depth of the stress block is deeper than the depth of the deck and the top flange together.

```
$DFl\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow \mathrm{length}\left(d_i\right) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e - 0.85 \cdot f_{c\_deck} \cdot \left(b_{eff} - b_v\right) \cdot deck_{eff} - 0.85 \cdot f_{c\_deck} \cdot \left(b_{ft} - b_v\right) \cdot d_{ft}}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_v}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$(DFl\_T) = \begin{pmatrix} -72.1350 \\ 0.0018 \end{pmatrix}$$

$$\left(c_{DFl\_T} := DFl\_T_0 \cdot in = -72.135 \cdot in\right)$$

$$\left(\rho_{DFl\_T} := DFl\_T_1 = 0.0018\right)$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu} \cdot \left(\frac{d_{i_0} - c}{c} \right)$$

```
Fl_C := | $c \leftarrow 1 \cdot in$
        | $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
        | $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
        | $N \leftarrow \text{length}(d_i) - 1$
        | while $\left| A_{eq\_s} - A_{eq\_f} \right| > 0.01 \cdot in^2$
        |     $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        |     $f(c) \leftarrow 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_{ft}) \cdot deck_{eff} + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{ft} \cdots$
        |         $+ (-E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e)$
        |     $c \leftarrow \text{root}(f(c), c, 0.1 \cdot in, d_{i_0})$
        |     $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        | $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
        | $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_C = \begin{pmatrix} 7.086548 \\ 0.001727 \end{pmatrix}$$

$$c_{Fl\_C} := Fl\_C_0 \cdot in = 7.087 \cdot in$$

$$\rho_{Fl\_C} := Fl\_C_1 = 0.0017$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Compression contorlled** section

```
$R\_C :=$
    $c \leftarrow 1 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow \mathrm{length}(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
        $g(c) \leftarrow 0.85 f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{eff} - E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e$
        $c \leftarrow \mathrm{root}\left(g(c), c, 0.1 \cdot in, d_{i_0}\right)$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$R\_C = \begin{pmatrix} 8.1876 \\ 0.0017 \end{pmatrix}$$

$$c_{R\_C} := R\_C_0 \cdot in = 8.188 \cdot in$$

$$\rho_{R\_C} := R\_C_1 = 0.0017$$

Depth of the N.A. and reinforcement ratio assuming **Double Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu} \cdot \left(\frac{d_{i_0} - c}{c} \right)$$

```
DFl_C := | $c \leftarrow 1 \cdot in$
         | $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
         | $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
         | $N \leftarrow length(d_i) - 1$
         | while $|A_{eq\_s} - A_{eq\_f}| > 0.01 \cdot in^2$
         |     $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
         |     $f(c) \leftarrow 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_{web}) \cdot deck_{eff} + 0.85 \cdot f_{c\_deck} \cdot (b_{ft} - b_{web}) \cdot d_{ft} \ldots$
         |         $+ 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{web} - E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e$
         |     $c \leftarrow root(f(c), c, 0.1 \cdot in, d_{i_0})$
         |     $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
         | $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
         | $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$DFl\_C = \begin{pmatrix} 4.154495 \\ 0.001735 \end{pmatrix}$$

$c_{DFl\_C} := DFl\_C_0 \cdot in = 4.154 \cdot in$

$\rho_{DFl\_C} := DFl\_C_1 = 0.0017$

Check the mode of failure

$$\text{Section\_Mode} := \left|\begin{array}{l} \text{"Rectangular\_Tension"} \ \text{ if } \ \beta_1 \cdot c_{R\_T} \le deck_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ \text{"Rectangular\_Compression"} \ \text{ if } \ \beta_1 \cdot c_{R\_C} \le deck_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ \text{"Flanged\_Tension"} \ \text{ if } \ \beta_1 \cdot c_{Fl\_T} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_T} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_T} < \rho_{Fl\_} \\ \text{"Flanged\_Compression"} \ \text{ if } \ \beta_1 \cdot c_{Fl\_C} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_C} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_C} > \\ \text{"Double\_Flanged\_Tension"} \ \text{ if } \ \beta_1 \cdot c_{DFl\_T} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_T} < \rho_{DFl\_bal} \\ \text{"Double\_Flanged\_Compression"} \ \text{ if } \ \beta_1 \cdot c_{DFl\_C} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_C} > \rho_{DFl\_bal} \end{array}\right.$$

$$(\text{Section\_Mode}) = \text{"Rectangular\_Tension"}$$

Select the correct depth of the N.A.

$$c := \left|\begin{array}{l} c_{R\_T} \ \text{ if } \ \beta_1 \cdot c_{R\_T} \le deck_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ c_{R\_C} \ \text{ if } \ \beta_1 \cdot c_{R\_C} \le deck_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ c_{Fl\_T} \ \text{ if } \ \beta_1 \cdot c_{Fl\_T} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_T} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_T} < \rho_{Fl\_bal} \\ c_{Fl\_C} \ \text{ if } \ \beta_1 \cdot c_{Fl\_C} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_C} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_C} > \rho_{Fl\_bal} \\ c_{DFl\_T} \ \text{ if } \ \beta_1 \cdot c_{DFl\_T} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_T} < \rho_{DFl\_bal} \\ c_{DFl\_C} \ \text{ if } \ \beta_1 \cdot c_{DFl\_C} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_C} > \rho_{DFl\_bal} \end{array}\right.$$

$$c = 6.366 \cdot in$$

Disclaimer: The design of the section as a dobule flanged section, while theoretically possible, indicates that the depth of the N.A. is in the web of the beam. That could lead to an over-reinforced section that has little or no ductility. Designer is advised to avoid designing the section as a dobule flanged section if possible to ensure proper ducitliy and significant cracking.deflection before failure

Calculate the strain in the extreme CFRP based on the mode of failure

$$\varepsilon_0 := \left|\begin{array}{l} \varepsilon_{pu} - \varepsilon_{pe} \ \text{ if } \ \text{Section\_Mode} = \text{"Rectangular\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} \ \text{ if } \ \text{Section\_Mode} = \text{"Flanged\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} \ \text{ if } \ \text{Section\_Mode} = \text{"Double\_Flanged\_Tension"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \ \text{ if } \ \text{Section\_Mode} = \text{"Rectangular\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \ \text{ if } \ \text{Section\_Mode} = \text{"Flanged\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \ \text{ if } \ \text{Section\_Mode} = \text{"Double\_Flanged\_Compression"} \end{array}\right. = 7.359 \times 10^{-3}$$

$$\varepsilon := \left| \begin{array}{l} \text{for } \ i \in 0 .. \text{length}(\text{Row}) - 1 \\ \quad \varepsilon_i \leftarrow \varepsilon_0 \cdot \left(\dfrac{d_{i_i} - c}{d_{i_0} - c} \right) \\ \varepsilon \end{array} \right. = \begin{pmatrix} 0.0074 \\ 0.0069 \\ 0.0065 \\ 0.0060 \end{pmatrix}$$

strain in ith layer of prestressing strands

$$\varepsilon_c := \varepsilon_0 \cdot \left(\frac{c}{d_{i_0} - c} \right) = 0.00139$$

strain in the concrete top of the deck

Strength limit state Flexural Resistance:

$$M_n := \left| \begin{array}{ll} E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ldots & \text{if } \ deck_{eff} < \beta_1 \cdot c \le deck_{eff} + d_{ft} \\ + 0.85 f_{c\_deck} \cdot \left(b_{eff} - b_{ft} \right) \cdot deck_{eff} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - \dfrac{deck_{eff}}{2} \right) & \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ldots & \text{if } \ \beta_1 \cdot c > deck_{eff} + d_{ft} \\ + 0.85 f_{c\_deck} \cdot \left(b_{eff} - b_v \right) \cdot deck_{eff} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - \dfrac{deck_{eff}}{2} \right) \ldots & \\ + 0.85 f_{c\_deck} \cdot \left(b_{ft} - b_v \right) \cdot d_{ft} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - deck_{eff} - \dfrac{d_{ft}}{2} \right) & \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ \ \text{if } \ \beta_1 \cdot c \le deck_{eff} & \end{array} \right.$$

$$M_n = 6208.403 \cdot kip \cdot ft$$

Nominal moment capacity

$$\phi := \left| \begin{array}{l} 0.85 \ \text{ if } \ \varepsilon_0 \geq 0.005 \\ 0.5167 + 66.67 \cdot \varepsilon_0 \ \text{ if } \ 0.002 \leq \varepsilon_0 \leq 0.005 \\ 0.65 \ \text{ if } \ \varepsilon_0 \leq 0.002 \end{array} \right. = 0.85$$

$M_r := \phi \cdot M_n = 5277.14 \cdot \text{kip} \cdot \text{ft}$

$M_{u\_strength} = 3261.28 \cdot \text{kip} \cdot \text{ft}$

$\text{if}\left(M_r > M_{u\_strength}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$

$$\frac{M_r}{M_{u\_strength}} = 1.62$$

Minimum reinforcement against cracking moment

$f_r := 0.24 \cdot \sqrt{f_{c\_beam} \cdot \text{ksi}} = 678.823\,\text{psi}$

Modulus of rupture of beam concrete, **AASHTO A 5.4.2.6**

$\gamma_1 := 1.6$

Flexural variability factor

$\gamma_2 := 1.1$

Prestress viariability factor

$\gamma_3 := 1.0$

Reinforcement strength ratio

$$f_{cpe} := \frac{P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_B} = 3282.56\,\text{psi}$$

Compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$$M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{bn} - M_{DC1} \cdot \left(\frac{S_{bn}}{S_B} - 1\right)\right] = 3628.36 \cdot \text{kip} \cdot \text{ft}$$

Cracking moment

$\text{if}\left(M_r > \min\left(M_{cr}, 1.33 \cdot M_{u\_strength}\right), \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

Approximate mid-span deflection at failure

The deflection calculations follows the approach outlined in the paper " Flexural behaviour of CFRP precast Decked Bulb T beams " by Grace et al. in May/June 2012, Journal of Composites for Construction. In order to calculate the deflection at failure, the moment capacity of the composite section is used as the bending moment. The stress level in the bottom most row is used to calculate the flexural rigidity. The deflection calculated below is approximate, but will give an indication of the deformbility and the level of warning exhibited near failure of the beam.

$$d_{i_0} = 40.00 \cdot in$$

Depth of the bottom row of strands to the extreme compression fiber

$$c = 6.37 \cdot in$$

Depth of the neutral axis to the extreme compression fiber

$$y_s := d_{i_0} - c = 33.63 \cdot in$$

Distance from neutral axis to the bottom row of strands

$$EI := \frac{M_n \cdot y_s}{\varepsilon_0} = 340524382.74 \cdot kip \cdot in^2$$

Flexural rigidity of the beam/deck section based on the stress level in the bottom row of prestressing strands

$$\omega_f := 8 \cdot \frac{M_n}{L^2} = 11.203 \cdot \frac{kip}{ft}$$

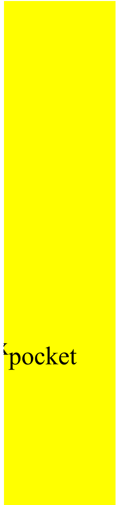
Failure load (dead and live loads) uniformly dirstibuted over the entire span

$$\delta_f := \frac{5 \cdot \omega_f \cdot L^4}{384EI} = 14.549 \cdot in$$

Midspan deflection at strength limit state

$$\frac{x - L_{end})^{2}}{2}\Bigg] \quad \text{if } L_{end} \le x < L_{var} + L_{end}$$

$$var \cdot \left(x - L_{end} - \frac{L_{var}}{3}\right)\Bigg] \quad \text{if } x \ge L_{var} + L_{end}$$


pocket

$$\wedge\ 0\cdot in \le x \le l_{ship}$$

$$nd \wedge 0\cdot in \le x \le l_{ship}$$

$$end \wedge l_{ship} \le x \le L_{ship}$$

$$\left.\right) + 0.5\cdot w_2\cdot\frac{\left(x - L_{end}\right)^2}{2}\Bigg]\ \text{ if }\ L_{end} \le x < L_{var} + L_{end} \wedge l_{ship} \le x \le \frac{L_{beam}}{2}$$

$$\left.\right) + 0.5\cdot w_2\cdot L_{var}\cdot\left(x - L_{end} - \frac{L_{var}}{3}\right)\Bigg]\ \text{ if }\ x \ge L_{var} + L_{end} \wedge l_{ship} \le x \le \frac{L_{beam}}{2}$$

= 0.0061

bal

ρ_{Fl_bal}

MDOT

Lawrence Tech

LRFD Design Example for:

CFCC Prestressed Precast Concrete Bulb T-Beam with Cast-In-Place Concrete Slab

Grace et al.

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7/1/2019

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About this Design Example

Description

This document provides guidance for the design of CFCC prestressed precast concrete beams according to AASHTO LRFD Bridge Design Specifications with the neccessary ammendmets where applicable, based on available literature and experimental data from tests conducted by Grace et. al at Lawrence Technological University. The example provided herein is a bulb T beam with a constant we thickness of 8 in.

Standards

The following design standards were utilized in this example:

- AASHTO LRFD Bridge Design Specification, 7th Edition, 2014
- Michigan Department of Transportation Bridge Design Manual, Volume 5
- Michigan Department of Transportation Bridge Design Guide
- ACI 440.4R-04, Prestressing Concrete Structures with FRP Tendons

Code & AASHTO LRFD UPDATES

This Mathcad sheet is developed based on available design guidelines and available AASHTO LRFD edition at the time of writing the sheet. Designer shall check and update design equations according to the latest edition of AASHTO LRFD

General notes

The following notes were considered in this design example:

1- Guarnateed strength of CFRP is reduced to account for environmental effect. The design guarnateec strength is taken as 0.9 x guarnateed strength recommended by manufacturer

2- Initial prestressing stress is limited to 65% of the design (reduced) guaranteed strength according to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations for initial/jacking stress in CFRP strands

3- CFCC strength immediately following transfer is limited to 60% of the design (reduced) guaranteed strength according ccording to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations

4- The depth of the haunch between the dck slab and the beam is ignored in calculating section properties or flexural capacity, while is included in calculating the dead loads

5- In strength limit state flanged section design, the concrete strength of the beam portion participating i the stress block was conservatively assumed equal to the concrete strength of the deck (AASHTO LRF C5.7.2.2)

6- Barrier weight was taken as 475 lb/ft. While, weight of midspan diaphragm was 500 lb/beam. Change according to the design

7- In the Mathcad sheet, the option of debonding as well as top prestressing strands are offered as means of reducing the end tensile stresses of the beams. designer may utilize either or both methods

8- In strength limit state check, the design addresses six different failure modes as follows:
Tension controlled rectangular section (depth of stress block is less than or equal the depth of the deck slab and the reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled rectangular section (depth of stress block is less than or equal the depth of the deck slab and the reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Tension controlled flanged section (depth of stress block is larger than the depth of the deck slab but less than the combined depth of the deck slab and beam top flange. The reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

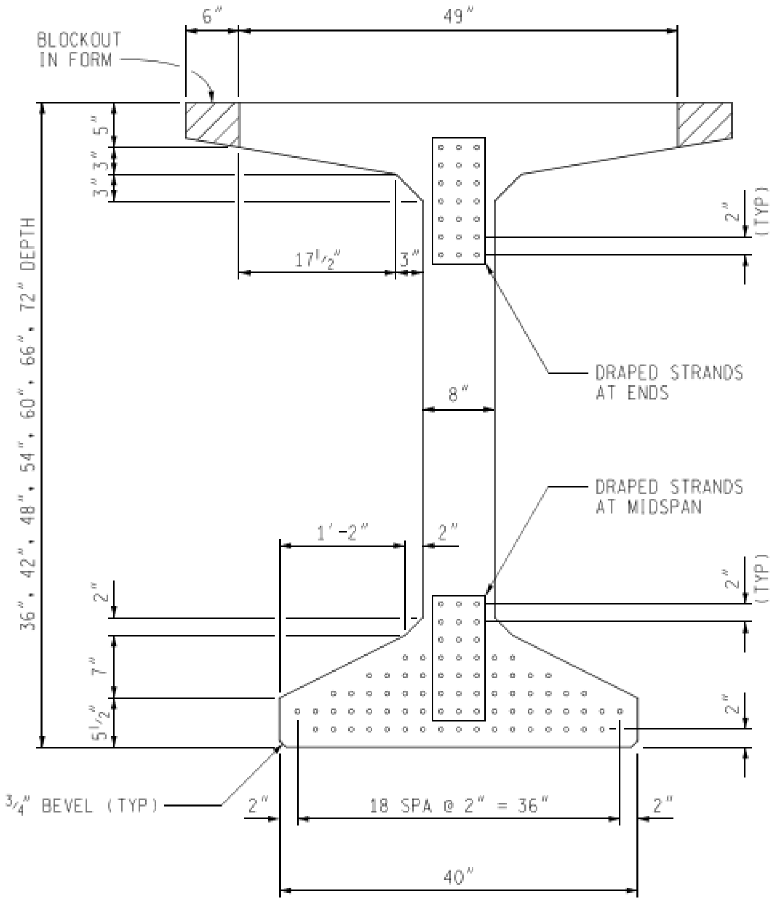
Compression controlled flanged section (depth of stress block is larger than the depth of the deck slab but less than the combined depth of the deck slab and beam top flange. The reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Tension controlled double flanged section (depth of stress block is larger than the combined depth of the deck slab and beam top flange. The reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled double flanged section (depth of stress block is larger than the combined depth of the deck slab and beam top flange. The reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Designer is advised to check the ductility of the beam and the deflection at failure in case of double flanged section because in that case, the N.A. of the section lies within the web of the beam and the ductility of the section may be compromised

9- This design example is developed based on allowable jacking strength and stress immediately after transfer according to the limits presented in the ACI 440.4R-04. The document can be updated using other prestress limits such as those presented in MDOT SPR-1690 research report and guide



$L_{RefToRef} := 140ft$	Distance between reference points
$D_{RefAtoBearing} := 18in$	Distance from reference point to bearing point
$D_{RefBtoBearing} := 18in$	Distance from reference point to bearing point
$L := L_{RefToRef} - D_{RefAtoBearing} - D_{RefBtoBearing} = 137\,ft$	Center to center span Length
$brg_{off} := 5.5in$	Center of bearing offset to end of beam (same value at both ends is assumed)
$L_{beam} := L + 2 \cdot brg_{off} = 137.917 \cdot ft$	Total length of beam
$l_{ship} := 42 \cdot in$	Distance from support to the end of the beam after force transfer and during shipping and handling
$L_{ship} := L_{beam} - l_{ship} \cdot 2 = 130.917\,ft$	Distance between supports during handling and shipping
$deck_{width} := 63ft + 3in$	Out to out deck width

$clear_{roadway} := 60ft + 0in$	Clear roadway width
$deck_{thick} := 9in$	Deck slab thickness
$t_{wear} := 0 \cdot in$	Wearing surface is included in the structural deck thickness only when designing the deck as per MDOT BDM 7.02.19.A.4. It is not used when designing the beam.
$t_{fws} := 2in$	Future wearing surface is applied as dead laod to accuant for additional deck thickness if a thicker rigid overlay is placed on deck
$walk_{width} := 0ft + 0in$	sidewalk width
$walk_{thick} := 0in$	sidewalk thickness (0" indicates no separate sidewalk pour)
$S := 6ft + 5in$	Center to center beam spacing
$NO_{beams} := 10$	Total number of beams
$haunch := 0in$	Average haunch thickness for section properties and strength calculations
$haunch_{d} := 2.0in$	Average haunch thickness for Load calculations
$overhang := 2ft + 9.75in$	Deck overhang width (same vaLue on both overhangs is assumed)
$barrier_{width} := 1ft + 8.25in$	Barrier width; include offset from back of barrier to edge of deck
$S_{exterior} := 57ft + 9in$	Hz distance between center of gravity of two exterior girders
$Lanes := \left(\frac{clear_{roadway}}{12ft}\right) = 5.00$	The number of design traffic Lanes can be caLcuLated as
$angle_{crossing} := 90deg$	Angle measured from centerline of bridge to the reference line
$\theta_{skew} := 90deg - angle_{crossing} = 0.00 \cdot deg$	Angle measured from a line perpendicular to the centerline of bridge to the reference line

Concrete Material Properties

$f_{c\_deck} := 5ksi$	Deck concrete compressive strength
$f_{c\_beam} := 10ksi$	Final beam concrete compressive strength

$f_{ci\_beam} := 0.8 f_{c\_beam} = 8 \cdot ksi$ Beam concrete compressive strength at reLease

$\omega_{conc} := 0.150 \frac{kip}{ft^3}$ Unit weight of reinforced concrete for load calculations

$barrier_{weight} := 0.475 \frac{kip}{ft}$ Weight per foot of barrier (aesthetic parapet tube, see MDOT BDG 6.29.10)

Unit weights of concrete used for modulus of eLasticity calculations, AASHTO Table 3.5.1-1

$$\gamma_c(f'_c) := \begin{cases} 0.145 \frac{kip}{ft^3} & \text{if } f'_c \le 5ksi \\ 0.140 \frac{kip}{ft^3} + 0.001 \cdot \left(\frac{f'_c}{ksi}\right) \frac{kip}{ft^3} & \text{otherwise} \end{cases}$$

$\gamma_{c.deck} := \gamma_c(f_{c\_deck}) = 145 \cdot pcf$

$\gamma_{c.beam} := \gamma_c(f_{c\_beam}) = 150 \cdot pcf$

$\gamma_{ci.beam} := \gamma_c(f_{ci\_beam}) = 148 \cdot pcf$

Concrete Modulus of Elasticity

Elastic modulus for concrete is as specified by **AASHTO A 5.4.2.4 (2015 Interim revision)** with a correction factor of 1.0

$$E_{c.beam\_i} := 120000 \cdot \left(\frac{\gamma_{ci.beam}}{\frac{kip}{ft^3}}\right)^{2.0} \cdot \left(\frac{f_{ci\_beam}}{ksi}\right)^{0.33} \cdot ksi = 5220.65 \cdot ksi$$

Beam concrete at reLease

$$E_{c.beam} := 120000 \cdot \left(\frac{\gamma_{c.beam}}{\frac{kip}{ft^3}}\right)^{2.0} \cdot \left(\frac{f_{c\_beam}}{ksi}\right)^{0.33} \cdot ksi = 5772.5 \cdot ksi$$

Beam concrete at 28 days

$$E_{c.deck} := 120000 \cdot \left(\frac{\gamma_{c.deck}}{\frac{kip}{ft^3}}\right)^{2.0} \cdot \left(\frac{f_{c\_deck}}{ksi}\right)^{0.33} \cdot ksi = 4291.19 \cdot ksi$$

Deck concrete at 28 days

CFCC Material Properties

$d_s := 15.2\text{mm} = 0.6\cdot\text{in}$	Prestressing strand diameter
$A_{strand} := 0.179\cdot\text{in}^2$	Effective cross sectionaL area
$E_p := 21000\text{ksi}$	Tensile elastic modulus
$T_{guts} := 60.70\text{kip}$	Guaranteed ultimate tensile capacity
$f'_{pu} := \dfrac{T_{guts}}{A_{strand}} = 339.11\cdot\text{ksi}$	Calculated ultimate tensile stress
$C_{Ese} := 0.9$	Environmental reduction factor for prestressed concrete exposed to weather for service limit state calculations
$f_{pu.service} := C_{Ese}\cdot f'_{pu} = 305.2\cdot\text{ksi}$	
$C_{Est} := 0.9$	Environmental reduction factor for prestressed concrete exposed to weather for strength limit state calculations
$f_{pu} := C_{Est}\cdot f'_{pu} = 305.2\cdot\text{ksi}$	

Modular Ratio

$n := \dfrac{E_{c.beam}}{E_{c.deck}} = 1.345$	Modular ratio for beam/deck slab
$n_p := \dfrac{E_p}{E_{c.deck}} = 4.89$	Modular ratio for Prestressing CFCC/beam

Bulb T Beam Section Properties: 72 inch beam depth

$A_{beam} := 1166.3in^2$	Minimum area of beam section
$d := 72in$	Depth of beam
$b_{web} := 8in$	Minimum web thickness
$b_{web.max} := 8in$	Maximum web thickness
$b_{ft} := 49in$	Width of top flange
$d_{ft} := 5in$	Thickness of top flange
$d_{h1} := 3 \cdot in$	Depth of the first haunch under the top flange
$b_{h1} := 14 \cdot in$	bottom width of the first haunch under the top flange
$d_{h2} := 3 \cdot in$	Depth of the second haunch under the top flange
$b_{fb} := 40in$	Width of bottom flange
$d_{fb} := 5.5in$	Thickness of bottom flange
$b_v := b_{web} = 8.00 \cdot in$	Shear width (equal to web thickness)
$\omega_{beam} := A_{beam} \cdot (150pcf) = 1214.9 \cdot plf$	Beam weight per foot
$I_{beam} := 844069in^4$	Minimum moment of inertia
$y_t := 36.2in$	Depth from centroid to top of beam
$y_b := 35.8in$	Depth from centroid to soffit of beam
$S_T := \dfrac{I_{beam}}{y_t} = 23316.82 \cdot in^3$	Minimum section modulus about top flange
$S_B := \dfrac{I_{beam}}{y_b} = 23577.35 \cdot in^3$	Minimum section modulus about bottom flange

Effective Flange Width of Concrete Deck Slab, AASHTO 4.6.2.6

$\text{Beam\_Design} := \text{"Interior"}$

Choose the design of the beam either "Interior" or "Exterior"

$b_{eff.int} := S = 6.42\,\text{ft}$

Effective flange width of deck slab for interior beams

$b_{eff.ext} := \frac{1}{2} \cdot S + \text{overhang} = 6.02\,\text{ft}$

Effective flange width of deck slab for exterior beams

$$b_{eff} := \left| \begin{array}{ll} b_{eff.int} & \text{if Beam\_Design} = \text{"Interior"} \\ b_{eff.ext} & \text{if Beam\_Design} = \text{"Exterior"} \end{array} \right. = 1.956$$

$d_{total} := deck_{thick} + d = 81 \cdot in$

Total depth of section including deck

Dynamic load Allowance

Dynamic load allowance from **AASHTO Table 3.6.2.1-1** is applied as an increment to the static wheel loads to account for wheel load impacts from moving vehicles.

$IM := 1 + 33\% = 1.33$

Design Factors

These factors are related to the ductility, redundancy and operational importance of the bridge structure components and are applied to the strength limit state.

Ductility

For Strength limit State, a factor of 1.05 is used for nonductile components and connections, 1.00 for conventional designs and details complying with these specifications, and 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by these specifications, **AASHTO A 1.3.3.**

$\eta_D := 1.00$

Redundancy

For Strength limit State, a factor of 1.05 is used for nonredundant members, 1.00 for conventional levels of redundancy, foundation elements where ϕ already accounts for redundancy as specified in **AASHTO A 10.5**, and 0.95 for exceptional levels of redundancy beyond girder continuity and a torsionally-closed cross-section, **AASHTO A 1.3.4.**

$\eta_R := 1.00$

Operational Importance

For the Strength limit State, a factor of 1.05 is used for critical or essential bridges, 1.00 for typical bridges, and 0.95 for relatively less important bridges, **AASHTO A 1.3.5.**

$\eta_I := 1.00$

Ductility, redundancy, and operational classification considered in the load modifier, **AASHTO Eqn. 1.3.2.1-2.**

$\eta_i := \eta_D \cdot \eta_R \cdot \eta_I = 1.00$

Composite Section Properties

This is the moment of inertia resisting superimposed dead loads.

Elastic Section Properties - Composite Section: k=2

$k_{sdl} := 2$

$A_{haunchkn} := \frac{b_{ft}}{k_{sdl} n} \cdot haunch = 0 \cdot in^2$ effective area of haunch

$d_{haunchkn} := d + \frac{haunch}{2} = 72 \cdot in$ Depth of centroid of haunch to bottom of beam

$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$

$b_{effkn} := \frac{b_{eff}}{k_{sdl} n} = 28.62 \cdot in$ Transformed deck width

$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 76.5 \cdot in$ Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 257.58 \cdot in^2$ Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 19705.08 \cdot in^3$ Static moment of inertia of transformed section about soffit of beam

$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 43.16 \cdot in$ Depth of CG of composite section from beam soffit

$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\ 3}}{12} = 1738.68 \cdot in^4$ Moment of inertia of transformed deck about centroid

$I_{haunchkn} := \frac{\frac{b_{ft}}{k_{sdl} \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$ Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_{3n} := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \cdots + A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2 = 1195303.3 \cdot in^4$$

$y_{b3n} := d_k = 43.163 \cdot in$

Depth of CG of composite section from beam soffit

$S_{b3n} := \frac{I_{3n}}{y_{b3n}} = 27692.97 \cdot in^3$

Section modulus about bottom of beam

$y_{t.bm.3n} := d - y_{b3n} = 28.84 \cdot in$

Depth of CG of composite section from top of beam

$S_{t.bm.3n} := \frac{I_{3n}}{y_{t.bm.3n}} = 41449.9 \cdot in^3$

Section modulus about top of beam

$y_{t3n} := d + haunch + deck_{thick} - t_{wear} - y_{b3n} = 37.84 \cdot in$

Depth of CG of composite section from top of deck

$S_{t3n} := \frac{I_{3n}}{y_{t3n}} = 31590.61 \cdot in^3$

Section modulus about top of deck

Elastic Section Properties - Composite Section: k=1

These properties are used to evaluate the moment of inertia for resisting live loads

Assumed wearing surface not included in the structural design deck thickness, per **MDOT BDM 7.02.19.A.4**.............

$k := 1$

$A_{haunchkn} := \frac{b_{ft}}{k\,n} \cdot haunch = 0 \cdot in^2$

effective area of haunch

$d_{haunchkn} := d + \frac{haunch}{2} = 72 \cdot in$

Depth of centroid of haunch to bottom of beam

$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$

$b_{effkn} := \frac{b_{eff}}{k\,n} = 57.24 \cdot in$

Transformed deck width

$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 76.5 \cdot in$

Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 515.17 \cdot in^2$

Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 39410.17 \cdot in^3$

Static moment of inertia of transformed section about soffit of beam

$$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 48.27 \cdot in$$

Depth of CG of composite section from beam soffit

$$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\,3}}{12} = 3477.37 \cdot in^4$$

Moment of inertia of transformed deck about centroid

$$I_{haunchkn} := \frac{\frac{b_{ft}}{k \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$$

Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_n := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \ldots = 1439459.5 \cdot in^4$$
$$+ A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2$$

$$y_{bn} := d_k = 48.27 \cdot in$$

Depth of CG of composite section from beam soffit

$$S_{bn} := \frac{I_n}{y_{bn}} = 29821.23 \cdot in^3$$

Section modulus about bottom of beam

$$y_{t.bm.n} := d - y_{bn} = 23.73 \cdot in$$

Depth of CG of composite section from top of beam

$$S_{t.bm.n} := \frac{I_n}{y_{t.bm.n}} = 60658.93 \cdot in^3$$

Section modulus about top of beam

$$y_{tn} := d + haunch + deck_{thick} - t_{wear} - y_{bn} = 32.73 \cdot in$$

Depth of CG of composite section from top of deck

$$S_{tn} := \frac{I_n}{y_{tn}} = 43979.31 \cdot in^3$$

Section modulus about top of deck

live load lateral Distribution Factors

Cross-section classification.. **Type K**

Distribution of live loads from the deck to the beams is evaluated based on the **AASHTO** specified distribution factors. These factors can only be used if generally, the following conditions are met;

- Width of deck is constant.
- Unless otherwise specified, the number of beams is not less than four.
- Beams are parallel and have approximately the same stiffness.
- Curvature in plan is less than the limit specified in **AASHTO A 4.6.1.2.4.**
- Unless otherwise specified, the roadway part of the overhang does not exceed 3.0 ft.
- Cross-section is consistent with one of the cross-sections shown in AASHTO **Table 4.6.2.2.1-1.**

Unless otherwise stated, stiffness parameters for area, moments of inertia and torsional stiffness used shall be taken as those of the cross-section to which traffic will be applied (composite section)

Distance between the centers of gravity of the basic beam and deck

$$e_g := d + \left(\frac{deck_{thick}}{2}\right) + haunch - y_b = 40.7 \cdot in$$

logitudinal stiffness parameter

$$K_g := n \cdot \left(I_{beam} + A_{beam} \cdot e_g^{\ 2}\right) = 3734316.14 \cdot in^4$$

Distribution of live loads for Moment in Interior Beams, AASHTO Table 4.6.2.2.2b-1

Range of Applicability.................................

$$\text{if}\left(3.5ft < S \le 16ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(4.5in < deck_{thick} \le 12in, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(20ft < L \le 240ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(NO_{beams} \ge 4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(10000\ in^4 < K_g \le 7000000\ in^4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

One lane loaded

$$M_{lane1\_int} := 0.06 + \left(\frac{S}{14ft}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12 \cdot L \cdot deck_{thick}^{\ 3}} \cdot \frac{ft}{in}\right)^{0.1} = 0.387$$

Two or more lanes loaded

$$M_{lane2\_int} := 0.075 + \left(\frac{S}{9.5ft}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot deck_{thick}{}^{3}} \cdot \frac{ft}{in}\right)^{0.1} = 0.555$$

live load moment disribution factor for interior beam

$$M_{lane\_int} := \max\left(M_{lane1\_int}, M_{lane2\_int}\right) = 0.555$$

Distribution of live loads for Moment in Exterior Beams, AASHTO Table 4.6.2.2.2d-1

One lane loaded (using the lever rule)

The lever rule involves the use of statics to determine the lateral distribution to the exterior girder by summing moments about the adjacent interior girder to determine the wheel-load reaction at the exterio girder assuming the concrete deck is hinged at the interior girder. A wheel cannot be closer than 2'-0" to the toe of barrier, and the standard wheel spacing is 6'-0". The evaluated factor is multiplied by the multiple presence factor, **AASHTO Table 3.6.1.1.2-1**.

Summing moments about the center of the interior beam

$$R := \frac{\left(S + overhang - barrier_{width} - 2 \cdot ft - \frac{6 \cdot ft}{2}\right)}{S} = 0.396$$

This factor is based on the lever arm rule considring the wheel load and not the resultant of both wheel

Moment distribution factor for exterior beam, one load loaded. The 1.2 accounts for the multiple presence factor, *m* from **AASHTO Table 3.6.1.1.2-1** for one lane loaded

$$M_{lane1\_ext} := R \cdot 1.2 = 0.475$$

Two or more lanes loaded

Horizontal distance from the centerline of the exterior web of exterior beam at deck level to the interior web edge of curb or traffic barrier must be greater than 0'-0"

$$d_e := \max\left(overhang - barrier_{width}, 0ft\right) = 1.13\,ft$$

Range of Applicability

$$\text{if}\left(-1ft \le d_e \le 5.5ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

lane fraction

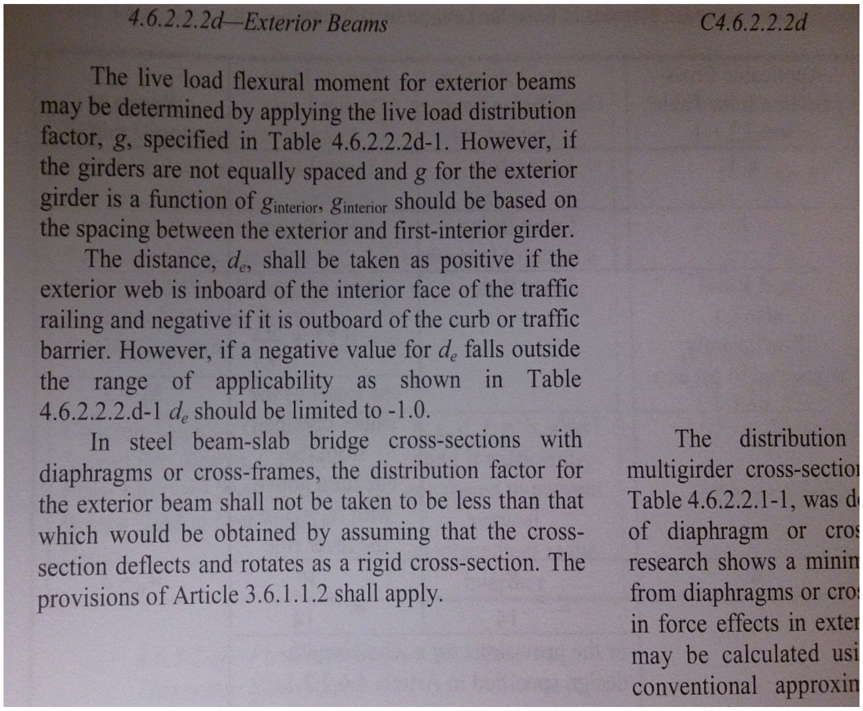
$$e := 0.77 + \frac{d_e}{9.1ft} = 0.894$$

Moment distribution factor for exterior beam, two or more lanes loaded

$$M_{lane2\_ext} := M_{lane\_int} \cdot e = 0.496$$

Distribution of live loads for Moment in Exterior Beams, AASHTO C4.6.2.2.2d

AASHTO LRFD 2014 recommends the rigid plate analysis only for **steel beam-slab bridges**. This was a change from ealier versions of AASHTO. It is up to the designed to ignore the rigid plate analysis or take it into consideration when calculating the DF for exterior beam



4.6.2.2.2d—Exterior Beams

The live load flexural moment for exterior beams may be determined by applying the live load distribution factor, g, specified in Table 4.6.2.2.2d-1. However, if the girders are not equally spaced and g for the exterior girder is a function of $g_{interior}$, $g_{interior}$ should be based on the spacing between the exterior and first-interior girder.

The distance, d_e, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier. However, if a negative value for d_e falls outside the range of applicability as shown in Table 4.6.2.2.2.d-1 d_e should be limited to -1.0.

In steel beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

C4.6.2.2.2d

The distribution
multigirder cross-sectio
Table 4.6.2.2.1-1, was d
of diaphragm or cros
research shows a minin
from diaphragms or cros
in force effects in exter
may be calculated usi
conventional approxin

Additional special analysis investigation is required because the distribution factor for multigirder in cross section was determined without consideration of diaphragm or cross frames. The multiple presence factors are used per **AASHTO Table 3.6.1.1.2-1.** This analysis should be done by sketching the cross section to determine the variables required for this example, the defined deck geometry is used. For any other geometry, these variables should be hand computed and input:

Horizontal distance from center of gravity of the pattern of girders to the exterior girder

$$X_{ext} := \frac{S_{exterior}}{2} = 28.88\,\text{ft}$$

Eccentricity of the center line of the standard wheel from the center of gravity of the pattern of girders

$$e_1 := X_{ext} + \text{overhang} - \text{barrier}_{width} - 2\text{ft} - \frac{6\text{ft}}{2} = 25\,\text{ft}$$

$$e_2 := e_1 - 12\text{ft} = 13\,\text{ft}$$

$$e_3 := e_2 - 12\text{ft} = 1\,\text{ft}$$

$$e_4 := e_3 - 12\text{ft} = -11\,\text{ft}$$

$$e_5 := e_4 - 12\text{ft} = -23\,\text{ft}$$

Summation of eccentricities for number of lanes considered:

$e_{NL1} := e_1 = 25\,\text{ft}$ One lane loaded

$e_{NL2} := e_1 + e_2 = 38\,ft$ Two lanes loaded

$e_{NL3} := e_{NL2} + e_3 = 39\,ft$ Three lanes loaded

$e_{NL4} := e_{NL3} + e_4 = 28\,ft$ Four lanes loaded

$e_{NL5} := e_{NL4} + e_5 = 5\,ft$ Five lanes loaded

Horizontal distances from the center of gravity of the pattern of girders to each girder

$$X_{beams} := \left| \begin{array}{l} \text{for } i \in 0 .. NO_{beams} - 1 \\ \quad X_i \leftarrow X_{ext} - (i \cdot S) \\ X \end{array} \right. =$$

	0
0	28.88
1	22.46
2	16.04
3	9.62
4	3.21
5	-3.21
6	-9.63
7	-16.04
8	-22.46
9	-28.88

ft

Summation of horizontal distances from the center of gravity of the pattern of girders to each girder

$$X_{NB} := \sum X_{beams}{}^2 = 3396.82 \cdot ft^2$$

$$m_{1R} := 1.2 \cdot \left(\frac{1}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL1}}{X_{NB}} \right) = 0.375$$

Reaction on exterior beam when one lane is loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{2R} := 1.0 \cdot \left(\frac{2}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL2}}{X_{NB}} \right) = 0.523$$

Reaction on exterior beam when two lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{3R} := 0.85 \cdot \left(\frac{3}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL3}}{X_{NB}} \right) = 0.537$$

Reaction on exterior beam when three lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{4R} := 0.65 \cdot \left(\frac{4}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL4}}{X_{NB}} \right) = 0.415$$

Reaction on exterior beam when four lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{5R} := 0.65 \cdot \left(\frac{5}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL5}}{X_{NB}} \right) = 0.353$$

Reaction on exterior beam when five lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

live load moment disribution factor for exterior beam

$$M_{lane\_ext} := \max\left(M_{lane1\_ext}, M_{lane2\_ext}, m_{1R}, m_{2R}, m_{3R}, m_{4R}, m_{5R}\right) = 0.537$$

Reduction of load Distribution Factors for Moment in longitudinal Beams on Skewed Supports

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10 degrees, the bending moments and shear forces are reduced in accordance with **AASHTO Table 4.6.2.2.2e-1 and 4.6.2.2.3c-1** respectively.

Moment

Range of Applicability

$$\text{if}\left(30deg \le \theta_{skew} \le 60deg, \text{"ok"}, \text{"Check C1 and θskew below"}\right) = \text{"Check C1 and θskew below"}$$

$$\text{if}\left(3.5ft < S \le 16ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(20ft < L \le 240ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(NO_{beams} \ge 4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\theta_{skew} := \left| \begin{array}{l} \theta_{skew} \ \text{if} \ \theta_{skew} \le 60 \cdot deg \\ 60 \cdot deg \ \text{if} \ \theta_{skew} > 60 \cdot deg \end{array} \right. = 0 \cdot deg$$

$$C_1 := \left| \begin{array}{l} 0 \ \text{if} \ \theta_{skew} < 30 \cdot deg \\ \left[0.25 \cdot \left(\frac{K_g}{12.0 \cdot L \cdot deck_{thick}^{\ 3}} \cdot \frac{ft}{in} \right)^{0.25} \cdot \left(\frac{S}{L} \right)^{0.5} \right] \ \text{otherwise} \end{array} \right. = 0$$

$$Mcorr_{factor} := 1 - C_1 \cdot \tan\left(\theta_{skew}\right)^{1.5} = 1$$

Correction factor for moment

Reduced distribution factors at strength limit state for interior girders due to skew

$$DF_{strength\_moment\_int} := M_{lane\_int} \cdot Mcorr_{factor} = 0.555$$

Moment

Reduced distribution factors at strength limit state for exterior girders due to skew

$$DF_{strength\_moment\_ext} := M_{lane\_ext} \cdot Mcorr_{factor} = 0.537$$

Moment

Design distribution factors for service and strength limit states

Distribution factor for moment at strength limit state

$$DF_{strength\_moment} := \left| \begin{array}{l} DF_{strength\_moment\_int} \ \text{ if } \ Beam\_Design = \text{"Interior"} \\ DF_{strength\_moment\_ext} \ \text{ if } \ Beam\_Design = \text{"Exterior"} \end{array} \right. = 0.555$$

live load Analysis

Flexure

As per **AASHTO A 3.6.1.2.1**, vehicular live loading designated by the standard HI-93 truck shall be a combination of the design truck or design tandem, and the design lane load. To produce extreme force effects, the spacing between the two 32-kip axles are taken as 14 ft.

Calculate the maximum moment due to the truck load. Maximum truck load moment occurs when the middle axle is positioned at distance 2.33 ft from the midspan. Maximum momment occurs under the middle axle load. Moment due to distributed load occurs at midspan.

Unless more detailed analysis is performed to determine the location and value for the maximum moment under combined truck and distributed loads at both service and strength limit state, the maximum moment from the truck load at distance 2.33 ft from midspan can be assumed to occur at the midspan and combined with the maximum moment from other dead and live distributed loads

Calculate the reaction at the end of the span

$$R := \frac{8kip \cdot \left(\frac{L}{2} - 16.33ft\right) + 32kip \cdot \left(\frac{L}{2} - 2.33ft\right) + 32kip \cdot \left(\frac{L}{2} + 11.67ft\right)}{L} = 37.228 \cdot kip$$

Calculate the maximum moment due to truck load

$$M_{truck} := R \cdot \left(\frac{L}{2} + 2.33ft\right) - 32 \cdot kip \cdot 14 \cdot ft = 2.189 \times 10^{3} \cdot kip \cdot ft$$

at distance 2.33 ft from midspan but can be assumed to occur at the midspan

Maximum moment due to design lane load, **AASHTO A 3.6.1.2.4**

$$X := \frac{L}{2} = 68.5\,ft$$

$$M_{lane} := \frac{0.64klf \cdot L \cdot X}{2} - 0.64klf \cdot \frac{X^2}{2} = 1501.52 \cdot kip \cdot ft$$

Maximum moment due to design tandem, **MDOT BDM 7.01.04.A**

$$M_{tandem} := \frac{60kip \cdot L}{4} = 2055 \cdot kip \cdot ft$$

Maximum moment due to vehicular live loading by the modified HI-93 design truck and tandem per **MDOT BDM 7.01.04.A**. Modification is by multiplying the load effects by a factor of 1.20. Dynamic load allowance is considered only for the design truck and tandem, **AASHTO A 3.6.1.2.2, 3.6.1.2.3 & 3.6.1.2.4.**

$$M_{LLI} := \left[1.20M_{lane} + IM\cdot\left(1.20\cdot\max\left(M_{truck}, M_{tandem}\right)\right)\right]\cdot DF_{strength\_moment} = 2938.78\cdot kip\cdot ft$$

Dead load Analysis

Noncomposite Dead load (DC$_1$)

$$M_{swbeam} := \frac{\omega_{beam}\cdot L^2}{8} = 2850.30\cdot kip\cdot ft$$

Total moment due to selfweight of beam

$$deck := \left(deck_{thick}\cdot b_{eff} + haunch_d\cdot b_{ft}\right)\cdot 0.15\frac{kip}{ft^3} = 0.82\cdot klf$$

Selfweight of deck and haunch on beam

$$M_{deck} := \frac{deck\cdot L^2}{8} = 1933.11\cdot kip\cdot ft$$

Moment due to selfweight of deck and haunch

$$sip := 15psf\cdot\left(b_{eff} - b_{ft}\right) = 0.035\cdot klf$$

15 psf weight included for stay-in-place forms per MDOT BDM 7.01.04.I

$$M_{sip} := \frac{sip\cdot L^2}{8} = 82.11\cdot kip\cdot ft$$

Moment due to stay-in-place forms

$$dia_{int} := 0.5\cdot kip$$

Weight of steel diaphragms at mid-span per each interior beam

$$dia_{ext} := 0.25\cdot kip$$

Weight of steel diaphragms at mid-span per each exterior beam

$$diaphragm := \left| \begin{array}{ll} dia_{int} & \text{if } Beam\_Design = \text{"Interior"} \\ dia_{ext} & \text{if } Beam\_Design = \text{"Exterior"} \end{array} \right. = 0.5\cdot kip$$

$$spa_{dia} := 2\left(S - b_{fb}\right)\cdot\tan\left(\theta_{skew}\right) = 0\,ft$$

One row of diaphragms at midspan are used.

$$M_{dia} := diaphragm\cdot\frac{L}{4} = 17.125\cdot kip\cdot ft$$

Moment due to diaphragm weight

Dead load (selfweight of beam+ deck+

$$DC_1 := \omega_{beam} + deck + sip = 2.074 \cdot klf$$

SIP forms) acting on non-composite section

$$M_{DC1} := M_{swbeam} + M_{deck} + M_{sip} + M_{dia} = 4882.65 \cdot kip \cdot ft$$

Total midspan moment acting on the non-composite section

Composite Dead load (DC_2)

$$util := \frac{1}{2} \cdot (0plf) = 0 \cdot klf$$

Weight of utilities upported by the superstructure

$$barrier1_{weight} := 0.475 \frac{kip}{ft}$$

Weight per foot of first barrier (aesthetics parapet tube, **MDOT BDG 6.29.10**)

$$barrier2_{weight} := 2.25 \cdot in \cdot 40 \cdot in \cdot \omega_{conc} + 0.475 \frac{kip}{ft} = 0.569 \cdot \frac{kip}{ft}$$

Weight per foot of second barrier (modified aesthetics parapet tube, **MDOT BDG 6.29.10**)

$$sidewalk := \frac{2 \cdot walk_{width} \cdot walk_{thick} \cdot \omega_{conc}}{NO_{beams}} = 0.00 \cdot klf$$

Weight to due extra thickness of sidewalk per beam

$$barrier := \frac{barrier1_{weight} + barrier2_{weight}}{NO_{beams}} = 0.10 \cdot klf$$

Total barrier weight per beam

$$soundwall_{weight} := 0 \cdot \frac{kip}{ft}$$

Weight of the sound wall, if there is a sound wall

Weight of the sound wall **for exterior beam** design assuming lever arm and an inetremiate hinge on the first interior beam

$$soundwall := \left| \begin{array}{ll} 0 \cdot \frac{kip}{ft} & \text{if } Beam\_Design = \text{"Interior"} \\ \left[soundwall_{weight} \cdot \frac{(S + overhang)}{S} \right] & \text{if } Beam\_Design = \text{"Exterior"} \end{array} \right. = 0 \cdot \frac{kip}{ft}$$

$$DC_2 := sidewalk + barrier + util + soundwall = 0.104 \cdot klf$$

Total dead load acting on the composite section

$$M_{DC2} := \frac{DC_2 \cdot L^2}{8} = 244.88 \cdot kip \cdot ft$$

Total midspan moment acting on the composite section

(DW) Wearing Surface load

$$DW := \left(b_{eff}\right) \cdot 0.025 \frac{kip}{ft^2} = 0.16 \cdot klf$$

Self weight of future wearing surface

Maximum unfactored dead load moments

$M_{DC} := M_{DC1} + M_{DC2} = 5127.52 \cdot kip \cdot ft$

Total midspan moment due to loads acting on the composite and non-composite section

$$M_{DW} := \frac{DW \cdot L^2}{8} = 376.36 \cdot kip \cdot ft$$

Midspan moment due to weight of future wearing surface

Wind load on the sound wall

If a tall sound wall is provided, wind effect shall be calculated and considered in the design. Assuming lever arm rule and an intermediate hinge at the first interior beam after the exterior beam, the wind load will affect the loads on the exterior beam and the first interior beam. In the following set of calculations, the wind effect was calculated as a concentrated moment at the end of the overhang of the bridge.

$$M_{wind} := 0.0 \cdot ft \cdot \frac{kip}{ft}$$

Moment due to wind acting at the sound wall

$$W := \frac{M_{wind}}{S} = 0 \cdot \frac{kip}{ft}$$

Extra load on the interior/exterior beam due to wind load assuming lever arm analysis and an intermediate hinge at the first interior beam

$$M_{WS} := \frac{W \cdot L^2}{8} = 0 \cdot kip \cdot ft$$

Interior beam moment due to wind acting at the sound wall

load Combinations

Load Combinations: Strength, Extreme Event, Service and Fatigue load combinations are defined per AASHTO 3.4.1. Verify which combination are appropriate. For this concrete box beam design, wind load is not evaluated, and no permit vehicle is specified. However, the design live loading is MDOT HL-93 Modified which accounts for Michigan's inventory of legal and permit vehicles.

Strength I, III, IV and Strength V limit states are considered for the design of this beam. Load combinations factors according to **AASHTO LRFD 2016 Interim revision are used (Check for latest AASHTO LRFD edition)**

$$M\_Strength_{I} := \eta_i \cdot \left(1.25 M_{DC} + 1.50 M_{DW} + 1.75 M_{LLI}\right) = 12116.81 \cdot kip \cdot ft$$

$$M\_Strength_{III} := \eta_i \cdot \left(1.25 M_{DC} + 1.50 M_{DW} + 1.0 M_{WS}\right) = 6973.94 \cdot kip \cdot ft$$

$$M\_Strength_{IV} := \eta_i \cdot \left[1.50 \cdot \left(M_{DC} + M_{DW}\right)\right] = 8255.82 \cdot kip \cdot ft$$

$$M\_Strength_{V} := \eta_i \cdot \left(1.25 M_{DC} + 1.50 M_{DW} + 1.35 M_{LLI} + 1.0 \cdot M_{WS}\right) = 10941.29 \cdot kip \cdot ft$$

$M_{u\_strength} := \max\left(M\_Strength_{I}, M\_Strength_{III}, M\_Strength_{IV}, M\_Strength_{V}\right) = 12116.81 \cdot kip \cdot ft$

Number of Prestressing Strands

The theoretical number of strands required is calculated using the **Service III** limit state

$$f_b := \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = 3.7 \cdot ksi$$

Tensile stress in bottom flange due to applied loads

Allowable stress limits for concrete

$f_{ti} := 0.24 \cdot \sqrt{f_{ci\_beam} \cdot ksi} = 0.68 \cdot ksi$ Initial allowable tensile stress

$f_{ci} := -0.65 \cdot f_{ci\_beam} = -5.20 \cdot ksi$ Initial allowable compressive stress (according to AASHTO LRFD 2016 interim revision)

$f_{tf} := 0 \cdot \sqrt{f_{c\_beam} \cdot ksi} = 0.00 \cdot ksi$ Final allowable tensile stress (allowing no tension)

No tension is allowed under service III limit state to avoid potential cracks and shear action on the strands

$f_{cfp} := -0.45 \cdot f_{c\_beam} = -4.50 \cdot ksi$ Final allowable compressive stress in the beam due to sum of effective prestress and permanent loads

$f_{cf.deckp} := -0.45 \cdot f_{c\_deck} = -2.25 \cdot ksi$ Final allowable compressive stress in the slab due to permanent loads

$f_{cf} := -0.6 \cdot f_{c\_beam} = -6.00 \cdot ksi$ Final allowable compressive stress in the beam due to sum o effective prestress, permanent loads, & transient loads

$f_{cf.deck} := -0.6 \cdot f_{c\_deck} = -3.00 \cdot ksi$ Final allowable compressive stress in the beam due to sum of permanent loads and transient loads

$f_p := f_b - f_{tf} = 3.7 \cdot ksi$ Excess tension in the bottom flange due to applied loads

Assuming strand pattern center of gravity is midway between the bottom two rows of strands, i.e. the same number of strands are used in the top and bottom rows of the bottom flange.

$y_{bs} := 3in$ Distance from soffit of beam to center of gravity of strands

$e_{st} := y_b - y_{bs} = 32.80 \cdot in$ Eccentricity of strands from the centroid of beam

Final prestressing force required to counteract excess tension in the bottom flange. Set allowable stress equal to the excess tension, solve for P_e.

$$P_{et} := \frac{f_p}{\left(\frac{1}{A_{beam}} + \frac{e_{st}}{S_B}\right)} = 1645.631 \cdot kip$$

$f_{j.max} := 0.65 \cdot f_{pu.service} = 198.377 \cdot ksi$ Maximum allowable Jacking stress, **ACI 440.4R Table 3.3**

$P_j := A_{strand} \cdot f_{j.max} = 35.51 \cdot kip$ Maximum Jacking force per strand

$f_t := 0.637 f_{pu.service} = 194.41 \cdot ksi$ Initial prestressing stress immediately **prior** to transfer. shall be less than or equal to the maximum jacking strength, and shall be adjusted accordingly to make sure the stress immedietely following transfer is not exceeding 0.6 times guaranteed strength as shown on the following page

$P_{in} := A_{strand} \cdot f_t = 34.80 \cdot kip$ Initial prestressing force per strand prior to transfer

$P_{pet} := A_{strand} \cdot f_t \cdot 0.75 = 26.10 \cdot kip$ Effective prestressing force assuming 25% final prestress losses per 0.6" diameter strand

$$NO_{strands\_i} := ceil\left(\frac{P_{et}}{P_{pet}}\right) = 64$$ Minimum number of strands required

Strand distribution per row. Row 0 is the bottom most row in the beam. Start adding strands from the bottom row going up until the number of strands is reached. do not skip rows inbetween. Extra rows with zero strands will be eliminated in the analysis.

$row_0 := 17$ $row_1 := 19$ $row_2 := 15$ $row_3 := 11$ $row_4 := 1$ $row_5 := 0$

$row_6 := 0$ $row_7 := 0$ $row_8 := 0$ $row_9 := 0$

$$row = \begin{pmatrix} 17 \\ 19 \\ 15 \\ 11 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
Row := | a ← 0
       | for i ∈ 0 .. length(row) − 1
       |     | a ← a + 1  if row_i > 0
       |     | a ← a  otherwise
       | for j ∈ 0 .. a − 1
       |     D_j ← row_j  if row_j ≠ 0
       | D
```

$$Row = \begin{pmatrix} 17 \\ 19 \\ 15 \\ 11 \\ 1 \end{pmatrix}$$

$NO_{strands} := \sum Row = 63.00$

Total number of prestressing strands

```
d_strand := | for i ∈ 0 .. length(Row) − 1
            |     d_s_i ← d − (2in) − (2in)i
            | d_s
```

$$= \begin{pmatrix} 70.00 \\ 68.00 \\ 66.00 \\ 64.00 \\ 62.00 \end{pmatrix} \cdot in$$

Depth of CFCC strands in each layer from the top of the beam section. This calculation assumes a 2" vertical spacing of the strand rows

$$CG := \frac{\overrightarrow{\left[Row \cdot \left(d - d_{strand}\right)\right]}}{\sum Row} = 4.73 \cdot in$$

Center of gravity of the strand group measured from the soffit of the beam section

$d_f := (d - CG) + haunch + deck_{thick} = 76.27 \cdot in$

Depth from extreme compression fiber to centroid of CFCC tension reinforcement

$e_s := y_b - CG = 31.07 \cdot in$

Eccentricity of strands from centroid of beam

$A_{ps} := A_{strand} \cdot NO_{strands} = 11.28 \cdot in^2$ Total area of prestressing CFCC strands

Prestress losses

loss due to Elastic Shortening, AASHTO Eqn. C5.9.5.2.3a-1

$$\Delta f_{PES} := \frac{A_{ps} \cdot f_t \cdot \left(I_{beam} + e_s^{\ 2} \cdot A_{beam}\right) - e_s \cdot M_{swbeam} \cdot A_{beam}}{A_{ps} \cdot \left(I_{beam} + e_s^{\ 2} \cdot A_{beam}\right) + \frac{A_{beam} \cdot I_{beam} \cdot E_{c.beam\_i}}{E_p}} = 11.54 \cdot ksi$$

$F_{pt} := f_t - \Delta f_{PES} = 182.87 \cdot ksi$ Prestressing stress immediately following transfer

$P_t := A_{ps} \cdot F_{pt} = 2062.271 \cdot kip$

According to ACI 440.4R, Table 3.3, the allowable stress immediately after transfer shall not exceed 0.6 fpu

$0.6 \cdot f_{pu.service} = 183.117 \cdot ksi$

$\text{if}\left(F_{pt} \le 0.6 \cdot f_{pu.service}, \text{"Ok"}, \text{"Not Ok"}\right) = \text{"Ok"}$

Approximate Estimate of Time dependent losses, AASHTO A 5.9.5.3

$H := 75$ Average annual ambient relative humidity

$\gamma_h := 1.7 - 0.01 \cdot H = 0.95$ Correction factor for relative humidity of ambient air

$$\gamma_{st} := \frac{5}{1 + \frac{f_{ci\_beam}}{ksi}} = 0.56$$

Correction factor for specified concrete strength at time of prestress transfer to the concrete member

$\Delta f_{pR} := f_t \cdot 1.75\% = 3.40 \cdot ksi$ Relaxation loss taken as 1.75% of the initial pull per experimental results from Grace et. al based on 1,000,000 hours (114 years)

$$\Delta f_{pLT} := 10 \cdot \frac{f_t \cdot A_{ps}}{A_{beam}} \cdot \gamma_h \cdot \gamma_{st} + 12ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} = 19.66 \cdot ksi$$

long term prestress loss

Difference in thermal coefficient expansion between concrete and CFCC

$\alpha := 6 \cdot 10^{-6} \cdot \frac{1}{F}$ Difference in coefficient of thermal expansion between concrete and CFCC

$t_{amb} := 68F$	Ambient temperature
$t_{low} := -10F$	lowest temperature in Michigan according to AASHTO IRFD 3.12.2
$\Delta t := t_{amb} - t_{low} = 78\,F$	Change in the temperature
$\Delta f_{pt} := \alpha \cdot \Delta t \cdot E_p = 9.83 \cdot ksi$	Prestress losses due to temp. effect
$f_{pe} := f_t - \Delta f_{pLT} - \Delta f_{PES} - \Delta f_{pt} = 153.39 \cdot ksi$	Effective prestressing stress after all losses
$P_e := A_{ps} \cdot f_{pe} = 1729.77 \cdot kip$	Effective prestressing force after all losses
$f_t = 194.41 \cdot ksi$	Initial prestress prior to transfer
$f_{pe} = 153.39 \cdot ksi$	Prestress level after all losses
$loss := \dfrac{f_t - f_{pe}}{f_t} = 21.10 \cdot \%$	Total prestress loss

Debonding Criteria

Estimate the location from each beam end where top prestressing or debonding is no longer needed The vectors are developed for possible two different deboning lengths per row. Enter the number of debonded strands and the estimated debonding length in the vectors below per each row location

Location: number of strands: debonding length:

$$Row_{db} := \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \\ 5 \\ 5 \\ 6 \\ 6 \end{pmatrix} \qquad n_{db} := \begin{pmatrix} 4 \\ 2 \\ 4 \\ 3 \\ 4 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad l_{db} := \begin{pmatrix} 16 \\ 20 \\ 12 \\ 8 \\ 8 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot ft$$

For debonding pattern, follow staggering guidelines in MDOT BDM 7.02.18.A.2

$$row_{db} := \left| \begin{array}{l} \text{for } i \in 0..2\,\text{length}(Row) - 1 \\ \quad D_i \leftarrow Row_{db_i} \\ D \end{array} \right.$$

$$N_{db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right) - 1 \\ \quad D_i \leftarrow n_{db_i} \\ D \end{array} \right.$$

$$L_{db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right) - 1 \\ \quad D_i \leftarrow l_{db_i} \\ D \end{array} \right.$$

$$row_{db} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \\ 5 \\ 5 \end{pmatrix} \qquad N_{db} = \begin{pmatrix} 4 \\ 2 \\ 4 \\ 3 \\ 4 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad L_{db} = \begin{pmatrix} 16 \\ 20 \\ 12 \\ 8 \\ 8 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ft}$$

$$\sum N_{db} = 19$$

$$Debond_{tot} := \frac{\sum N_{db}}{NO_{strands}} = 30.16 \cdot \%$$

Portion of partially debonded strands in beam section

$$\text{if}\left(Debond_{tot} \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

Total number of debonded strands in rows

```
$N_{db.row} :=$ for $i \in 0..\text{length}(Row) - 1$
    $a_i \leftarrow 0$
    for $j \in 0..\text{length}(N_{db}) - 1$
        $a_i \leftarrow a_i + N_{db_j}$  if  $row_{db_j} = i + 1$
a
```

$$= \begin{pmatrix} 6.00 \\ 7.00 \\ 6.00 \\ 0.00 \\ 0.00 \end{pmatrix}$$

```
$Debond_{row} :=$ for $i \in 0..\text{length}(Row) - 1$
    $a_i \leftarrow 0$
    $a_i \leftarrow \dfrac{N_{db.row_i}}{Row_i}$  if  $Row_i > 0$
    0  otherwise
a
```

$$= \begin{pmatrix} 35.29 \\ 36.84 \\ 40.00 \\ 0.00 \\ 0.00 \end{pmatrix} \cdot \%$$

$\text{if}\left(\max\left(Debond_{row}\right) \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$

The limit of 40% is taken according to MDOT BDM 7.02.18. A2

Optional: only needed if debonding scheme is not sufficient to eliminate the tensile stresses at beam ends either at transfer or due to handling and shipping

CFCC strand transfer length, **ACI 440.4R Table 6.1**

$L_t := 50 d_s = 2.49\,\text{ft}$

Number of top prestressing strands in the top flange

$$Row_{top} := \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Depth of the top prestressing strands from the top surface of the beam

$$d_{top} := \begin{pmatrix} 3 \\ 5 \end{pmatrix} \cdot in$$

Initial prestressing stress/force at the top prestressing strands

$F_{p\_top} := 50 \cdot ksi$

Distance from the end of the beam to the point where the top prestressing is no longer needed

$x_{p\_top} := 10\cdot ft$

Top prestressing strands shall not extend the the middle third of the beam. Otherwise, it could affect the stresses at service limit state

$$Check\_Top\_prestressing\_Length := \begin{vmatrix} \text{"Okay"} \quad \text{if } x_{p\_top} \le \frac{L_{beam}}{3} \\ \text{"Check service stress @ x.p\_top"} \quad \text{if } x_{p\_top} > \frac{L_{beam}}{3} \end{vmatrix} = \text{"Okay"}$$

Distance from the end of the beam to the pocket where top prestressing strand is cut after concrete pouring. The middle region between the cut pockets shall be dobonded to avoid force transfer to the middle region

$x_{pocket} := x_{p\_top} + L_t = 12.493\, ft$

Serviceability Checks

Stress check locations along the beam

Stress locations after the transfer length for bonded and de-bonded strands

$$X_{release} := sort\left[stack\left[L_t, \left(L_{db} + L_t\right), x_{p\_top}, x_{pocket}\right]\right] = \begin{pmatrix} 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 10 \\ 10.493 \\ 10.493 \\ 10.493 \\ 12.493 \\ 14.493 \\ 18.493 \\ 22.493 \end{pmatrix} ft$$

Extracting repreated X from the vector

```
$x_{release} :=$
    $k \leftarrow 0$
    $x_0 \leftarrow L_t$
    for $i \in 1 .. \text{length}(X_{release}) - 1$
        $k \leftarrow k + 1$ if $(X_{release_i} \neq X_{release_{i-1}})$
        $x_k \leftarrow X_{release_i}$
    $x$
```

$$x_{release} = \begin{pmatrix} 2.493 \\ 10 \\ 10.493 \\ 12.493 \\ 14.493 \\ 18.493 \\ 22.493 \end{pmatrix} \cdot ft$$

Area of strands in each row at each stress check location

```
$A_{db} :=$
    for $i \in 0 .. \text{length}(x_{release}) - 1$
        for $z \in 0 .. \text{length}(Row) - 1$
            $A_{i,z} \leftarrow Row_z \cdot A_{strand}$
        for $j \in 0 .. \text{length}(N_{db}) - 1$
            $n \leftarrow N_{db_j}$
            $row \leftarrow row_{db_j}$
            $L \leftarrow L_{db_j}$
            $A_{i,row-1} \leftarrow (A_{i,row-1} - n \cdot A_{strand}) \cdot \frac{x_{release_i}}{L_t}$ if $x_{release_i} < L_t$
            $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand}$ if $L_t \leq x_{release_i} \leq L$
            $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand} \cdots + n \cdot A_{strand} \cdot \frac{(x_{release_i} - L)}{L_t}$ if $L < x_{release_i} \leq L + L_t$
    $A$
```

$$A_{db} = \begin{pmatrix} 1.97 & 2.15 & 1.61 & 1.97 & 0.18 \\ 1.97 & 2.58 & 2.47 & 1.97 & 0.18 \\ 1.97 & 2.69 & 2.69 & 1.97 & 0.18 \\ 1.97 & 2.83 & 2.69 & 1.97 & 0.18 \\ 1.97 & 3.40 & 2.69 & 1.97 & 0.18 \\ 2.68 & 3.40 & 2.69 & 1.97 & 0.18 \\ 3.04 & 3.40 & 2.69 & 1.97 & 0.18 \end{pmatrix} \cdot in^2$$

Beam stresses at release due to prestressing only

Sign convention; negative and positive stresses/forces for compression and tension respectively

$$P_{ps} := -F_{pt} \cdot A_{db} = \begin{pmatrix} -360.08 & -392.81 & -294.61 & -360.08 & -32.73 \\ -360.08 & -471.58 & -452.15 & -360.08 & -32.73 \\ -360.08 & -491.02 & -491.02 & -360.08 & -32.73 \\ -360.08 & -516.93 & -491.02 & -360.08 & -32.73 \\ -360.08 & -621.95 & -491.02 & -360.08 & -32.73 \\ -491.02 & -621.95 & -491.02 & -360.08 & -32.73 \\ -556.49 & -621.95 & -491.02 & -360.08 & -32.73 \end{pmatrix} \cdot kip$$

Midspan moment due to prestressing at release

$$M_{ps} := P_{ps} \cdot \left(d_{strand} - y_t\right) = \begin{pmatrix} -3691.356 \\ -4291.316 \\ -4439.338 \\ -4508.004 \\ -4786.323 \\ -5155.131 \\ -5339.535 \end{pmatrix} \cdot kip \cdot ft$$

Top and bottom concrete stresses at check locations due to prestressing ONLY

```
$f_{ps} :=$ for $i \in 0..\text{length}(x_{release}) - 1$
    $M \leftarrow M_{ps_i}$
    $P \leftarrow \sum_{j=0}^{\text{cols}(P_{ps})-1} P_{ps_{i,j}}$
    $A \leftarrow A_{beam}$
    $S_{top} \leftarrow S_T$
    $S_{bott} \leftarrow S_B$
    $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
    $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
  $f$
```

$$f_{ps} = \begin{array}{c} \begin{array}{cc} \text{top} & \text{bottom} \end{array} \\ \begin{pmatrix} 665 & -3114 \\ 771 & -3622 \\ 797 & -3747 \\ 810 & -3804 \\ 863 & -4036 \\ 941 & -4336 \\ 980 & -4486 \end{pmatrix} \end{array} \cdot psi$$

Beam stresses at release due to selfweight

Moment due to self weight of beam at check locations

$$M_{sw}(x) := \frac{\omega_{beam} \cdot x}{2} \cdot (L_{beam} - x)$$

Top and bottom concrete stresses at check locations due to beam self weight ONIY

```
$f_{sw} :=$ for $i \in 0..\text{length}(x_{release}) - 1$
    $M \leftarrow M_{sw}(x_{release_i})$
    $f_{i,0} \leftarrow \frac{-M}{S_T}$
    $f_{i,1} \leftarrow \frac{M}{S_B}$
  $f$
```

$$f_{sw} = \begin{array}{c} \begin{array}{cc} \text{top} & \text{bottom} \end{array} \\ \begin{pmatrix} -106 & 104 \\ -400 & 395 \\ -418 & 413 \\ -490 & 484 \\ -559 & 553 \\ -690 & 683 \\ -812 & 803 \end{pmatrix} \end{array} \cdot psi$$

Area of **top prestressing** strands at distance X.release from the end

```
$A_{top} :=$ for $i \in 0..\text{length}(x_{release}) - 1$
               for $z \in 0..\text{length}(Row_{top}) - 1$
                   $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} \cdot \frac{x_{release_i}}{L_t}$ if $x_{release_i} \le L_t$
                   $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand}$ if $L_t < x_{release_i} \le x_{p\_top}$
                   $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} - \frac{x_{release_i} - x_{p\_top}}{L_t} \cdot (Row_{top_z} \cdot A_{strand})$ if $x_{p\_top} < x_{release_i} \le x$
                   $A_{i,z} \leftarrow 0$ if $x_{release_i} > x_{p\_top} + L_t$
             A
```

$$A_{top} = \begin{pmatrix} 0.358 & 0.358 \\ 0.358 & 0.358 \\ 0.287 & 0.287 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot in^2 \qquad x_{release} = \begin{pmatrix} 2.493 \\ 10 \\ 10.493 \\ 12.493 \\ 14.493 \\ 18.493 \\ 22.493 \end{pmatrix} ft$$

$$P_{p\_top} := -F_{p\_top} \cdot A_{top} = \begin{pmatrix} -17.90 & -17.90 \\ -17.90 & -17.90 \\ -14.36 & -14.36 \\ -0.00 & -0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \end{pmatrix} \cdot kip$$

$$M_{p\_top} := P_{p\_top} \cdot \left(d_{top} - y_t\right) = \begin{pmatrix} 96.063 \\ 96.063 \\ 77.053 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot kip \cdot ft$$

```
$f_{p\_top} :=$ for $i \in 0 .. \text{length}(x_{release}) - 1$
    $M \leftarrow M_{p\_top_i}$
    $P \leftarrow \sum_{j=0}^{\text{cols}(P_{p\_top})-1} P_{p\_top_{i,j}}$
    $A \leftarrow A_{beam}$
    $S_{top} \leftarrow S_T$
    $S_{bott} \leftarrow S_B$
    $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
    $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
$f$
```

Stresses in the beam due to the top prestressing strands only

$$f_{p\_top} = \begin{pmatrix} -80.134 & 18.197 \\ -80.134 & 18.197 \\ -64.276 & 14.596 \\ -2.821 \times 10^{-14} & 6.407 \times 10^{-15} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{psi}$$

<u>Check for beam stresses at release against allowable stresses</u>

Beam stresses at release

top bottom

$$f_{c.release} := f_{ps} + f_{sw} + f_{p\_top} = \begin{pmatrix} 479.113 & -2991.114 \\ 290.936 & -3208.005 \\ 314.871 & -3319.018 \\ 320.41 & -3319.715 \\ 304.237 & -3482.828 \\ 250.563 & -3653.041 \\ 168.123 & -3683.159 \end{pmatrix} \cdot psi \qquad x_{release} = \begin{pmatrix} 2.49 \\ 10.00 \\ 10.49 \\ 12.49 \\ 14.49 \\ 18.49 \\ 22.49 \end{pmatrix} ft$$

$f_{ti.release} := \max(f_{c.release}) = 479\,psi$ Maximum tensile stress at release

$f_{ci.release} := \min(f_{c.release}) = -3683\,psi$ Maximum compressive stress at release

$\text{if}\left(f_{ti} \geq f_{ti.release}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable tension check $f_{ti} = 679\,psi$

$\text{if}\left(-f_{ci} \geq -f_{ci.release}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable compression check $f_{ci} = -5200\,psi$

Camber immediately after transfer

Camber due to prestressing assuming constant maximum force (ignore debonding)

$$\frac{-\min(M_{ps}) \cdot L_{beam}^{2}}{8 \cdot E_{c.beam\_i} \cdot I_{beam}} = 4.978 \cdot in$$

Deflection due to top prestressing assuming constant maximum force (including debonding transfer length)

$$\delta_{p\_top} := \frac{M_{p\_top_0} \cdot x_{p\_top}^{2}}{2 \cdot \left(E_{c.beam\_i} \cdot I_{beam}\right)} = 1.884 \times 10^{-3} \cdot in$$

Deflection due to selfweight of the beam

$$\frac{-5 \cdot \omega_{beam} \cdot L_{beam}^{4}}{384 \cdot E_{c.beam\_i} \cdot I_{beam}} = -2.244 \cdot in$$

Considering the reduced camber due to the effect of debonding

$$d_{strand.db} := \left| \begin{array}{l} \text{for } \ i \in 0 .. \text{length}\left(row_{db}\right) - 1 \\ \quad d_{s_i} \leftarrow d - (2in)row_{db_i} \\ d_s \end{array} \right. = \begin{pmatrix} 70.00 \\ 70.00 \\ 68.00 \\ 68.00 \\ 66.00 \\ 66.00 \\ 64.00 \\ 64.00 \\ 62.00 \\ 62.00 \end{pmatrix} \cdot in$$

$$\delta_{db} := \frac{\overrightarrow{\left[N_{db} \cdot A_{strand} \cdot F_{pt} \cdot \left(d_{strand.db} - y_t\right) \cdot \left(L_{db} + L_t\right)^2\right]}}{2 \cdot E_{c.beam\_i} \cdot I_{beam}} = \begin{pmatrix} 0.025 \\ 0.018 \\ 0.014 \\ 5.618 \times 10^{-3} \\ 7.02 \times 10^{-3} \\ 3.51 \times 10^{-3} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot in$$

$$\sum \delta_{db} = 0.073 \cdot in$$

$$Camber_{tr} := \frac{-\min\left(M_{ps}\right) \cdot L_{beam}^{\ 2}}{8 \cdot E_{c.beam\_i} \cdot I_{beam}} - \frac{5 \cdot \omega_{beam} \cdot L_{beam}^{\ 4}}{384 \cdot E_{c.beam\_i} \cdot I_{beam}} - \sum \delta_{db} - \delta_{p\_top} = 2.659 \cdot in$$

Positive sign indicates camber upwards. Negative sign indeicates deflection

Check the stresses of the beam during shipping and handling, where the supports are not at the ends of the beam (Find the exact location of the supports during shipping and handling)

Moment due to self weight of beam at check locations

$$M_{sw.ship}(x) := \left| \begin{array}{ll} \dfrac{-\omega_{beam} \cdot x^2}{2} & \text{if } 0 \cdot in \le x \le l_{ship} \\ \dfrac{\omega_{beam} \cdot L_{beam} \cdot \left(x - l_{ship}\right)}{2} - \dfrac{\left(\omega_{beam} \cdot x^2\right)}{2} & \text{if } l_{ship} \le x \le \dfrac{L_{beam}}{2} \end{array} \right.$$

Top and bottom concrete stresses at check locations due to beam self weight ONlY

```
$f_{sw.ship} :=$ for $i \in 0 .. \text{length}\left(x_{release}\right) - 1$
    $M \leftarrow M_{sw.ship}\left(x_{release_i}\right)$
    $f_{i,0} \leftarrow \dfrac{-M}{S_T}$
    $f_{i,1} \leftarrow \dfrac{M}{S_B}$
  $f$
```

$$f_{sw.ship} = \begin{array}{c} \begin{array}{cc} \text{top} & \text{bottom} \end{array} \\ \begin{pmatrix} 2 & -2 \\ -249 & 246 \\ -267 & 264 \\ -339 & 335 \\ -408 & 404 \\ -540 & 534 \\ -661 & 653 \end{pmatrix} \end{array} \cdot psi$$

Check for beam stresses during handling & shipping against allowable stresses

Beam stresses during shipping @ handling

$$f_{c.ship} := f_{ps} + f_{sw.ship} + f_{p\_top} = \begin{array}{c} \begin{array}{cc} \text{top} & \text{bottom} \end{array} \\ \begin{pmatrix} 586.62 & -3097.433 \\ 441.841 & -3357.243 \\ 465.776 & -3468.257 \\ 471.315 & -3468.953 \\ 455.143 & -3632.066 \\ 401.469 & -3802.279 \\ 319.028 & -3832.398 \end{pmatrix} \end{array} \cdot psi \qquad x_{release} = \begin{pmatrix} 2.49 \\ 10.00 \\ 10.49 \\ 12.49 \\ 14.49 \\ 18.49 \\ 22.49 \end{pmatrix} ft$$

$f_{ti.ship} := \max\left(f_{c.ship}\right) = 587\,psi$ Maximum tensile stress at release

$f_{ci.ship} := \min\left(f_{c.ship}\right) = -3832\,psi$ Maximum compressive stress at release

$\text{if}\left(f_{ti} \ge f_{ti.ship}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable tension check $f_{ti} = 679\,psi$

$\text{if}\left(-f_{ci} \ge -f_{ci.ship}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable compression check $f_{ci} = -5200\,psi$

Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent loads only

Compressive stress at top of deck due to loads on composite section

$$f_{cf\_actual\_mid} := \frac{-(M_{DC2} + M_{DW})}{S_{t3n} \cdot k_{sdl} \cdot n} = -88\,psi$$

$$if(-f_{cf.deckp} > -f_{cf\_actual\_mid}, "ok", "no\ good") = "ok"$$

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress and permament loads only

Compressive stress at top flange of beam due to prestressing and permanent loads

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} = -1871\,psi$$

$$if(-f_{cfp} > -f_{cf\_actual\_mid}, "ok", "not\ ok") = "ok"$$

Allowable stress check

Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent and transient loads

Compressive stress at top of deck due to loads on composite section **including wind effect according to AASHTO LRFD 2016 Interim revision**

$$f_{cf\_actual\_mid} := \frac{-(M_{DC2} + M_{DW})}{S_{t3n} \cdot k_{sdl} \cdot n} - \frac{1.0 M_{LLI}}{S_{tn} \cdot k \cdot n} - \frac{1.0 M_{WS}}{S_{tn} \cdot k \cdot n} = -684\,psi$$

$$if(-f_{cf.deck} > -f_{cf\_actual\_mid}, "ok", "no\ good") = "ok"$$

Allowable stress check

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress, permanent, and transient loads

Compressive stress at top flange of beam due to prestressing and all loads................

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} - \frac{M_{LLI}}{S_{t.bm.n}} - \frac{1.0 \cdot M_{WS}}{S_{t.bm.n}} = -2452\,psi$$

$$if(-f_{cf} > -f_{cf\_actual\_mid}, "ok", "not\ ok") = "ok"$$

Allowable stress check

Service III limit State - Check for tensile stresses at bottom flange of beam at service conditions

Tensile stress at bottom flange of beam due to prestressing and all loads

$$f_{tf\_actual\_mid} := \frac{-P_e}{A_{beam}} - \frac{P_e \cdot e_s}{S_B} + \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = -62 \cdot psi$$

$$\text{if}\left(f_{tf} > f_{tf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

Allowable stress check

Calculate bar area required to resist tension in the top flange at release, AASHTO Table 5.9.4.1.2-1:

$$f_{ti.ship} = 586.62\,psi$$

Maximum top flange tensile stress at release or handling, whichever is larger (usually, handling stresses are larger)

$$f_c := \text{vlookup}\left(f_{ti.ship}, f_{c.ship}, 1\right)_0 = -3.097 \times 10^3\ psi$$

Bottom flange compressive stress corresponding to the maximum top flange tensile stress at release/shipping

$$slope_m := \frac{f_{ti.ship} - f_c}{d} = 51.167 \cdot \frac{psi}{in}$$

Slope of the section stress over the depth of the beam

$$x_o := \frac{f_{ti.ship}}{slope_m} = 11.465 \cdot in$$

Distance measured from the top of the beam to the point of zero stress

Calculate the width of the beam where the tensile stresses are acting

```
$b_{ten} :=$  for $i \in 0 .. \text{ceil}\left(\frac{x_o}{in}\right)$
                  $x_i \leftarrow \frac{x_o \cdot i}{\text{ceil}\left(\frac{x_o}{in}\right)}$
                  $b_i \leftarrow b_{ft}$  if $0 \le x_i \le d_{ft}$
                  $b_i \leftarrow \left[b_{ft} - \frac{x_i - d_{ft}}{d_{h1}} \cdot \left(b_{ft} - b_{h1}\right)\right]$  if $d_{ft} < x_i \le d_{ft} + d_{h1}$
                  $b_i \leftarrow \left[b_{h1} - \frac{x_i - d_{ft} - d_{h1}}{d_{h2}} \cdot \left(b_{h1} - b_v\right)\right]$  if $d_{ft} + d_{h1} < x_i \le d_{ft} + d_{h1} + d_{h2}$
                  $b_i \leftarrow b_v$  if $d_{ft} + d_{h1} + d_{h2} < x_i$
              $b$
```

Calculate the tensile stress values every inch of depth starting from the top surface of the beam

$$f := \left| \begin{array}{l} \text{for } i \in 0..\text{ceil}\left(\frac{x_o}{in}\right) \\ \quad \left| \begin{array}{l} x_i \leftarrow \dfrac{x_o \cdot i}{\text{ceil}\left(\frac{x_o}{in}\right)} \\ f_i \leftarrow f_{ti.ship} - slope_m \cdot x_i \end{array}\right. \\ f \end{array}\right.$$

$$f = \begin{pmatrix} 586.62 \\ 537.735 \\ 488.85 \\ 439.965 \\ 391.08 \\ 342.195 \\ 293.31 \\ 244.425 \\ 195.54 \\ 146.655 \\ 97.77 \\ 48.885 \\ 0 \end{pmatrix} psi \qquad b_{ten} = \begin{pmatrix} 49 \\ 49 \\ 49 \\ 49 \\ 49 \\ 49 \\ 40.456 \\ 29.31 \\ 18.163 \\ 12.803 \\ 10.892 \\ 8.981 \\ 8 \end{pmatrix} \cdot in$$

Calculate the tensile force that shall be resisted by top reinforcement

$$T := \sum_{i=0}^{\text{length}(f)-2} \left[\frac{1}{4} \cdot \left(f_i + f_{i+1}\right) \cdot \left(b_{ten_i} + b_{ten_{i+1}}\right) \cdot \frac{x_o}{\text{ceil}\left(\frac{x_o}{in}\right)} \right] = 141.041 \cdot kip$$

$$A_{s.top} := \frac{T}{30 \cdot ksi} = 4.701 \cdot in^2$$

Calculate area of tensile reinforcement required in the top of the beam. The stress in bars is limited to 30ksi per AASHTO 5.9.4.1.2. See Figure C.5.9.4.1.2-1 which is based upon .5 f.y of steel rebar

$$A_{bar.top} := 0.44 \cdot in^2$$

Cross sectional area of No. 6 steel rebars

$$n_{bar.release} := \text{Ceil}\left(\frac{A_{s.top}}{A_{bar.top}}, 1\right) = 11$$

number of No. 6 bars provided in the top flange to resist tension at release in the beam ends.

Calculation of minimum length of top tensile reinforcement

AASHTO LRFD Table 5.9.4.1.2-1 specifies a maximum concrete tensile stress of

$0.0948 \cdot \sqrt{f_{ci\_beam}} \leq 0.2\ \text{ksi}$ for tensile zones without bonded reinforcement

$$f_{t.max} := \min\left(0.0948 \cdot \sqrt{\frac{f_{ci\_beam}}{\text{ksi}}}, 0.2\right) \cdot \text{ksi} = 0.2 \cdot \text{ksi}$$

Calculate the minimum required length of top reinforcement based on the stress calculated at distances x.release during release or shipping and handling, whichever is greater. If all the stresses are larger tha f.t.max, estimate the stress after the last point of debonding

```
L_topr :=  | $h \leftarrow x_{release}$
           | $f \leftarrow f_{c.ship}^{\langle 0 \rangle}$
           | $i \leftarrow \text{length}(f) - 1$
           | while $f_i < f_{t.max}$
           |     | break if $i = 0$
           |     | $i \leftarrow i - 1$
           | $x \leftarrow 1 \cdot \text{ft}$
           | $f_{ps} \leftarrow f_{ps_{\text{rows}(f_{ps})-1,0}}$
           | $S(x) \leftarrow f_{ps} - f_{t.max} - \dfrac{\dfrac{\omega_{beam} \cdot L_{beam} \cdot (x - l_{ship})}{2} - \dfrac{(\omega_{beam} \cdot x^2)}{2}}{S_T}$
           | $g \leftarrow \text{root}(S(x), x)$
           | $g$ if $f_{\text{length}(f)-1} > f_{t.max}$
           | $\dfrac{L_{beam}}{2}$ if $\text{Im}(g) \neq 0 \wedge f_{\text{length}(f)-1} > f_{t.max}$
           | $h_{i+1}$ otherwise
```

$$L_{topr} = 26.789\,\text{ft}$$

$$l_d := 1.4 \cdot \frac{1.25 \cdot \dfrac{\pi \cdot \left(\dfrac{6}{8}\right)^2}{4} \cdot 60}{\sqrt{\dfrac{f_{c\_beam}}{\text{ksi}}}} \cdot \text{in} = 1.222 \cdot \text{ft}$$

Calculate the tension development length required for the tensile reinforcement in the top of the beam. As provided AASHTO 5.11.2.1.1 taking into account 1.4 modification factor per AASHTO 5.11.2.1.2

$$L_{topR} := L_{topr} + l_d = 28.012\,\text{ft}$$

Minimum length required for the top reinforcement from each end, **if larger than half the length of the beam, then the top reinforcement shall continue through the enitre beam length from end to end.**

Flexural Capacity

Stress block factor, **AASHTO 5.7.2.2.** Assuming depth of neutral axis lies within the deck

$$\beta_1 := \begin{vmatrix} 0.65 \ \text{ if } \ f_{c\_deck} \geq 8000psi \\ 0.85 \ \text{ if } \ f_{c\_deck} \leq 4000psi \\ \left[0.85 - \left(\frac{f_{c\_deck} - 4000psi}{1000psi}\right)0.05\right] \ \text{ otherwise} \end{vmatrix} = 0.8$$

$$\varepsilon_{cu} := 0.003$$

Maximum usable concrete compressive strain

$$\varepsilon_{pu} := \frac{f_{pu}}{E_p} = 0.0145$$

Ultimate tensile strain of CFCC strand

$$\varepsilon_{pe} := \frac{f_{pe}}{E_p} = 0.0073$$

Effective CFCC prestressing strain

$$\varepsilon_0 := \varepsilon_{pu} - \varepsilon_{pe} = 0.0072$$

Reserve strain in CFCC

$$d_i := d_{strand} + haunch + deck_{thick} = \begin{pmatrix} 79.00 \\ 77.00 \\ 75.00 \\ 73.00 \\ 71.00 \end{pmatrix} \cdot in$$

Depth of prestressing strands
from top of concrete deck

$$A_f := A_{strand} \cdot Row = \begin{pmatrix} 3.04 \\ 3.40 \\ 2.69 \\ 1.97 \\ 0.18 \end{pmatrix} \cdot in^2$$

Area of strands in rows

$$P_{row} := A_f \cdot f_{pe} = \begin{pmatrix} 466.76 \\ 521.68 \\ 411.85 \\ 302.02 \\ 27.46 \end{pmatrix} \cdot kip$$

Effective prestressing force of strands in rows

$$s_i := \left| \begin{array}{l} \text{for } i \in 0..\,\text{length}(\text{Row}) - 1 \\ \quad s_i \leftarrow d_{i_0} - d_{i_i} \\ s \end{array} \right. = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \\ 8 \end{pmatrix} \cdot \text{in}$$

Distance from each layer of prestressing strands to the bottom prestressting layer

$$\text{deck}_{\text{eff}} := \text{deck}_{\text{thick}} - t_{\text{wear}} = 9 \cdot \text{in}$$

Effective deck thickness (total thickness minus assumed sacrificial wearing surface thickness)

Balanced reinforcement ratio

$$c_{\text{bal}} := \frac{\varepsilon_{\text{cu}}}{\varepsilon_{\text{cu}} + \varepsilon_0} \cdot d_{i_0} = 23.17 \cdot \text{in}$$

Depth of neutral axis at balanced failure

Balanced reinforcement ratio assuming **Rectangular section**

$$\rho_{\text{R\_bal}} := \frac{0.85 \cdot f_{\text{c\_deck}} \cdot \beta_1 \cdot b_{\text{eff}} \cdot c_{\text{bal}} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{\text{eff}} \cdot d_{i_0}} = 0.0047$$

Balanced reinforcement ratio assuming **Flanged section**

$$\rho_{\text{Fl\_bal}} := \frac{0.85 \cdot f_{\text{c\_deck}} \cdot \text{deck}_{\text{eff}} \cdot \left(b_{\text{eff}} - b_{\text{ft}}\right) + 0.85 \cdot f_{\text{c\_deck}} \cdot \beta_1 \cdot b_{\text{ft}} \cdot c_{\text{bal}} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{\text{eff}} \cdot d_{i_0}} = 0.0035$$

Balanced reinforcement ratio assuming **Double Flanged section**

$$\rho_{\text{DFl\_bal}} := \frac{0.85 \cdot f_{\text{c\_deck}} \cdot \text{deck}_{\text{eff}} \cdot \left(b_{\text{eff}} - b_{\text{web}}\right) + 0.85 \cdot f_{\text{c\_deck}} \cdot d_{\text{ft}} \cdot \left(b_{\text{ft}} - b_{\text{web}}\right) + 0.85 \cdot f_{\text{c\_deck}} \cdot \beta_1 \cdot b_{\text{web}} \cdot c_{\text{b}}}{E_p \cdot \varepsilon_0 \cdot b_{\text{eff}} \cdot d_{i_0}}$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Tension contorlled** section

```
$Fl\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $|A_{eq\_s} - A_{eq\_f}| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e - 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_{ft}) \cdot deck_{eff}}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{ft}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_T = \begin{pmatrix} 13.7991 \\ 0.0018 \end{pmatrix}$$

$$c_{Fl\_T} := Fl\_T_0 \cdot in = 13.799 \cdot in$$

$$\rho_{Fl\_T} := Fl\_T_1 = 0.0018$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Tension contorlled** section

```
$R\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow \mathrm{length}(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{eff}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$R\_T = \begin{pmatrix} 12.876102 \\ 0.001777 \end{pmatrix}$$

$$c_{R\_T} := R\_T_0 \cdot in = 12.876 \cdot in$$

$$\rho_{R\_T} := R\_T_1 = 0.0018$$

Depth of the N.A. and reinforcement ratio assuming **Double-Flanged Tension contorlled** section. Th depth of the stress block is deeper than the depth of the deck and the top flange together.

```
$DFl\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow \text{length}(d_i) - 1$
    while $|A_{eq\_s} - A_{eq\_f}| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e - 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_{web}) \cdot deck_{eff} - 0.85 \cdot f_{c\_deck} \cdot (b_{ft} - b_{web}) \cdot d_{ft}}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{web}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$DFl\_T = \begin{pmatrix} -4.5819 \\ 0.0018 \end{pmatrix}$$

$$c_{DFl\_T} := DFl\_T_0 \cdot in = -4.582 \cdot in$$

$$\rho_{DFl\_T} := DFl\_T_1 = 0.0018$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu}\cdot\left(\frac{d_{i_0} - c}{c}\right)$$

```
$Fl\_C :=$
    $c \leftarrow 1\cdot in$
    $A_{eq\_s} \leftarrow 1.0\cdot in^2$
    $A_{eq\_f} \leftarrow 2.0\cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
        $f(c) \leftarrow 0.85\cdot f_{c\_deck}\cdot(b_{eff} - b_{ft})\cdot deck_{eff} + 0.85\cdot f_{c\_deck}\cdot\beta_1\cdot c\cdot b_{ft}\ \ldots$
            $+\left(-E_p\cdot\varepsilon_0(c)\cdot A_{eq\_s} - P_e\right)$
        $c \leftarrow root\left(f(c), c, 0.1\cdot in, d_{i_0}\right)$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_C = \begin{pmatrix} 17.879783 \\ 0.001771 \end{pmatrix}$$

$$c_{Fl\_C} := Fl\_C_0\cdot in = 17.88\cdot in$$

$$\rho_{Fl\_C} := Fl\_C_1 = 0.0018$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Compression contorlled** section

```
$R\_C :=$
    $c \leftarrow 1 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $|A_{eq\_s} - A_{eq\_f}| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        $g(c) \leftarrow 0.85 f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{eff} - E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e$
        $c \leftarrow root\left(g(c), c, 0.1 \cdot in, d_{i_0}\right)$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$R\_C = \begin{pmatrix} 16.4643 \\ 0.0018 \end{pmatrix}$$

$$c_{R\_C} := R\_C_0 \cdot in = 16.464 \cdot in$$

$$\rho_{R\_C} := R\_C_1 = 0.0018$$

Depth of the N.A. and reinforcement ratio assuming **Double Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu} \cdot \left(\frac{d_{i_0} - c}{c} \right)$$

```
DFl_C := | $c \leftarrow 1 \cdot in$
         | $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
         | $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
         | $N \leftarrow length(d_i) - 1$
         | while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
         |     $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_i}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
         |     $f(c) \leftarrow 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_{web}) \cdot deck_{eff} + 0.85 \cdot f_{c\_deck} \cdot (b_{ft} - b_{web}) \cdot d_{ft} \ldots$
         |         $+ 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{web} - E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e$
         |     $c \leftarrow root\left(f(c), c, 0.1 \cdot in, d_{i_0}\right)$
         |     $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_i}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
         | $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
         | $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$DFl\_C = \begin{pmatrix} 18.151975 \\ 0.001771 \end{pmatrix}$$

$$c_{DFl\_C} := DFl\_C_0 \cdot in = 18.152 \cdot in$$

$$\rho_{DFl\_C} := DFl\_C_1 = 0.0018$$

Check the mode of failure

```
Section_Mode := | "Rectangular_Tension"  if $\beta_1 \cdot c_{R\_T} \le deck_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal}$
                | "Rectangular_Compression"  if $\beta_1 \cdot c_{R\_C} \le deck_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal}$
                | "Flanged_Tension"  if $\beta_1 \cdot c_{Fl\_T} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_T} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_T} < \rho_{Fl\_}$
                | "Flanged_Compression"  if $\beta_1 \cdot c_{Fl\_C} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_C} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_C} >$
                | "Double_Flanged_Tension"  if $\beta_1 \cdot c_{DFl\_T} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_T} < \rho_{DFl\_bal}$
                | "Double_Flanged_Compression"  if $\beta_1 \cdot c_{DFl\_C} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_C} > \rho_{DFl\_bal}$
```

Section_Mode = "Flanged_Tension"

Select the correct depth of the N.A.

$$c := \begin{cases} c_{R\_T} & \text{if } \beta_1 \cdot c_{R\_T} \le deck_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ c_{R\_C} & \text{if } \beta_1 \cdot c_{R\_C} \le deck_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ c_{Fl\_T} & \text{if } \beta_1 \cdot c_{Fl\_T} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_T} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_T} < \rho_{Fl\_bal} \\ c_{Fl\_C} & \text{if } \beta_1 \cdot c_{Fl\_C} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_C} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_C} > \rho_{Fl\_bal} \\ c_{DFl\_T} & \text{if } \beta_1 \cdot c_{DFl\_T} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_T} < \rho_{DFl\_bal} \\ c_{DFl\_C} & \text{if } \beta_1 \cdot c_{DFl\_C} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_C} > \rho_{DFl\_bal} \end{cases}$$

$$c = 13.799 \cdot in$$

Disclaimer: The design of the section as a dobule flanged section, while theoretically possible, indicates that the depth of the N.A. is in the web of the beam. That could lead to an over-reinforced section that has little or no ductility. Designer is advised to avoid designing the section as a dobule flanged section if possible to ensure proper ducitliy and significant cracking.deflection before failure

Calculate the strain in the extreme CFRP based on the mode of failure

$$\varepsilon_0 := \begin{cases} \varepsilon_{pu} - \varepsilon_{pe} & \text{if } Section\_Mode = \text{"Rectangular\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} & \text{if } Section\_Mode = \text{"Flanged\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} & \text{if } Section\_Mode = \text{"Double\_Flanged\_Tension"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} & \text{if } Section\_Mode = \text{"Rectangular\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} & \text{if } Section\_Mode = \text{"Flanged\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} & \text{if } Section\_Mode = \text{"Double\_Flanged\_Compression"} \end{cases} = 7.229 \times 10^{-3}$$

$$\varepsilon := \left| \begin{array}{l} \text{for } \ i \in 0\,..\,\text{length}(\text{Row}) - 1 \\ \quad \varepsilon_i \leftarrow \varepsilon_0 \cdot \left(\dfrac{d_{i_i} - c}{d_{i_0} - c} \right) \\ \varepsilon \end{array} \right. = \begin{pmatrix} 0.0072 \\ 0.0070 \\ 0.0068 \\ 0.0066 \\ 0.0063 \end{pmatrix}$$

strain in ith layer of prestressing strands

$$\varepsilon_c := \varepsilon_0 \cdot \left(\frac{c}{d_{i_0} - c} \right) = 0.00153$$

strain in the concrete top of the deck

Strength limit state Flexural Resistance:

$$M_n := \left| \begin{array}{ll} E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ldots & \text{if } \ deck_{eff} < \beta_1 \cdot c \le deck_{eff} + d_{ft} \\ + 0.85 f_{c\_deck} \cdot \left(b_{eff} - b_{ft} \right) \cdot deck_{eff} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - \dfrac{deck_{eff}}{2} \right) & \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ldots & \text{if } \ \beta_1 \cdot c > deck_{eff} + d_{ft} \\ + 0.85 f_{c\_deck} \cdot \left(b_{eff} - b_{web} \right) \cdot deck_{eff} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - \dfrac{deck_{eff}}{2} \right) \ldots & \\ + 0.85 f_{c\_deck} \cdot \left(b_{ft} - b_{web} \right) \cdot d_{ft} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - deck_{eff} - \dfrac{d_{ft}}{2} \right) & \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ \text{ if } \ \beta_1 \cdot c \le deck_{eff} & \end{array} \right.$$

$$M_n = 19970.701 \cdot \text{kip} \cdot \text{ft}$$

Nominal moment capacity

$$\phi := \left| \begin{array}{l} 0.85 \ \text{ if } \ \varepsilon_0 \ge 0.005 \\ 0.5167 + 66.67 \cdot \varepsilon_0 \ \text{ if } \ 0.002 \le \varepsilon_0 \le 0.005 \\ 0.65 \ \text{ if } \ \varepsilon_0 \le 0.002 \end{array} \right. = 0.85$$

$$M_r := \phi \cdot M_n = 16975.10 \cdot \text{kip} \cdot \text{ft}$$

$$M_{u\_strength} = 12116.81 \cdot \text{kip} \cdot \text{ft}$$

$$\text{if}\left(M_r > M_{u\_strength}, \text{"ok"}, \text{"no good"} \right) = \text{"ok"}$$

$$\frac{M_r}{M_{u\_strength}} = 1.40$$

Minimum reinforcement against cracking moment

$f_r := 0.24 \cdot \sqrt{f_{c\_beam} \cdot ksi} = 758.947\,psi$	Modulus of rupture of beam concrete, **AASHTO A 5.4.2.6**
$\gamma_1 := 1.6$	Flexural variability factor
$\gamma_2 := 1.1$	Prestress viariability factor
$\gamma_3 := 1.0$	Reinforcement strength ratio
$f_{cpe} := \frac{P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_B} = 3762.60\,psi$	Compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)
$M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe} \right) \cdot S_{bn} - M_{DC1} \cdot \left(\frac{S_{bn}}{S_B} - 1 \right) \right] = 12010.14 \cdot kip \cdot ft$	Cracking moment

$$\text{if}\left(M_r > \min\left(M_{cr}, 1.33 \cdot M_{u\_strength}\right), \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

Approximate mid-span deflection at failure

The deflection calculations follows the approach outlined in the paper " Flexural behaviour of CFRP precast Decked Bulb T beams " by Grace et al. in May/June 2012, Journal of Composites for Construction. In order to calculate the deflection at failure, the moment capacity of the composite section is used as the bending moment. The stress level in the bottom most row is used to calculate the flexural rigidity. The deflection calculated below is approximate, but will give an indication of the deformbility and the level of warning exhibited near failure of the beam.

$d_{i_0} = 79.00 \cdot in$	Depth of the bottom row of strands to the extreme compression fiber
$c = 13.80 \cdot in$	Depth of the neutral axis to the extreme compression fiber
$y_s := d_{i_0} - c = 65.20 \cdot in$	Distance from neutral axis to the bottom row of strands
$EI := \frac{M_n \cdot y_s}{\varepsilon_0} = 2161517999.84 \cdot kip \cdot in^2$	Flexural rigidity of the beam/deck section based on the stress level in the bottom row of prestressing strands
$\omega_f := 8 \cdot \frac{M_n}{L^2} = 8.512 \cdot \frac{kip}{ft}$	Failure load (dead and live loads) uniformly dirstibuted over the entire span
$\delta_f := \frac{5 \cdot \omega_f \cdot L^4}{384EI} = 31.214 \cdot in$	Midspan deflection at strength limit state

f

pocket

$$\frac{\mathrm{al} - \mathrm{P_e}}{} = 0.0026$$

ne

bal

ρ_{Fl_bal}

MDOT

Lawrence Tech

LRFD Design Example for:

CFCC Prestressed Precast Concrete Double-T beam with Cast-In-Place Concrete Slab

Grace et al.

7/1/2019

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Disclaimer

"This material is based upon work supported by the Federal Highway Administration under SPR-1690. Any opinions, findings and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the Federal Highway Administration."

About this Design Example

Description

This document provides guidance for the design of CFCC prestressed precast concrete beams according to AASHTO LRFD Bridge Design Specifications with the neccessary ammendmets where applicable, based on available literature and experimental data from tests conducted by Grace et. al at Lawrence Technological University. The example provided herein is a double-T beam. The cross-section of the bridge is **Type I** as described by **AASHTO Table 4.6.2.2.1-1**.

Standards

The following design standards were utilized in this example:

- AASHTO LRFD Bridge Design Specification, 7th Edition, 2014
- Michigan Department of Transportation Bridge Design Manual, Volume 5
- Michigan Department of Transportation Bridge Design Guide
- ACI 440.4R-04, Prestressing Concrete Structures with FRP Tendons

Code & AASHTO LRFD UPDATES

This Mathcad sheet is developed based on available design guidelines and available AASHTO LRFD edition at the time of writing the sheet. Designer shall check and update design equations according to the latest edition of AASHTO LRFD

General notes

The following notes were considered in this design example:

1- Guarnateed strength of CFRP is reduced to account for environmental effect. The design guarnateec strength is taken as 0.9 x guarnateed strength recommended by manufacturer

2- Initial prestressing stress is limited to 65% of the design (reduced) guaranteed strength according to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations for initial/jacking stress in CFRP strands

3- CFCC strength immediately following transfer is limited to 60% of the design (reduced) guaranteed strength according ccording to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations

4- The depth of the haunch is ignored in calculating section properties or flexural capacity, while is included in calculating the dead loads

5- In strength limit state flanged section design, the concrete strength of the beam portion participating i the stress block was conservatively assumed equal to the concrete strength of the deck (AASHTO LRF C5.7.2.2)

6- Barrier weight was taken as 475 lb/ft. While, weight of midspan diaphragm was 500 lb/beam

7- In srength limit state, the effective height considered for the flanged section equal to the effective deck thickness and the beam top flange

8- In the Mathcad sheet, the option of debonding as well as top prestressing strands are offered as means of reducing the end tensile stresses of the beams

9- This design example is developed based on allowable jacking strength and stress immediately after transfer according to the limits presented in the ACI 440.4R-04. The document can be updated using other prestress limits such as those presented in MDOT SPR-1690 research report and guide

$L_{RefToRef} := 66ft$

$D_{RefAtoBearing} := 18in$

$D_{RefBtoBearing} := 18in$

$L := L_{RefToRef} - D_{RefAtoBearing} - D_{RefBtoBearing} = 63\,ft$ Center to center span Length

$brg_{off} := 6in$ Center of bearing offset to end of beam (same vaLue at both ends is assumed)

$L_{beam} := L + 2 \cdot brg_{off} = 64 \cdot ft$ TotaL length of beam

$l_{ship} := 24 \cdot in$ Distance from support to the end of the beam after force transfer and during shipping and handling

$L_{ship} := L_{beam} - l_{ship} \cdot 2 = 60\,ft$ Distance between supports during handling and shipping

$deck_{width} := 28ft + 0in$ Out to out deck width

$clear_{roadway} := 20ft + 0in$ CLear roadway width

$deck_{thick} := 3in$ Deck slab thickness

$t_{wear} := 0 \cdot in$ Wearing surface is included in the structural deck thickness only when designing the deck as per MDOT BDM 7.02.19.A.4. It is not used when designing the beam.

$t_{fws} := 2in$ Future wearing surface is applied as dead laod to accuant for additional deck thickness if a thicker rigid overlay is placed on deck

$walk_{width} := 5ft + 7in$ sidewalk width

$walk_{thick} := 9in$ sidewalk thickness (0" indicates no separate sidewalk pour)

$S := 7ft + 0in$ Center to center beam spacing

$NO_{beams} := 4$ Total number of beams

$haunch := 0in$ Average haunch thickness for section properties and strength calculations

$haunch_d := 0.0in$ Average haunch thickness for load calculations

$\text{overhang} := 3\text{ft} + 6\text{in}$ Deck overhang width (same value on both overhangs is assumed)

$\text{barrier}_{\text{width}} := 1\text{ft} + 2.5\text{in}$ Barrier width; include offset from back of barrier to edge of deck

$S_{\text{exterior}} := 21\text{ft} + 0\text{in}$ Hz distance between center of gravity of two exterior girders

$\text{Lanes} := \text{floor}\left(\frac{\text{clear}_{\text{roadway}}}{12\text{ft}}\right) = 1.00$ The number of design traffic Lanes

$\text{angle}_{\text{crossing}} := 60\text{deg}$ Angle measured from centerline of bridge to the reference line

$\theta_{\text{skew}} := 90\text{deg} - \text{angle}_{\text{crossing}} = 30.00 \cdot \text{deg}$ Angle measured from a line perpendicular to the centerline of bridge to the reference line

Concrete Material Properties

$f_{c\_deck} := 5\text{ksi}$ Deck concrete compressive strength

$f_{c\_beam} := 8\text{ksi}$ FinaL beam concrete compressive strength

$f_{ci\_beam} := 0.8 f_{c\_beam} = 6.4 \cdot \text{ksi}$ Beam concrete compressive strength at reLease

$\omega_{\text{conc}} := 0.150 \frac{\text{kip}}{\text{ft}^3}$ Unit weight of reinforced concrete for load calculations

$\text{barrier}_{\text{weight}} := 0.475 \frac{\text{kip}}{\text{ft}}$ Weight per foot of barrier (aesthetic parapet tube, see MDOT BDG 6.29.10)

Unit weights of concrete used for modulus of eLasticity calculations, AASHTO Table 3.5.1-1

$$\gamma_c(f'_c) := \begin{cases} 0.145 \frac{\text{kip}}{\text{ft}^3} & \text{if } f'_c \le 5\text{ksi} \\ 0.140 \frac{\text{kip}}{\text{ft}^3} + 0.001 \cdot \left(\frac{f'_c}{\text{ksi}}\right) \frac{\text{kip}}{\text{ft}^3} & \text{otherwise} \end{cases}$$

$\gamma_{c.deck} := \gamma_c(f_{c\_deck}) = 145 \cdot \text{pcf}$

$\gamma_{c.beam} := \gamma_c(f_{c\_beam}) = 148 \cdot \text{pcf}$

$\gamma_{ci.beam} := \gamma_c(f_{ci\_beam}) = 146.4 \cdot \text{pcf}$

Concrete Modulus of Elasticity

Elastic modulus for concrete is as specified by **AASHTO A 5.4.2.4** with a correction factor of 1.0

$E_{c.beam\_i} := 120000 \cdot \left(\frac{\gamma_{ci.beam}}{\frac{kip}{ft^3}} \right)^{2.0} \cdot \left(\frac{f_{ci\_beam}}{ksi} \right)^{0.33} \cdot ksi = 4745.73 \cdot ksi$ Beam concrete at reLease

$E_{c.beam} := 120000 \cdot \left(\frac{\gamma_{c.beam}}{\frac{kip}{ft^3}} \right)^{2.0} \cdot \left(\frac{f_{c\_beam}}{ksi} \right)^{0.33} \cdot ksi = 5220.65 \cdot ksi$ Beam concrete at 28 days

$E_{c.deck} := 120000 \cdot \left(\frac{\gamma_{c.deck}}{\frac{kip}{ft^3}} \right)^{2.0} \cdot \left(\frac{f_{c\_deck}}{ksi} \right)^{0.33} \cdot ksi = 4291.19 \cdot ksi$ Deck concrete at 28 days

CFCC Material Properties

$d_s := 15.2mm = 0.6 \cdot in$ Prestressing strand diameter

$A_{strand} := 0.179 \cdot in^2$ Effective cross sectionaL area

$E_p := 21000ksi$ Tensile elastic modulus

$T_{guts} := 60.70kip$ Guaranteed ultimate tensile capacity

$f'_{pu} := \frac{T_{guts}}{A_{strand}} = 339.11 \cdot ksi$ Calculated ultimate tensile stress

$C_{Ese} := 0.9$

$f_{pu.service} := C_{Ese} \cdot f'_{pu} = 305.2 \cdot ksi$

Environmental reduction factor for prestressed concrete exposed to weather for service limit state calculations

$C_{Est} := 0.9$

$f_{pu} := C_{Est} \cdot f'_{pu} = 305.2 \cdot ksi$

Environmental reduction factor for prestressed concrete exposed to weather for strength limit state calculations

Modular Ratio

$n := \frac{E_{c.beam}}{E_{c.deck}} = 1.217$ Modular ratio for beam

$n_p := \frac{E_p}{E_{c.deck}} = 4.89$ Modular ratio for Prestressing CFCC

Double-T Beam Section Properties:

$A_{beam} := 1470in^2$	Minimum area of beam section
$d := 48in$	Depth of beam
$b_{web} := 11in$	Minimum web thickness (at the bottom)
$b_{web.max} := 12in$	Maximum web thickness (at the top)
$b_{ft} := 84in$	Width of top flange
$d_{ft} := 6in$	Thickness of top flange
$b_{fb} := 0.0in$	Width of bottom flange
$d_{fb} := 0.0in$	Thickness of bottom flange
$b_v := \frac{b_{web} + b_{web.max}}{2} \cdot 2 = 23.00 \cdot in$	Total web shear width
$\omega_{beam} := A_{beam} \cdot (150pcf) = 1531.25 \cdot plf$	Beam weight per foot
$I_{beam} := 328802.29in^4$	Minimum moment of inertia
$y_t := 18.81in$	Depth from centroid to top of beam
$y_b := 29.19in$	Depth from centroid to soffit of beam
$S_T := \frac{I_{beam}}{y_t} = 17480.19 \cdot in^3$	Minimum section modulus about top flange
$S_B := \frac{I_{beam}}{y_b} = 11264.21 \cdot in^3$	Minimum section modulus about bottom flange

Effective Flange Width of Concrete Deck Slab, AASHTO A 4.6.2.6

$\text{Beam\_Design} := \text{"Interior"}$

Choose the design of the beam either "Interior" or "Exterior"

$b_{eff.int} := S = 7.00\,ft$

Effective flange width of deck slab for interior beams

$b_{eff.ext} := \frac{1}{2}\cdot S + overhang = 7.00\,ft$

Effective flange width of deck slab for exterior beams

$$b_{eff} := \left| \begin{array}{l} b_{eff.int} \text{ if } \text{Beam\_Design} = \text{"Interior"} \\ b_{eff.ext} \text{ if } \text{Beam\_Design} = \text{"Exterior"} \end{array} \right. = 2.134$$

$d_{total} := deck_{thick} + d = 51\cdot in$

Total depth of section including deck

Dynamic load Allowance

Dynamic load allowance from **AASHTO Table 3.6.2.1-1** is applied as an increment to the static wheel loads to account for wheel load impacts from moving vehicles.

$IM := 1 + 33\% = 1.33$

Design Factors

These factors are related to the ductility, redundancy and operational importance of the bridge structure components and are applied to the strength limit state.

Ductility

For Strength limit State, a factor of 1.05 is used for nonductile components and connections, 1.00 for conventional designs and details complying with these specifications, and 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by these specifications, **AASHTO A 1.3.3.**

$\eta_D := 1.00$

Redundancy

For Strength limit State, a factor of 1.05 is used for nonredundant members, 1.00 for conventional level of redundancy, foundation elements where ϕ already accounts for redundancy as specified in **AASHT A 10.5**, and 0.95 for exceptional levels of redundancy beyond girder continuity and a torsionally-closed cross-section, **AASHTO A 1.3.4.**

$\eta_R := 1.00$

Operational Importance

For the Strength limit State, a factor of 1.05 is used for critical or essential bridges, 1.00 for typical bridges, and 0.95 for relatively less important bridges, **AASHTO A 1.3.5.**

$\eta_I := 1.00$

Ductility, redundancy, and operational classification considered in the load modifier, **AASHTO Eqn. 1.3.2.1-2.**

$\eta_i := \eta_D\cdot\eta_R\cdot\eta_I = 1.00$

Composite Section Properties

This is the moment of inertia resisting superimposed dead loads.

Elastic Section Properties - Composite Section: k=2

$k_{sdl} := 2$

$A_{haunchkn} := \frac{b_{ft}}{k_{sdl}\,n} \cdot haunch = 0 \cdot in^2$ effective area of haunch

$d_{haunchkn} := d + \frac{haunch}{2} = 48 \cdot in$ Depth of centroid of haunch to bottom of beam

$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$

$b_{effkn} := \frac{b_{eff}}{k_{sdl}\,n} = 34.52 \cdot in$ Transformed deck width

$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 49.5 \cdot in$ Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 103.57 \cdot in^2$ Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 5126.59 \cdot in^3$ Static moment of inertia of transformed section about soffit of beam

$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 30.53 \cdot in$ Depth of CG of composite section from beam soffit

$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\;3}}{12} = 77.68 \cdot in^4$ Moment of inertia of transformed deck about centroid

$I_{haunchkn} := \frac{\frac{b_{ft}}{k_{sdl} \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$ Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_{3n} := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \cdots + A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2 = 368789.4 \cdot in^4$$

$y_{b3n} := d_k = 30.527 \cdot in$ Depth of CG of composite section from beam soffit

$S_{b3n} := \frac{I_{3n}}{y_{b3n}} = 12080.86 \cdot in^3$ Section modulus about bottom of beam

$y_{t.bm.3n} := d - y_{b3n} = 17.47 \cdot in$

Depth of CG of composite section from top of beam

$$S_{t.bm.3n} := \frac{I_{3n}}{y_{t.bm.3n}} = 21105.93 \cdot in^3$$

Section modulus about top of beam

$y_{t3n} := d + haunch + deck_{thick} - t_{wear} - y_{b3n} = 20.47 \cdot in$

Depth of CG of composite section from top of deck

$$S_{t3n} := \frac{I_{3n}}{y_{t3n}} = 18013.23 \cdot in^3$$

Section modulus about top of deck

Elastic Section Properties - Composite Section: k=1

These properties are used to evaluate the moment of inertia for resisting live loads

Assumed wearing surface not included in the structural design deck thickness, per **MDOT BDM 7.02.19.A.4**............

$k := 1$

$$A_{haunchkn} := \frac{b_{ft}}{k\,n} \cdot haunch = 0 \cdot in^2$$

effective area of haunch

$$d_{haunchkn} := d + \frac{haunch}{2} = 48 \cdot in$$

Depth of centroid of haunch to bottom of beam

$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$

$$b_{effkn} := \frac{b_{eff}}{k\,n} = 69.05 \cdot in$$

Transformed deck width

$$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 49.5 \cdot in$$

Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 207.14 \cdot in^2$

Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 10253.18 \cdot in^3$

Static moment of inertia of transformed section about soffit of beam

$$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 31.7 \cdot in$$

Depth of CG of composite section from beam soffit

$$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\,3}}{12} = 155.35 \cdot in^4$$

Moment of inertia of transformed deck about centroid

$$I_{haunchkn} := \frac{\frac{b_{ft}}{k \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$$

Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_n := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \cdots + A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2 = 403847.4 \cdot in^4$$

$$y_{bn} := d_k = 31.698 \cdot in$$

Depth of CG of composite section from beam soffit

$$S_{bn} := \frac{I_n}{y_{bn}} = 12740.31 \cdot in^3$$

Section modulus about bottom of beam

$$y_{t.bm.n} := d - y_{bn} = 16.30 \cdot in$$

Depth of CG of composite section from top of beam

$$S_{t.bm.n} := \frac{I_n}{y_{t.bm.n}} = 24773.47 \cdot in^3$$

Section modulus about top of beam

$$y_{tn} := d + haunch + deck_{thick} - t_{wear} - y_{bn} = 19.3 \cdot in$$

Depth of CG of composite section from top of deck

$$S_{tn} := \frac{I_n}{y_{tn}} = 20922.99 \cdot in^3$$

Section modulus about top of deck

live load lateral Distribution Factors

Cross-section classification...................... **Type I with beams sufficiently connected to act as a unit**

Distribution of live loads from the deck to the beams is evaluated based on the **AASHTO** specified distribution factors. These factors can only be used if generally, the following conditions are met;

- Width of deck is constant.
- Unless otherwise specified, the number of beams is not less than four.
- Beams are parallel and have approximately the same stiffness.
- Curvature in plan is less than the limit specified in **AASHTO A 4.6.1.2.4.**
- Unless otherwise specified, the roadway part of the overhang does not exceed 3.0 ft.
- Cross-section is consistent with one of the cross-sections shown in AASHTO **Table 4.6.2.2.1-1.**

Unless otherwise stated, stiffness parameters for area, moments of inertia and torsional stiffness used shall be taken as those of the cross-section to which traffic will be applied (composite section)

Distance between the centers of gravity of the basic beam and deck

$$e_g := d + \left(\frac{deck_{thick}}{2}\right) + haunch - y_b = 20.31 \cdot in$$

logitudinal stiffness parameter

$$K_g := n \cdot \left(I_{beam} + A_{beam} \cdot e_g^{\ 2}\right) = 1137727.64 \cdot in^4$$

Distribution of live loads for Moment in Interior Beams, AASHTO Table 4.6.2.2.2b-1

Range of Applicability.................................

$$\text{if}(3.5ft < S \le 16ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$$

$$\text{if}\left(4.5in < deck_{thick} \le 12in, \text{"ok"}, \text{"not ok"}\right) = \text{"not ok"}$$

$$\text{if}(20ft < L \le 240ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$$

$$\text{if}\left(NO_{beams} \ge 4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

$$\text{if}\left(10000\ in^4 < K_g \le 7000000\ in^4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$$

One lane loaded

$$M_{lane1\_int} := 0.06 + \left(\frac{S}{14ft}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12 \cdot L \cdot deck_{thick}^{\ 3}} \cdot \frac{ft}{in}\right)^{0.1} = 0.646$$

live load moment disribution factor for interior beam

$$M_{lane\_int} := \max\left(M_{lane1\_int}\right) = 0.646$$

Distribution of live loads for Moment in Exterior Beams, AASHTO Table 4.6.2.2.2d-1

One lane loaded (using the lever rule)

The lever rule involves the use of statics to determine the lateral distribution to the exterior girder by summing moments about the adjacent interior girder to determine the wheel-load reaction at the exterio girder assuming the concrete deck is hinged at the interior girder. A wheel cannot be closer than 2'-0" to the toe of barrier, and the standard wheel spacing is 6'-0". The evaluated factor is multiplied by the multiple presence factor, **AASHTO Table 3.6.1.1.2-1**.

Summing moments about the center of the interior beam

$$R := \frac{\left(S + overhang - barrier_{width} - 2 \cdot ft - \frac{6 \cdot ft}{2}\right)}{S} = 0.613$$

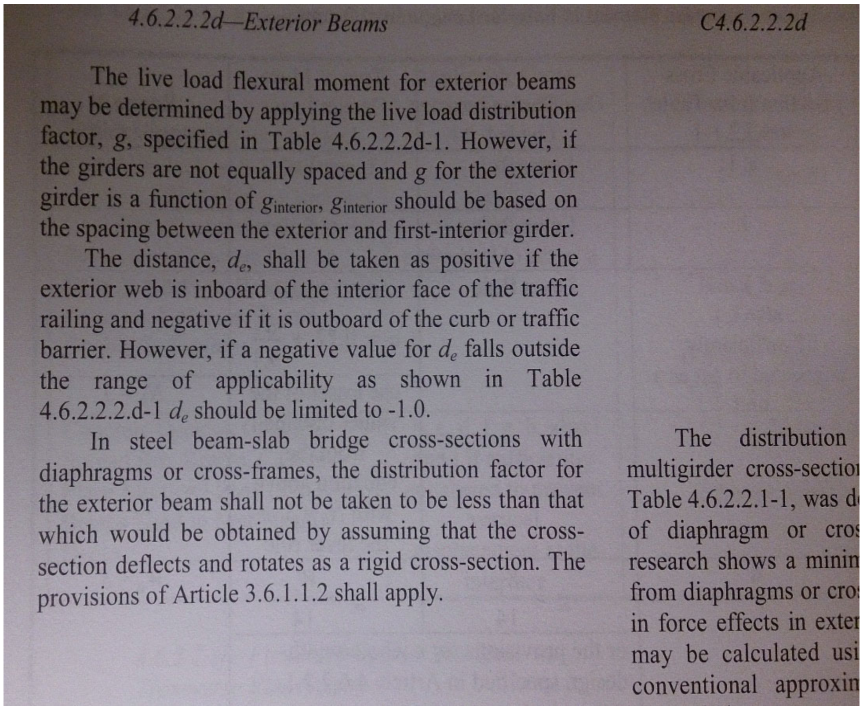
This factor is based on the lever arm rule considring the wheel load and not the resultant of both wheel

Moment distribution factor for exterior beam, one load loaded. The 1.2 accounts for the multiple presence factor, *m* from **AASHTO Table 3.6.1.1.2-1** for one lane loaded

$M_{lane1\_ext} := R \cdot 1.2 = 0.736$

Distribution of live loads for Moment in Exterior Beams, AASHTO C4.6.2.2.2d

AASHTO LRFD 2014 recommends the rigid plate analysis only for **steel beam-slab bridges**. This was a change from ealier versions of AASHTO. It is up to the designed to ignore the rigid plate analysis or take it into consideration when calculating the DF for exterior beam



4.6.2.2.2d—Exterior Beams

The live load flexural moment for exterior beams may be determined by applying the live load distribution factor, *g*, specified in Table 4.6.2.2.2d-1. However, if the girders are not equally spaced and *g* for the exterior girder is a function of $g_{interior}$, $g_{interior}$ should be based on the spacing between the exterior and first-interior girder.

The distance, d_e, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier. However, if a negative value for d_e falls outside the range of applicability as shown in Table 4.6.2.2.2.d-1 d_e should be limited to -1.0.

In steel beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

C4.6.2.2.2d

The distribution
multigirder cross-sectio
Table 4.6.2.2.1-1, was d
of diaphragm or cros
research shows a minin
from diaphragms or cro
in force effects in exte
may be calculated usi
conventional approxin

Additional special analysis investigation is required because the distribution factor for multigirder in cross section was determined without consideration of diaphragm or cross frames. The multiple presence factors are used per **AASHTO Table 3.6.1.1.2-1.** This analysis should be done by sketching the cross section to determine the variables required for this example, the defined deck geometry is used. For any other geometry, these variables should be hand computed and input:

Horizontal distance from center of gravity of the pattern of girders to the exterior girder

$$X_{ext} := \frac{S_{exterior}}{2} = 10.50\,ft$$

Eccentricity of the center line of the standard wheel from the center of gravity of the pattern of girders

$e_1 := X_{ext} + overhang - barrier_{width} - 2ft - \frac{6ft}{2} = 7.792\,ft$

Summation of eccentricities for number of lanes considered:

$e_{NL1} := e_1 = 7.792\,ft$ One lane loaded

$$X_{beams} := \left| \begin{array}{l} \text{for } i \in 0..NO_{beams} - 1 \\ \quad X_i \leftarrow X_{ext} - (i \cdot S) \\ X \end{array} \right. = \begin{pmatrix} 10.50 \\ 3.50 \\ -3.50 \\ -10.50 \end{pmatrix} ft$$

Summation of horizontal distances from the center of gravity of the pattern of girders to each girder

$X_{NB} := \sum X_{beams}^{\,2} = 245.00 \cdot ft^2$

$m_{1R} := 1.2 \cdot \left(\frac{1}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL1}}{X_{NB}} \right) = 0.701$ Reaction on exterior beam when one lane is loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

live load moment disribution factor for exterior beam

$M_{lane\_ext} := \max\left(M_{lane1\_ext}, m_{1R}\right) = 0.736$

Reduction of load Distribution Factors for Moment in longitudinal Beams on Skewed Supports

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10 degrees, the bending moments and shear forces are reduced in accordance with **AASHTO Table 4.6.2.2.2e-1 and 4.6.2.2.3c-1** respectively.

Moment

Range of Applicability

$\text{if}\left(30deg \le \theta_{skew} \le 60deg, \text{"ok"}, \text{"Check below for adjustments of C1 and θskew"}\right) = \text{"ok"}$

$\text{if}\left(3.5ft < S \le 16ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

$\text{if}\left(20ft < L \le 240ft, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

$\text{if}\left(NO_{beams} \ge 4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

$$\theta_{skew} := \begin{cases} \theta_{skew} & \text{if } \theta_{skew} \le 60\cdot deg \\ 60\cdot deg & \text{if } \theta_{skew} > 60\cdot deg \end{cases} = 30\cdot deg$$

$$C_1 := \begin{cases} 0 & \text{if } \theta_{skew} < 30\cdot deg \\ \left[0.25\cdot\left(\frac{K_g}{12.0\cdot L\cdot deck_{thick}{}^3}\cdot\frac{ft}{in}\right)^{0.25}\cdot\left(\frac{S}{L}\right)^{0.5}\right] & \text{otherwise} \end{cases} = 0.228$$

$$Mcorr_{factor} := 1 - C_1\cdot\tan\left(\theta_{skew}\right)^{1.5} = 0.9$$

Correction factor for moment

Reduced distribution factors at strength limit state for interior girders due to skew

$$DF_{strength\_moment\_int} := M_{lane\_int}\cdot Mcorr_{factor} = 0.582$$

Moment

Reduced distribution factors at strength limit state for exterior girders due to skew

$$DF_{strength\_moment\_ext} := M_{lane\_ext}\cdot Mcorr_{factor} = 0.662$$

Moment

Design distribution factors for service and strength limit states

Distribution factor for moment at strength limit state

$$DF_{strength\_moment} := \begin{cases} DF_{strength\_moment\_int} & \text{if } Beam\_Design = \text{"Interior"} \\ DF_{strength\_moment\_ext} & \text{if } Beam\_Design = \text{"Exterior"} \end{cases} = 0.582$$

live load Analysis

Flexure

As per **AASHTO A 3.6.1.2.1**, vehicular live loading designated by the standard HI-93 truck shall be a combination of the design truck or design tandem, and the design lane load. To produce extreme force effects, the spacing between the two 32-kip axles are taken as 14 ft.

Calculate the maximum moment due to the truck load. Maximum truck load moment occurs when the middle axle is positioned at distance 2.33 ft from the midspan. Maximum momment occurs under the middle axle load. Moment due to distributed load occurs at midspan.

Unless more detailed analysis is performed to determine the location and value for the maximum moment under combined truck and distributed loads at both service and strength limit state, the maximum moment from the truck load at distance 2.33 ft from midspan can be assumed to occur at the midspan and combined with the maximum moment from other dead and live distributed loads

Calculate the reaction at the end of the span

$$R := \frac{8kip\cdot\left(\frac{L}{2} - 16.33ft\right) + 32kip\cdot\left(\frac{L}{2} - 2.33ft\right) + 32kip\cdot\left(\frac{L}{2} + 11.67ft\right)}{L} = 38.67\cdot kip$$

Calculate the maximum moment

$$M_{truck} := R\cdot\left(\frac{L}{2} + 2.33ft\right) - 32\cdot kip\cdot 14\cdot ft = 860.222\cdot kip\cdot ft$$

Maximum moment due to design lane load, **AASHTO A 3.6.1.2.4**

$$X := \frac{L}{2} = 31.5\,ft$$

$$M_{lane} := \frac{0.64klf\cdot L\cdot X}{2} - 0.64klf\cdot\frac{X^2}{2} = 317.52\cdot kip\cdot ft$$

Maximum moment due to design tandem, **MDOT BDM 7.01.04.A**

$$M_{tandem} := \frac{60kip\cdot L}{4} = 945\cdot kip\cdot ft$$

Maximum moment due to vehicular live loading by the modified HI-93 design truck and tandem per **MDOT BDM 7.01.04.A**. Modification is by multiplying the load effects by a factor of 1.20. Dynamic load allowance is considered only for the design truck and tandem, **AASHTO A 3.6.1.2.2, 3.6.1.2.3 & 3.6.1.2.4.**

$$M_{LLI} := \left[1.20M_{lane} + IM\cdot\left(1.20\cdot max\left(M_{truck}, M_{tandem}\right)\right)\right]\cdot DF_{strength\_moment} = 1098.62\cdot kip\cdot ft$$

Dead load Analysis

Dead load calculations are slightly adjusted for exterior beam design.

Noncomposite Dead load (DC$_1$)

$$M_{swbeam} := \frac{\omega_{beam}\cdot L^2}{8} = 759.69\cdot kip\cdot ft$$

Total moment due to selfweight of beam

$$deck := \left(deck_{thick}\cdot b_{eff} + haunch_d\cdot b_{ft}\right)\cdot 0.15\frac{kip}{ft^3} = 0.26\cdot klf$$

Selfweight of deck and haunch on beam

$$M_{deck} := \frac{deck\cdot L^2}{8} = 130.23\cdot kip\cdot ft$$

Moment due to selfweight of deck and haunch

$$sip := 15psf\cdot\left(b_{eff} - b_{ft}\right) = 0\cdot klf$$

15 psf weight included for stay-in-place forms per MDOT BDM 7.01.04.I

$$M_{sip} := \frac{sip \cdot L^{2}}{8} = 0.00 \cdot kip \cdot ft$$

Moment due to stay-in-place forms

$$dia_{int} := 0.5 \cdot kip$$

Weight of steel diaphragms at mid-span per each interior beam

$$dia_{ext} := 0.25 \cdot kip$$

Weight of steel diaphragms at mid-span per each exterior beam

$$diaphragm := \left| \begin{array}{l} dia_{int} \text{ if } Beam\_Design = \text{"Interior"} \\ dia_{ext} \text{ if } Beam\_Design = \text{"Exterior"} \end{array} \right. = 0.5 \cdot kip$$

$$spa_{dia} := 2\left(S - b_{v}\right) \cdot \tan\left(\theta_{skew}\right) = 5.87\,ft$$

One row of diaphragms at midspan are used.

$$M_{dia} := diaphragm \cdot \frac{L}{4} = 7.875 \cdot kip \cdot ft$$

$$DC_{1} := \omega_{beam} + deck + sip = 1.794 \cdot klf$$

Dead load (o.wt of beam+ deck+ SIP forms) acting on non-composite section

$$M_{DC1} := M_{swbeam} + M_{deck} + M_{sip} + M_{dia} = 897.80 \cdot kip \cdot ft$$

Total midspan moment acting on the non-composite section

Composite Dead load (DC_2)

$$util := \frac{1}{2} \cdot (0plf) = 0 \cdot klf$$

No utilities are supported by the superstructure

$$barrier1_{weight} := 0.475 \frac{kip}{ft}$$

Weight per foot of first barrier (aesthetics parapet tube, **MDOT BDG 6.29.10**)

$$barrier2_{weight} := 2.25 \cdot in \cdot 40 \cdot in \cdot \omega_{conc} + 0.475 \frac{kip}{ft} = 0.569 \cdot \frac{kip}{ft}$$

Weight per foot of second barrier (modified aesthetics parapet tube, **MDOT BDG 6.29.10**)

$$sidewalk := \frac{2 \cdot walk_{width} \cdot walk_{thick} \cdot \omega_{conc}}{NO_{beams}} = 0.31 \cdot klf$$

Weight to due extra thickness of sidewalk per beam

$$barrier := \frac{barrier1_{weight} + barrier2_{weight}}{NO_{beams}} = 0.26 \cdot klf$$

Total barrier weight per beam

$$soundwall_{weight} := 0.0 \cdot \frac{kip}{ft}$$

Weight of the sound wall, if there is a sound wall

Weight of the sound wall **for exterior beam** design assuming lever arm and an inetremiate hinge on the first interior beam

$$soundwall := \begin{cases} 0 \cdot \frac{kip}{ft} & \text{if } Beam\_Design = \text{"Interior"} \\ \left[soundwall_{weight} \cdot \frac{(S + overhang)}{S} \right] & \text{if } Beam\_Design = \text{"Exterior"} \end{cases} = 0 \cdot \frac{kip}{ft}$$

$DC_2 := sidewalk + barrier + util + soundwall = 0.575 \cdot klf$

Total dead load acting on the composite section

$$M_{DC2} := \frac{DC_2 \cdot L^2}{8} = 285.27 \cdot kip \cdot ft$$

Total midspan moment acting on the composite section

(DW) Wearing Surface load

$$DW := \left(b_{eff}\right) \cdot 0.025 \frac{kip}{ft^2} = 0.175 \cdot klf$$

Self weight of future wearing surface

Maximum unfactored dead load moments

$M_{DC} := M_{DC1} + M_{DC2} = 1183.07 \cdot kip \cdot ft$

Total midspan moment due to loads acting on the composite and non-composite section

$$M_{DW} := \frac{DW \cdot L^2}{8} = 86.82 \cdot kip \cdot ft$$

Midspan moment due to weight of future wearing surface

Wind load on the sound wall

$$M_{wind} := 0.0 \cdot ft \cdot \frac{kip}{ft}$$

Moment due to wind acting at the sound wall

$$W := \frac{M_{wind}}{S} = 0 \cdot \frac{kip}{ft}$$

Extra load on the interior beam due to wind load assuming lever arm analysis and an intermediate hinge at the first interior beam

$$M_{WS} := \frac{W \cdot L^2}{8} = 0 \cdot kip \cdot ft$$

Interior beam moment due to wind acting at the sound wall

load Combinations

Load Combinations: Strength, Extreme Event, Service and Fatigue load combinations are defined per AASHTO 3.4.1. Verify which combination are appropriate. For this concrete box beam design, wind load is not evaluated, and no permit vehicle is specified. However, the design live loading is MDOT HL-93 Modified which accounts for Michigan's inventory of legal and permit vehicles.

Strength I, III, IV and Strength V limit states are considered for the design of this beam. Load combinations factors according to AASHTO LRFD 2016 Interim revision are used

$$M\_Strength_I := \eta_i \cdot \left(1.25M_{DC} + 1.50M_{DW} + 1.75M_{LLI}\right) = 3531.65 \cdot kip \cdot ft$$

$$M\_Strength_{III} := \eta_i \cdot \left(1.25M_{DC} + 1.50M_{DW} + 1.0M_{WS}\right) = 1609.07 \cdot kip \cdot ft$$

$$M\_Strength_{IV} := \eta_i \cdot \left[1.50 \cdot \left(M_{DC} + M_{DW}\right)\right] = 1904.84 \cdot kip \cdot ft$$

$$M\_Strength_V := \eta_i \cdot \left(1.25M_{DC} + 1.50M_{DW} + 1.35M_{LLI} + 1.0 \cdot M_{WS}\right) = 3092.21 \cdot kip \cdot ft$$

$$M_{u\_strength} := \max\left(M\_Strength_I, M\_Strength_{III}, M\_Strength_{IV}, M\_Strength_V\right) = 3531.65 \cdot kip \cdot ft$$

Number of Prestressing Strands

The theoretical number of strands required is calculated using the **Service III** limit state

$$f_b := \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = 2.15 \cdot ksi$$

Tensile stress in bottom flange due to applied loads

Allowable stress limits for concrete (ACI 440.4R Table 3.2)

$f_{ti} := 0.24 \cdot \sqrt{f_{ci\_beam} \cdot ksi} = 0.61 \cdot ksi$ Initial allowable tensile stress

$f_{ci} := -0.65 \cdot f_{ci\_beam} = -4.16 \cdot ksi$ Initial allowable compressive stress (according to AASHTO LRFD 2016 interim revision)

$f_{tf} := 0 \cdot \sqrt{f_{c\_beam} \cdot ksi} = 0.00 \cdot ksi$ Final allowable tensile stress (allowing no tension)

No tension is allowed under service III limit state to avoid potential cracks and shear action on the strands

$f_{cfp} := -0.45 \cdot f_{c\_beam} = -3.60 \cdot ksi$ Final allowable compressive stress in the beam due to sum of effective prestress and permanent loads

$f_{cf.deckp} := -0.45 \cdot f_{c\_deck} = -2.25 \cdot ksi$ Final allowable compressive stress in the slab due to permanent loads

$f_{cf} := -0.6 \cdot f_{c\_beam} = -4.80 \cdot ksi$ Final allowable compressive stress in the beam due to sum of effective prestress, permanent loads, & transient loads

$f_{cf.deck} := -0.6 \cdot f_{c\_deck} = -3.00 \cdot ksi$ Final allowable compressive stress in the beam due to sum of permanent loads and transient loads

$f_p := f_b - f_{tf} = 2.15 \cdot ksi$

Excess tension in the bottom flange due to applied loads

Assuming strand pattern center of gravity is midway between the bottom two rows of strands, i.e. the same number of strands are used in the top and bottom rows of the bottom flange.

$y_{bs} := 3in$

Distance from soffit of beam to center of gravity of strands

$e_{st} := y_b - y_{bs} = 26.19 \cdot in$

Eccentricity of strands from the centroid of beam

Final prestressing force required to counteract excess tension in the bottom flange. Set allowable stress equal to the excess tension, solve for P_e.

$$P_{et} := \frac{f_p}{\left(\frac{1}{A_{beam}} + \frac{e_{st}}{S_B}\right)} = 716.682 \cdot kip$$

$f_{j.act} := 0.65 \cdot f_{pu.service} = 198.377 \cdot ksi$

Actual Jacking stress

$f_{j.max} := 0.65 \cdot f_{pu.service} = 198.377 \cdot ksi$

Maximum allowable Jacking stress, **ACI 440.4R Table 3.3**

$if\left(f_{j.act} \le f_{j.max}, \text{"Ok"}, \text{"Not Ok"}\right) = \text{"Ok"}$

$P_j := A_{strand} \cdot f_{j.act} = 35.51 \cdot kip$

Maximum Jacking prestressing force per strand

$f_t := 0.622 f_{pu.service} = 189.83 \cdot ksi$

Initial prestressing stress immediately **prior** to transfer. shall be less than or equal to the maximum jacking strength, and shall be adjusted accordingly to make sure the stress immedietely following transfer is not exceeding 0.6 times guaranteed strength as shown on the following page

$P_{in} := A_{strand} \cdot f_t = 33.98 \cdot kip$

Initial prestressing force per strand prior to transfer

$P_{pet} := A_{strand} \cdot f_t \cdot 0.75 = 25.48 \cdot kip$

Effective prestressing force assuming 25% final prestress losses per 0.6" diameter strand

$$NO_{strands\_i} := ceil\left(\frac{P_{et}}{P_{pet}}\right) = 29$$

Minimum number of strands required

Strand distribution per row. Row 0 is the bottom most row in the beam. Start adding strands from the

bottom row going up until the number of strands is reached. do not skip rows inbetween. Extra rows wit
zero strands will be eliminated in the analysis.

$row_0 := 8$ $row_1 := 8$ $row_2 := 8$ $row_3 := 4$ $row_4 := 0$ $row_5 := 0$

$row_6 := 0$ $row_7 := 0$ $row_8 := 0$ $row_9 := 0$

$$row = \begin{pmatrix} 8 \\ 8 \\ 8 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
Row := | $a \leftarrow 0$
       | for $i \in 0..length(row) - 1$
       |     | $a \leftarrow a + 1$  if $row_i > 0$
       |     | $a \leftarrow a$  otherwise
       | for $j \in 0..a - 1$
       |     $D_j \leftarrow row_j$
       | D
```

$$Row = \begin{pmatrix} 8 \\ 8 \\ 8 \\ 4 \end{pmatrix}$$

$NO_{strands} := \sum Row = 28.00$ Total number of prestressing strands

```
d_strand := | for $i \in 0..length(Row) - 1$
            |     $d_{s_i} \leftarrow d - (2in) - (2in)i$
            | $d_s$
```

$$= \begin{pmatrix} 46.00 \\ 44.00 \\ 42.00 \\ 40.00 \end{pmatrix} \cdot in$$

Depth of CFCC strands in each layer from the top of the beam section. This calculation assumes a 2" vertical spacing of the strand rows

$$CG := \frac{\left[Row\cdot\left(d - d_{strand}\right)\right]}{\sum Row} = 4.57\cdot in$$

Center of gravity of the strand group measured from the soffit of the beam section

$d_f := (d - CG) + haunch + deck_{thick} = 46.43 \cdot in$

Depth from extreme compression fiber to centroid of CFCC tension reinforcement

$e_s := y_b - CG = 24.62 \cdot in$

Eccentricity of strands from centroid of beam

$A_{ps} := A_{strand} \cdot NO_{strands} = 5.01 \cdot in^2$

Total area of prestressing CFCC strands

Prestress losses

loss due to Elastic Shortening, AASHTO Eqn. C5.9.5.2.3a-1

$$\Delta f_{PES} := \frac{A_{ps} \cdot f_t \cdot \left(I_{beam} + e_s^{\ 2} \cdot A_{beam}\right) - e_s \cdot M_{swbeam} \cdot A_{beam}}{A_{ps} \cdot \left(I_{beam} + e_s^{\ 2} \cdot A_{beam}\right) + \dfrac{A_{beam} \cdot I_{beam} \cdot E_{c.beam\_i}}{E_p}} = 7.20 \cdot ksi$$

$F_{pt} := f_t - \Delta f_{PES} = 182.63 \cdot ksi$

Prestressing stress immediately following transfer

$P_t := A_{ps} \cdot F_{pt} = 915.344 \cdot kip$

According to ACI 440.4R, Table 3.3, the allowable stress immediately after transfer shall not exceed 0.6 fpu

$0.6 \cdot f_{pu.service} = 183.117 \cdot ksi$

$if\left(F_{pt} \le 0.6 \cdot f_{pu.service}, \text{"Ok"}, \text{"Not Ok"}\right) = \text{"Ok"}$

Approximate Estimate of Time dependent losses, AASHTO A 5.9.5.3

$H := 75$

Average annual ambient relative humidity

$\gamma_h := 1.7 - 0.01 \cdot H = 0.95$

Correction factor for relative humidity of ambient air

$$\gamma_{st} := \frac{5}{1 + \frac{f_{ci\_beam}}{ksi}} = 0.68$$

Correction factor for specified concrete strength at time of prestress transfer to the concrete member

$$\Delta f_{pR} := f_t \cdot 1.75\% = 3.32 \cdot ksi$$

Relaxation loss taken as 1.75% of the initial pull per experimental results from Grace et. al based on 1,000,000 hours (114 years)

$$\Delta f_{pLT} := 10 \cdot \frac{f_t \cdot A_{ps}}{A_{beam}} \cdot \gamma_h \cdot \gamma_{st} + 12ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} = 15.18 \cdot ksi$$

long term prestress loss

Difference in thermal coefficient expansion between concrete and CFCC

$$\alpha := 6 \cdot 10^{-6} \cdot \frac{1}{F}$$

Difference in coefficient of thermal expansion between concrete and CFCC

$$t_{amb} := 68F$$

Ambient temperature

$$t_{low} := -10F$$

lowest temperature in Michigan according to AASHTO IRFD 3.12.2

$$\Delta t := t_{amb} - t_{low} = 78\,F$$

Change in the temperature

$$\Delta f_{pt} := \alpha \cdot \Delta t \cdot E_p = 9.83 \cdot ksi$$

Prestress losses due to temp. effect

$$f_{pe} := f_t - \Delta f_{pLT} - \Delta f_{PES} - \Delta f_{pt} = 157.62 \cdot ksi$$

Effective prestressing stress after all losses

$$P_e := A_{ps} \cdot f_{pe} = 790.01 \cdot kip$$

Effective prestressing force after all losses

$$f_t = 189.83 \cdot ksi$$

Initial prestress prior to transfer, not including anchorage losses

$$f_{pe} = 157.62 \cdot ksi$$

Prestress level after all losses

$$loss := \frac{f_t - f_{pe}}{f_t} = 16.97 \cdot \%$$

Total prestress loss

Debonding Criteria

Estimate the location from each beam end where top prestressing or debonding is no longer needed The vectors are developed for possible two different deboning lengths per row. Enter the number of debonded strands and the estimated debonding length in the vectors below per each row location

Location: number of strands: debonding length:

$$Row_{db} := \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \\ 5 \\ 5 \\ 6 \\ 6 \end{pmatrix} \qquad n_{db} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad l_{db} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot ft$$

For debonding pattern, follow staggering guidelines in MDOT BDM 7.02.18.A.2

$$row_{db} := \left| \begin{array}{l} \text{for } i \in 0..2\,\text{length}(Row) - 1 \\ \quad D_i \leftarrow Row_{db_i} \\ D \end{array} \right.$$

$$N_{db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right) - 1 \\ \quad D_i \leftarrow n_{db_i} \\ D \end{array} \right.$$

$$L_{db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right) - 1 \\ \quad D_i \leftarrow l_{db_i} \\ D \end{array} \right.$$

$$row_{db} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \end{pmatrix} \qquad N_{db} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad L_{db} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\sum N_{db} = 0$$

$$\text{Debond}_{tot} := \frac{\sum N_{db}}{NO_{strands}} = 0.00\cdot\%$$

Portion of partially debonded strands in beam section

$$\text{if}\left(\text{Debond}_{tot} \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

Total number of debonded strands in rows

$N_{db.row} :=$

```
for $i \in 0..\text{length}(Row) - 1$
    $a_i \leftarrow 0$
    for $j \in 0..\text{length}(N_{db}) - 1$
        $a_i \leftarrow a_i + N_{db_j}$ if $row_{db_j} = i + 1$
a
```

$$= \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix}$$

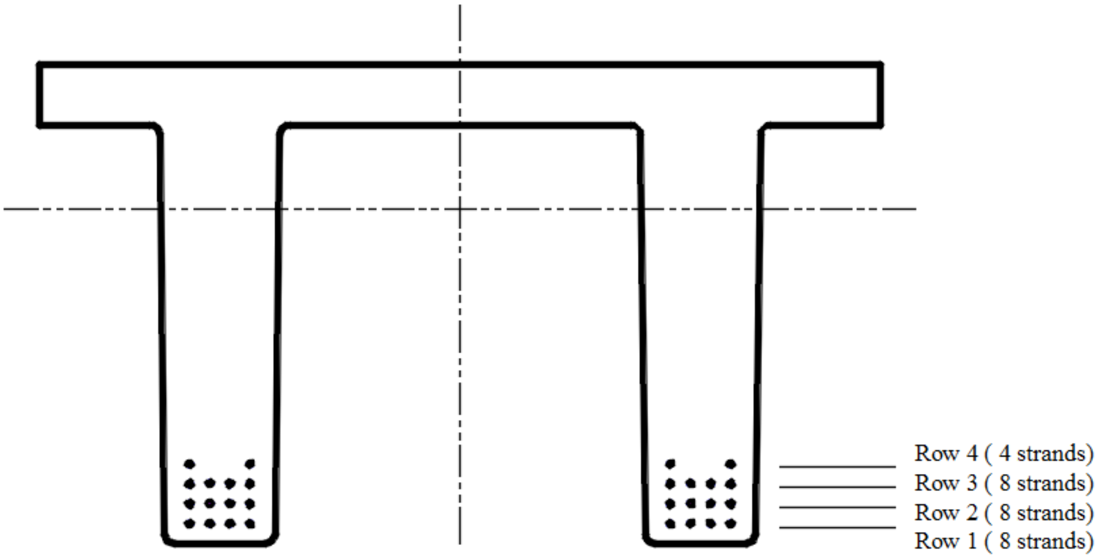
$\text{Debond}_{row} :=$

```
for $i \in 0..\text{length}(Row) - 1$
    $a_i \leftarrow 0$
    $a_i \leftarrow \frac{N_{db.row_i}}{Row_i}$ if $Row_i > 0$
    $0$ otherwise
a
```

$$= \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix}\cdot\%$$

$$\text{if}\left(\max\left(\text{Debond}_{row}\right) \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

The limit of 40% is taken according to MDOT BDM 7.02.18. A2



Optional: only needed if debonding scheme is not sufficient to eliminate the tensile stresses at beam ends either at transfer or due to handling and shipping

CFCC strand transfer length, **ACI 440.4R Table 6.1** $L_t := 50d_s = 2.49\,\text{ft}$

Number of top prestressing strands in the top flange

$$Row_{top} := \begin{pmatrix}2\\2\end{pmatrix}$$

Depth of the top prestressing strands from the top surface of the beam

$$d_{top} := \begin{pmatrix}3\\5\end{pmatrix}\cdot \text{in}$$

Initial prestressing stress/force at the top prestressing strands

$$F_{p\_top} := 50\cdot \text{ksi}$$

Distance from the end of the beam to the point where the top prestressing is no longer needed

$$x_{p\_top} := 10\cdot \text{ft}$$

Top prestressing strands shall not extend the the middle third of the beam. Otherwise, it could affect th stresses at service limit state

$$\text{Check\_Top\_prestressing\_Length} := \begin{array}{|l} \text{"Okay"} \quad \text{if } x_{p\_top} \le \frac{L_{beam}}{3} \\ \text{"Check service stress @ x.p\_top"} \quad \text{if } x_{p\_top} > \frac{L_{beam}}{3} \end{array} = \text{"Okay"}$$

Distance from the end of the beam to the pocket where top prestressing strand is cut after concrete pouring. The middle region between the cut pockets shall be dobonded to avoid force transfer to the middle region

$$x_{pocket} := x_{p\_top} + L_t = 12.493\,\text{ft}$$

Serviceability Checks

CFCC strand transfer length, **ACI 440.4R Table 6.1**

Stress check locations along the beam

Stress locations after the transfer length for bonded and de-bonded strands

$$X_{release} := \text{sort}\left[\text{stack}\left[L_t, \left(L_{db} + L_t\right), x_{p\_top}, x_{pocket}\right]\right] = \begin{pmatrix} 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 10 \\ 12.493 \end{pmatrix} \text{ft}$$

Extracting repreated X from the vector

```
$x_{release} :=$
    $k \leftarrow 0$
    $x_0 \leftarrow L_t$
    for $i \in 1 .. \text{length}(X_{release}) - 1$
        $k \leftarrow k + 1$ if $(X_{release_i} \neq X_{release_{i-1}})$
        $x_k \leftarrow X_{release_i}$
    $x$
```

$$x_{release} = \begin{pmatrix} 2.493 \\ 10 \\ 12.493 \end{pmatrix} \cdot ft$$

Area of strands in each row at each stress check location

```
$A_{db} :=$
    for $i \in 0 .. \text{length}(x_{release}) - 1$
        for $z \in 0 .. \text{length}(Row) - 1$
            $A_{i,z} \leftarrow Row_z \cdot A_{strand}$
        for $j \in 0 .. \text{length}(N_{db}) - 1$
            $n \leftarrow N_{db_j}$
            $row \leftarrow row_{db_j}$
            $L \leftarrow L_{db_j}$
            $A_{i,row-1} \leftarrow (A_{i,row-1} - n \cdot A_{strand}) \cdot \frac{x_{release_i}}{L_t}$ if $x_{release_i} < L_t$
            $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand}$ if $L_t \leq x_{release_i} \leq L$
            $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand} \cdots + n \cdot A_{strand} \cdot \frac{(x_{release_i} - L)}{L_t}$ if $L < x_{release_i} \leq L + L_t$
    $A$
```

$$A_{db} = \begin{pmatrix} 1.43 & 1.43 & 1.43 & 0.72 \\ 1.43 & 1.43 & 1.43 & 0.72 \\ 1.43 & 1.43 & 1.43 & 0.72 \end{pmatrix} \cdot in^2$$

Beam stresses at release due to prestressing only

Sign convention; negative and positive stresses/forces for compression and tension respectively

$$P_{ps} := -F_{pt} \cdot A_{db} = \begin{pmatrix} -261.53 & -261.53 & -261.53 & -130.76 \\ -261.53 & -261.53 & -261.53 & -130.76 \\ -261.53 & -261.53 & -261.53 & -130.76 \end{pmatrix} \cdot kip$$

Midspan moment due to prestressing at release

$$M_{ps} := P_{ps} \cdot \left(d_{strand} - y_t\right) = \begin{pmatrix} -1877.872 \\ -1877.872 \\ -1877.872 \end{pmatrix} \cdot kip \cdot ft$$

Top and bottom concrete stresses at check locations due to prestressing ONLY

```
$f_{ps} :=$ for $i \in 0..\text{length}(x_{release}) - 1$
    $M \leftarrow M_{ps_i}$
    $P \leftarrow \sum_{j=0}^{\text{cols}(P_{ps})-1} P_{ps_{i,j}}$
    $A \leftarrow A_{beam}$
    $S_{top} \leftarrow S_T$
    $S_{bott} \leftarrow S_B$
    $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
    $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
  $f$
```

$$f_{ps} = \left(\begin{array}{cc} \text{top} & \text{bottom} \\ 666 & -2623 \\ 666 & -2623 \\ 666 & -2623 \end{array}\right) \cdot psi$$

Beam stresses at release due to selfweight

Moment due to self weight of beam at check locations

$$M_{sw}(x) := \frac{\omega_{beam} \cdot x}{2} \cdot \left(L_{beam} - x\right)$$

Top and bottom concrete stresses at check locations due to beam self weight ONIY

```
$f_{sw} :=$ for $i \in 0..\text{length}(x_{release}) - 1$
    $M \leftarrow M_{sw}\left(x_{release_i}\right)$
    $f_{i,0} \leftarrow \frac{-M}{S_T}$
    $f_{i,1} \leftarrow \frac{M}{S_B}$
  $f$
```

$$f_{sw} = \left(\begin{array}{cc} \text{top} & \text{bottom} \\ -81 & 125 \\ -284 & 440 \\ -338 & 525 \end{array}\right) \cdot psi$$

Area of **top prestressing** strands at distance X.release from the end

```
$A_{top} :=$ for $i \in 0 .. \text{length}(x_{release}) - 1$
               for $z \in 0 .. \text{length}(Row_{top}) - 1$
                   $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} \cdot \frac{x_{release_i}}{L_t}$  if $x_{release_i} \le L_t$
                   $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand}$  if $L_t < x_{release_i} \le x_{p\_top}$
                   $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} - \frac{x_{release_i} - x_{p\_top}}{L_t} \cdot (Row_{top_z} \cdot A_{strand})$  if $x_{p\_top} < x_{release_i} \le$
                   $A_{i,z} \leftarrow 0$  if $x_{release_i} > x_{p\_top} + L_t$
           $A$
```

$$A_{top} = \begin{pmatrix} 0.358 & 0.358 \\ 0.358 & 0.358 \\ 0 & 0 \end{pmatrix} \cdot in^2 \qquad x_{release} = \begin{pmatrix} 2.493 \\ 10 \\ 12.493 \end{pmatrix} ft$$

$$P_{p\_top} := -F_{p\_top} \cdot A_{top} = \begin{pmatrix} -17.90 & -17.90 \\ -17.90 & -17.90 \\ -0.00 & -0.00 \end{pmatrix} \cdot kip$$

$$M_{p\_top} := P_{p\_top} \cdot (d_{top} - y_t) = \begin{pmatrix} 44.183 \\ 44.183 \\ 0 \end{pmatrix} \cdot kip \cdot ft$$

```
$f_{p\_top} :=$ for $i \in 0..\text{length}(x_{release}) - 1$
    $M \leftarrow M_{p\_top_i}$
    $P \leftarrow \sum_{j=0}^{\text{cols}(P_{p\_top})-1} P_{p\_top_{i,j}}$
    $A \leftarrow A_{beam}$
    $S_{top} \leftarrow S_T$
    $S_{bott} \leftarrow S_B$
    $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
    $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
  $f$
```

Stresses in the beam due to the top prestressing strands only

$$f_{p\_top} = \begin{pmatrix} -54.685 & 22.716 \\ -54.685 & 22.716 \\ -1.925 \times 10^{-14} & 7.997 \times 10^{-15} \end{pmatrix} \text{psi}$$

Check for beam stresses at release against allowable stresses

Beam stresses at release

$$f_{c.release} := f_{ps} + f_{sw} + f_{p\_top} = \begin{array}{c} \begin{array}{cc} \text{top} & \text{bottom} \end{array} \\ \begin{pmatrix} 531.169 & -2475.416 \\ 327.954 & -2160.061 \\ 328.243 & -2098.363 \end{pmatrix} \end{array} \cdot \text{psi}$$

$$x_{release} = \begin{pmatrix} 2.49 \\ 10.00 \\ 12.49 \end{pmatrix} \text{ft}$$

$f_{ti.release} := \max(f_{c.release}) = 531\,\text{psi}$ Maximum tensile stress at release

$f_{ci.release} := \min(f_{c.release}) = -2475\,\text{psi}$ Maximum compressive stress at release

$\text{if}(f_{ti} \geq f_{ti.release}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$ Allowable tension check $f_{ti} = 607\,\text{psi}$

$\text{if}\left(-f_{ci} \geq -f_{ci.release}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable compression check $f_{ci} = -4160\,psi$

Camber immediately after transfer

Camber due to prestressing assuming constant maximum force (ignore debonding)

$$\frac{-\min\left(M_{ps}\right)\cdot L_{beam}^{2}}{8\cdot E_{c.beam\_i}\cdot I_{beam}} = 1.065\cdot in$$

Deflection due to top prestressing assuming constant maximum force (including debonding transfer length)

$$\delta_{p\_top} := \frac{M_{p\_top_0}\cdot x_{p\_top}^{2}}{2\cdot\left(E_{c.beam\_i}\cdot I_{beam}\right)} = 2.446\times10^{-3}\cdot in$$

Deflection due to selfweight of the beam

$$\frac{-5\cdot\omega_{beam}\cdot L_{beam}^{4}}{384\cdot E_{c.beam\_i}\cdot I_{beam}} = -0.37\cdot in$$

Considering the reduced camber due to the effect of debonding

$$d_{strand.db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right)-1 \\ \quad d_{s_i} \leftarrow d-(2in)row_{db_i} \\ d_s \end{array} \right. = \begin{pmatrix} 46.00 \\ 46.00 \\ 44.00 \\ 44.00 \\ 42.00 \\ 42.00 \\ 40.00 \\ 40.00 \end{pmatrix}\cdot in$$

$$\delta_{db} := \frac{\overrightarrow{\left[N_{db} \cdot A_{strand} \cdot F_{pt} \cdot \left(d_{strand.db} - y_t\right) \cdot \left(L_{db} + L_t\right)^2\right]}}{2 \cdot E_{c.beam\_i} \cdot I_{beam}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot in$$

$$\sum \delta_{db} = 0 \cdot in$$

$$Camber_{tr} := \frac{-\min\left(M_{ps}\right) \cdot L_{beam}^{\ 2}}{8 \cdot E_{c.beam\_i} \cdot I_{beam}} - \frac{5 \cdot \omega_{beam} \cdot L_{beam}^{\ 4}}{384 \cdot E_{c.beam\_i} \cdot I_{beam}} - \sum \delta_{db} - \delta_{p\_top} = 0.692 \cdot in$$

Positive sign indicates camber upwards. Negative sign indeicates deflection

Check the stresses of the beam during shipping and handling, where the supports are not at the ends of the beam (Find the exact location of the supports during shipping and handling)

Moment due to self weight of beam at check locations

$$M_{sw.ship}(x) := \left| \begin{array}{ll} \dfrac{-\omega_{beam} \cdot x^2}{2} & \text{if } 0 \cdot in \le x \le l_{ship} \\ \dfrac{\omega_{beam} \cdot L_{beam} \cdot \left(x - l_{ship}\right)}{2} - \dfrac{\left(\omega_{beam} \cdot x^2\right)}{2} & \text{if } l_{ship} \le x \le \dfrac{L_{beam}}{2} \end{array} \right.$$

Top and bottom concrete stresses at check locations due to beam self weight ONIY

```
f_sw.ship :=  | for i ∈ 0 .. length(x_release) − 1
              |   | M ← M_sw.ship(x_release_i)
              |   | f_i,0 ← −M / S_T
              |   | f_i,1 ← M / S_B
              | f
```

$$f_{sw.ship} = \begin{array}{c} \begin{array}{cc} \text{top} & \text{bottom} \end{array} \\ \begin{pmatrix} -13 & 21 \\ -217 & 336 \\ -271 & 420 \end{pmatrix} \cdot psi \end{array}$$

Check for beam stresses during handling & shipping against allowable stresses

Beam stresses during shipping @ handling

$$f_{c.ship} := f_{ps} + f_{sw.ship} + f_{p\_top} = \begin{array}{cc} \text{top} & \text{bottom} \\ \left(\begin{array}{cc} 598.445 & -2579.818 \\ 395.23 & -2264.463 \\ 395.519 & -2202.765 \end{array}\right) \end{array} \cdot psi$$

$$x_{release} = \begin{pmatrix} 2.49 \\ 10.00 \\ 12.49 \end{pmatrix} ft$$

$f_{ti.ship} := \max\left(f_{c.ship}\right) = 598\,psi$ Maximum tensile stress at release

$f_{ci.ship} := \min\left(f_{c.ship}\right) = -2580\,psi$ Maximum compressive stress at release

$\text{if}\left(f_{ti} \geq f_{ti.ship}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable tension check $f_{ti} = 607\,psi$

$\text{if}\left(-f_{ci} \geq -f_{ci.ship}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable compression check $f_{ci} = -4160\,psi$

Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent loads only

Compressive stress at top of deck due to loads on composite section

$$f_{cf\_actual\_mid} := \frac{-\left(M_{DC2} + M_{DW}\right)}{S_{t3n} \cdot k_{sdl} \cdot n} = -102\,psi$$

$$\text{if}\left(-f_{cf.deckp} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$$

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress and permament loads only

Compressive stress at top flange of beam due to prestressing and permanent loads

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} = -253\,psi$$

$\text{if}\left(-f_{cfp} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent and transient loads

Compressive stress at top of deck due to loads on composite section **including wind effect**

according to AASHTO LRFD 2016 Interim revision

$$f_{cf\_actual\_mid} := \frac{-\left(M_{DC2}+M_{DW}\right)}{S_{t3n}\cdot k_{sdl}\cdot n} - \frac{1.0M_{LLI}}{S_{tn}\cdot k\cdot n} - \frac{1.0M_{WS}}{S_{tn}\cdot k\cdot n} = -620\,psi$$

$\text{if}\left(-f_{cf.deck} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$ Allowable stress check

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress, permanent, and transient loads

Compressive stress at top flange of beam due to prestressing and all loads................

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e\cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2}+M_{DW}}{S_{t.bm.3n}} - \frac{M_{LLI}}{S_{t.bm.n}} - \frac{1.0\cdot M_{WS}}{S_{t.bm.n}} = -785\,psi$$

$\text{if}\left(-f_{cf} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Service III limit State - Check for tensile stresses at bottom flange of beam at service conditions

Tensile stress at bottom flange of beam due to prestressing and all loads

$$f_{tf\_actual\_mid} := \frac{-P_e}{A_{beam}} - \frac{P_e\cdot e_s}{S_B} + \frac{M_{DC1}}{S_B} + \frac{M_{DC2}+M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = -110\cdot psi$$

$\text{if}\left(f_{tf} > f_{tf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Calculate bar area required to resist tension in the top flange at release, AASHTO Table 5.9.4.1.2-1:

$f_{ti.ship} = 598.445\,psi$

Maximum top flange tensile stress at release or handling, whichever is larger (usually, handling stresses are larger)

$f_c := \text{vlookup}\left(f_{ti.ship}, f_{c.ship}, 1\right)_0 = -2.58\times 10^3\,psi$

Bottom flange compressive stress corresponding to the maximum top flange tensile stress at release/shipping

$$slope_m := \frac{f_{ti.ship} - f_c}{d} = 66.214\cdot\frac{psi}{in}$$

Slope of the section stress over the depth of the beam

$$x_o := \frac{f_{ti.ship}}{slope_m} = 9.038\cdot in$$

Distance measured from the top of the beam to the point of zero stress

Calculate the width of the beam where the tensile stresses are acting

```
$b_{ten} :=$ for $i \in 0 .. \text{ceil}\left(\frac{x_o}{in}\right)$
    $x_i \leftarrow \frac{x_o \cdot i}{\text{ceil}\left(\frac{x_o}{in}\right)}$
    $b_i \leftarrow b_{ft}$  if $0 \le x_i \le d_{ft}$
    $b_i \leftarrow b_v$  if $d_{ft} < x_i$
  $b$
```

Calculate the tensile stress values every inch of depth starting from the top surface of the beam

```
$f :=$ for $i \in 0 .. \text{ceil}\left(\frac{x_o}{in}\right)$
    $x_i \leftarrow \frac{x_o \cdot i}{\text{ceil}\left(\frac{x_o}{in}\right)}$
    $f_i \leftarrow f_{ti.ship} - slope_m \cdot x_i$
  $f$
```

$$f = \begin{pmatrix} 598.445 \\ 538.6 \\ 478.756 \\ 418.911 \\ 359.067 \\ 299.222 \\ 239.378 \\ 179.533 \\ 119.689 \\ 59.844 \\ 6.754 \times 10^{-14} \end{pmatrix} \quad {}_1b_{ten} = \begin{pmatrix} 84 \\ 84 \\ 84 \\ 84 \\ 84 \\ 84 \\ 84 \\ 23 \\ 23 \\ 23 \\ 23 \end{pmatrix} \cdot in$$

Calculate the tensile force that shall be resisted by top reinforcement

$$T := \sum_{i=0}^{\text{length}(f)-2} \left[\frac{1}{4} \cdot \left(f_i + f_{i+1}\right) \cdot \left(b_{ten_i} + b_{ten_{i+1}}\right) \cdot \frac{x_o}{\text{ceil}\left(\frac{x_o}{in}\right)} \right] = 206.548 \cdot kip$$

$A_{s.top} := \frac{T}{30 \cdot ksi} = 6.885 \cdot in^2$

Calculate area of tensile reinforcement required in the top of the beam. The stress in bars is limited to 30ksi per AASHTO 5.9.4.1.2. See Figure C.5.9.4.1.2-1 which is based upon .5 f.y of steel rebar

$A_{bar.top} := 0.44 \cdot in^2$

Cross sectional area of No. 6 steel rebars

$n_{bar.release} := \text{Ceil}\left(\frac{A_{s.top}}{A_{bar.top}}, 1\right) = 16$

number of No. 6 bars provided in the top flange to resist tension at release in the beam ends.

Calculation of minimum length of top tensile reinforcement

AASHTO LRFD Table 5.9.4.1.2-1 specifies a maximum concrete tensile stress of $0.0948 \cdot \sqrt{f_{ci\_beam}} \leq 0.2 \text{ ksi}$ for tensile zones without bonded reinforcement

$f_{t.max} := \min\left(0.0948 \cdot \sqrt{\frac{f_{ci\_beam}}{ksi}}, 0.2\right) \cdot ksi = 0.2 \cdot ksi$

Calculate the minimum required length of top reinforcement based on the stress calculated at distance
x.release during release or shipping and handling, whichever is greater. If all the stresses are larger tha
f.t.max, estimate the stress after the last point of debonding

```
$L_{topr} :=$
    $h \leftarrow x_{release}$
    $f \leftarrow f_{c.ship}^{\langle 0 \rangle}$
    $i \leftarrow \text{length}(f) - 1$
    while $f_i < f_{t.max}$
        break if $i = 0$
        $i \leftarrow i - 1$
    $x \leftarrow 1 \cdot ft$
    $f_{ps} \leftarrow f_{ps_{\text{rows}(f_{ps})-1, 0}}$
    $S(x) \leftarrow f_{ps} - f_{t.max} - \frac{\frac{\omega_{beam} \cdot L_{beam} \cdot (x - l_{ship})}{2} - \frac{(\omega_{beam} \cdot x^2)}{2}}{S_T}$
    $g \leftarrow \text{root}(S(x), x)$
    $g$ if $f_{\text{length}(f)-1} > f_{t.max}$
    $\frac{L_{beam}}{2}$ if $\text{Im}(g) \neq 0 \wedge f_{\text{length}(f)-1} > f_{t.max}$
    $h_{i+1}$ otherwise
```

$L_{topr} = 29.083\,ft$

$$l_d := 1.4 \cdot \frac{1.25 \cdot \frac{\pi \cdot \left(\frac{6}{8}\right)^2}{4} \cdot 60}{\sqrt{\frac{f_{c\_beam}}{ksi}}} \cdot in = 1.367 \cdot ft$$

Calculate the tension development length required for the tensile reinforcement in the top of the beam. As provided AASHTO 5.11.2.1.1 taking into account 1.4 modification factor per AASHTO 5.11.2.1.2

$L_{topR} := L_{topr} + l_d = 30.45\,ft$

Minimum length required for the top reinforcement from each end

Flexural Capacity

Stress block factor, **AASHTO 5.7.2.2.** Assuming depth of neutral axis lies within the deck

$$\beta_1 := \begin{array}{|l} 0.65 \ \text{ if } f_{c\_deck} \geq 8000psi \\ 0.85 \ \text{ if } f_{c\_deck} \leq 4000psi \\ \left[0.85 - \left(\frac{f_{c\_deck} - 4000psi}{1000psi}\right)0.05\right] \ \text{ otherwise} \end{array} = 0.8$$

$\varepsilon_{cu} := 0.003$

Maximum usable concrete compressive strain

$$\varepsilon_{pu} := \frac{f_{pu}}{E_p} = 0.0145$$

Ultimate tensile strain of CFCC strand

$$\varepsilon_{pe} := \frac{f_{pe}}{E_p} = 0.0075$$

Effective CFCC prestressing strain

$\varepsilon_0 := \varepsilon_{pu} - \varepsilon_{pe} = 0.007$

Reserve strain in CFCC

$$d_i := d_{strand} + haunch + deck_{thick} = \begin{pmatrix} 49.00 \\ 47.00 \\ 45.00 \\ 43.00 \end{pmatrix} \cdot in$$

Depth of prestressing strands
from top of concrete deck

$$A_f := A_{strand} \cdot Row = \begin{pmatrix} 1.43 \\ 1.43 \\ 1.43 \\ 0.72 \end{pmatrix} \cdot in^2$$

Area of strands in rows

$$P_{row} := A_f \cdot f_{pe} = \begin{pmatrix} 225.72 \\ 225.72 \\ 225.72 \\ 112.86 \end{pmatrix} \cdot kip$$

Effective prestressing force of strands in rows

$$s_i := \left| \begin{array}{l} \text{for } i \in 0 .. \text{length}(Row) - 1 \\ \quad s_i \leftarrow d_{i_0} - d_{i_i} \\ s \end{array} \right. = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \end{pmatrix} \cdot in$$

Distance from each layer of prestressing
strands to the bottom prestressting layer

$$h_{eff} := deck_{thick} + d_{ft} - t_{wear} = 9 \cdot in$$

Effective thickness (total thickness minus
assumed sacrificial wearing surface thickness)

Balanced reinforcement ratio

$$c_{bal} := \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_0} \cdot d_{i_0} = 14.66 \cdot in$$

Depth of neutral axis at balanced failure

Balanced reinforcement ratio assuming flanged section

$$\rho_{Fl\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot h_{eff} \cdot \left(b_{eff} - b_v\right) + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_v \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} = 0.0044$$

Balanced reinforcement ratio assuming rectangular section

$$\rho_{R\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{eff} \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} = 0.0056$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Tension contorlled** section

```
$Fl\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow \text{length}(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e - 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_v) \cdot h_{eff}}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_v}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_T = \begin{pmatrix} -10.6832 \\ 0.0012 \end{pmatrix}$$

$$c_{Fl\_T} := Fl\_T_0 \cdot in = -10.683 \cdot in$$

$$\rho_{Fl\_T} := Fl\_T_1 = 0.0012$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Tension contorlled** section

```
$R\_T :=$
    $c \leftarrow 1.0\cdot in$
    $A_{eq\_s} \leftarrow 1.0\cdot in^2$
    $A_{eq\_f} \leftarrow 2.0\cdot in^2$
    $N \leftarrow \text{length}(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p\cdot\varepsilon_0\cdot A_{eq\_s} + P_e}{0.85\cdot f_{c\_deck}\cdot\beta_1\cdot b_{eff}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
    $\begin{pmatrix}\frac{c}{in}\\ \rho\end{pmatrix}$
```

$$(R\_T) = \begin{pmatrix}5.203783\\ 0.001146\end{pmatrix}$$

$$c_{R\_T} := R\_T_0\cdot in = 5.204\cdot in$$

$$\rho_{R\_T} := R\_T_1 = 0.0011$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu} \cdot \left(\frac{d_{i_0} - c}{c} \right)$$

```
$Fl\_C :=$
    $c \leftarrow 1 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left| A_{eq\_s} - A_{eq\_f} \right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        $f(c) \leftarrow 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_v) \cdot h_{eff} + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_v \cdots$
            $+ (-E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e)$
        $c \leftarrow root(f(c), c, 0.1 \cdot in, d_{i_0})$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_C = \begin{pmatrix} 6.245089 \\ 0.001144 \end{pmatrix}$$

$$c_{Fl\_C} := Fl\_C_0 \cdot in = 6.245 \cdot in$$

$$\rho_{Fl\_C} := Fl\_C_1 = 0.0011$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Compression contorlled** section

```
R_C := | $c \leftarrow 1\cdot in$
       | $A_{eq\_s} \leftarrow 1.0\cdot in^2$
       | $A_{eq\_f} \leftarrow 2.0\cdot in^2$
       | $N \leftarrow length\left(d_i\right) - 1$
       | while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^2$
       |     | $A_{eq\_s} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
       |     | $g(c) \leftarrow 0.85 f_{c\_deck}\cdot\beta_1\cdot c\cdot b_{eff} - E_p\cdot\varepsilon_0(c)\cdot A_{eq\_s} - P_e$
       |     | $c \leftarrow root\left(g(c), c, 0.1\cdot in, d_{i_0}\right)$
       |     | $A_{eq\_f} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
       | $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
       | $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$R\_C = \begin{pmatrix} 8.0426 \\ 0.0011 \end{pmatrix}$$

$$c_{R\_C} := R\_C_0\cdot in = 8.043\cdot in$$

$$\rho_{R\_C} := R\_C_1 = 0.0011$$

Check the mode of failure

$$\text{Section\_Mode} := \left|\begin{array}{l} \text{"Rectangular\_Tension"} \quad \text{if } \beta_1 \cdot c_{R\_T} \le h_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ \text{"Rectangular\_Compression"} \quad \text{if } \beta_1 \cdot c_{R\_C} \le h_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ \text{"Flanged\_Tension"} \quad \text{if } \beta_1 \cdot c_{Fl\_T} > h_{eff} \wedge \rho_{Fl\_T} < \rho_{Fl\_bal} \\ \text{"Flanged\_Compression"} \quad \text{if } \beta_1 \cdot c_{Fl\_C} > h_{eff} \wedge \rho_{Fl\_C} > \rho_{Fl\_bal} \end{array}\right.$$

$$(\text{Section\_Mode}) = \text{"Rectangular\_Tension"}$$

Select the correct depth of the N.A.

$$c := \left|\begin{array}{l} c_{R\_T} \quad \text{if } \beta_1 \cdot c_{R\_T} \le h_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ c_{R\_C} \quad \text{if } \beta_1 \cdot c_{R\_C} \le h_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ c_{Fl\_T} \quad \text{if } \beta_1 \cdot c_{Fl\_T} > h_{eff} \wedge \rho_{Fl\_T} < \rho_{Fl\_bal} \\ c_{Fl\_C} \quad \text{if } \beta_1 \cdot c_{Fl\_C} > h_{eff} \wedge \rho_{Fl\_C} > \rho_{Fl\_bal} \end{array}\right.$$

$$c = 5.204 \cdot in$$

Calculate the strain in the extreme CFRP based on the mode of failure

$$\varepsilon_0 := \left|\begin{array}{l} \varepsilon_{pu} - \varepsilon_{pe} \quad \text{if Section\_Mode} = \text{"Rectangular\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} \quad \text{if Section\_Mode} = \text{"Flanged\_Tension"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \quad \text{if Section\_Mode} = \text{"Rectangular\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \quad \text{if Section\_Mode} = \text{"Flanged\_Compression"} \end{array}\right. = 7.027 \times 10^{-3}$$

$$\varepsilon := \left|\begin{array}{l} \text{for } i \in 0 .. \text{length(Row)} - 1 \\ \quad \varepsilon_i \leftarrow \varepsilon_0 \cdot \left(\dfrac{d_{i_i} - c}{d_{i_0} - c}\right) \\ \varepsilon \end{array}\right. = \begin{pmatrix} 0.0070 \\ 0.0067 \\ 0.0064 \\ 0.0061 \end{pmatrix}$$

strain in ith layer of prestressing strands

$\varepsilon_c := \varepsilon_0 \cdot \left(\frac{c}{d_{i_0} - c} \right) = 0.00083$

strain in the concrete top of the deck

Strength limit state Flexural Resistance:

$$M_n := \left| \begin{array}{l} E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \frac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \frac{\beta_1 \cdot c}{2} \right) ... \quad \text{if } \beta_1 \cdot c > h_{eff} \\ +0.85 f_{c\_deck} \cdot (b_{eff} - b_v) \cdot h_{eff} \cdot \left(\frac{\beta_1 \cdot c}{2} - \frac{h_{eff}}{2} \right) \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \frac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \frac{\beta_1 \cdot c}{2} \right) \quad \text{if } \beta_1 \cdot c \le h_{eff} \end{array} \right.$$

$M_n = 5498.407 \cdot kip \cdot ft$

Nominal moment capacity

$$\phi := \left| \begin{array}{l} 0.85 \quad \text{if } \varepsilon_0 \ge 0.005 \\ 0.5167 + 66.67 \cdot \varepsilon_0 \quad \text{if } 0.002 \le \varepsilon_0 \le 0.005 \\ 0.65 \quad \text{if } \varepsilon_0 \le 0.002 \end{array} \right. = 0.85$$

$M_r := \phi \cdot M_n = 4673.65 \cdot kip \cdot ft$

$M_{u\_strength} = 3531.65 \cdot kip \cdot ft$

$\text{if}\left(M_r > M_{u\_strength}, \text{"ok"}, \text{"no good"} \right) = \text{"ok"}$

$\frac{M_r}{M_{u\_strength}} = 1.32$

Minimum reinforcement against cracking moment

$f_r := 0.24 \cdot \sqrt{f_{c\_beam} \cdot ksi} = 678.823\,psi$

Modulus of rupture of beam concrete, **AASHTO A 5.4.2.6**

$\gamma_1 := 1.6$

Flexural variability factor

$\gamma_2 := 1.1$

Prestress viariability factor

$\gamma_3 := 1.0$

Reinforcement strength ratio

$$f_{cpe} := \frac{P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_B} = 2264.03\,psi$$

Compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$$M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe} \right) \cdot S_{bn} - M_{DC1} \cdot \left(\frac{S_{bn}}{S_B} - 1 \right) \right] = 3679.54 \cdot kip \cdot ft$$

Cracking moment

$$\text{if}\left(M_r > \min\left(M_{cr}, 1.33 \cdot M_{u\_strength} \right), \text{"ok"}, \text{"not ok"} \right) = \text{"ok"}$$

Approximate mid-span deflection at failure

The deflection calculations follows the approach outlined in the paper " Flexural behaviour of CFRP precast Decked Bulb T beams " by Grace et al. in May/June 2012, Journal of Composites for Construction. In order to calculate the deflection at failure, the moment capacity of the composite section is used as the bending moment. The stress level in the bottom most row is used to calculate the flexural rigidity. The deflection calculated below is approximate, but will give an indication of the deformbility and the level of warning exhibited near failure of the beam.

$d_{i_0} = 49.00 \cdot in$ — Depth of the bottom row of strands to the extreme compression fiber

$c = 5.20 \cdot in$ — Depth of the neutral axis to the extreme compression fiber

$y_s := d_{i_0} - c = 43.80 \cdot in$ — Distance from neutral axis to the bottom row of strands

$$EI := \frac{M_n \cdot y_s}{\varepsilon_0} = 411215285.27 \cdot kip \cdot in^2$$

Flexural rigidity of the beam/deck section based on the stress level in the bottom row of prestressing strands

$$\omega_f := 8 \cdot \frac{M_n}{L^2} = 11.083 \cdot \frac{kip}{ft}$$

Failure load (dead and live loads) uniformly dirstibuted over the entire span

$$\delta_f := \frac{5 \cdot \omega_f \cdot L^4}{384EI} = 9.553 \cdot in$$

Midspan deflection at strength limit state

pocket

LRFD Design Example for:

CFCC Prestressed Precast Concrete I-Beam with Cast-In-Place Concrete Slab

Grace et al.

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About this Design Example

Description

This document provides guidance for the design of CFCC prestressed precast concrete beams according to AASHTO LRFD Bridge Design Specifications with the neccessary ammendmets where applicable, based on available literature and experimental data from tests conducted by Grace et. al at Lawrence Technological University. The example provided herein is an I beam with a constant web thickness of 7 in. The cross-section of the bridge is **Type K** as described by **AASHTO Table 4.6.2.2.1-1**.

Standards

The following design standards were utilized in this example:

- AASHTO LRFD Bridge Design Specification, 7th Edition, 2014
- Michigan Department of Transportation Bridge Design Manual, Volume 5
- Michigan Department of Transportation Bridge Design Guide
- ACI 440.4R-04, Prestressing Concrete Structures with FRP Tendons

Code & AASHTO LRFD UPDATES

This Mathcad sheet is developed based on available design guidelines and available AASHTO LRFD edition at the time of writing the sheet. Designer shall check and update design equations according to the latest edition of AASHTO LRFD

General notes

The following notes were considered in this design example:

1- Guarnateed strength of CFRP is reduced to account for environmental effect. The design guarnateec strength is taken as 0.9 x guarnateed strength recommended by manufacturer

2- Initial prestressing stress is limited to 65% of the design (reduced) guaranteed strength according to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations for initial/jacking stress in CFRP strands

3- CFCC strength immediately following transfer is limited to 60% of the design (reduced) guaranteed strength according ccording to current ACI 440.4R-04. This limit is subject to change. Check the latest recommendations

4- The depth of the haunch is ignored in calculating section properties or flexural capacity, while is included in calculating the dead loads

5- In strength limit state flanged section design, the concrete strength of the beam portion participating i the stress block was conservatively assumed equal to the concrete strength of the deck (AASHTO LRF C5.7.2.2)

6- Barrier weight was taken as 475 lb/ft. While, weight of midspan diaphragm was 500 lb/beam

7- In the Mathcad sheet, the option of debonding as well as top prestressing strands are offered as means of reducing the end tensile stresses of the beams

8- In strength limit state check, the design addresses six different failure modes as follows:
Tension controlled rectangular section (depth of stress block is less than or equal the depth of the deck slab and the reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled rectangular section (depth of stress block is less than or equal the depth of the deck slab and the reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Tension controlled flanged section (depth of stress block is larger than the depth of the deck slab but less than the combined depth of the deck slab and beam top flange. The reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled flanged section (depth of stress block is larger than the depth of the deck slab but less than the combined depth of the deck slab and beam top flange. The reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Tension controlled double flanged section (depth of stress block is larger than the combined depth of the deck slab and beam top flange. The reinforcement ratio is less than balanced reinforcement ratio, CFRP ruptures before concrete crushing)

Compression controlled double flanged section (depth of stress block is larger than the combined depth of the deck slab and beam top flange. The reinforcement ratio is larger than balanced reinforcement ratio, Concrete crushes before CFRP rupture)

Designer is advised to check the ductility of the beam and the deflection at failure in case of double flanged section because in that case, the N.A. of the section lies within the web of the beam and the ductility of the section may be compromised

9- This design example is developed based on allowable jacking strength and stress immediately after transfer according to the limits presented in the ACI 440.4R-04. The document can be updated using other prestress limits such as those presented in MDOT SPR-1690 research report and guide

$L_{RefToRef} := 68ft$

$D_{RefAtoBearing} := 18in$

$D_{RefBtoBearing} := 18in$

$L := L_{RefToRef} - D_{RefAtoBearing} - D_{RefBtoBearing} = 65\,ft$ Center to center span Length

$brg_{off} := 5.5in$ Center of bearing offset to end of beam (same vaLue at both ends is assumed)

$L_{beam} := L + 2 \cdot brg_{off} = 65.917 \cdot ft$ TotaL length of beam

$l_{ship} := 24 \cdot in$ Distance from support to the end of the beam after force transfer and during shipping and handling

$L_{ship} := L_{beam} - l_{ship} \cdot 2 = 61.917\,ft$ Distance between supports during handling and shipping

$deck_{width} := 63ft + 3in$ Out to out deck width

$clear_{roadway} := 52ft + 0in$ CLear roadway width

$deck_{thick} := 9in$ Deck slab thickness

$t_{wear} := 0 \cdot in$ Wearing surface is included in the structural deck thickness only when designing the deck as per MDOT BDM 7.02.19.A.4. It is not used when designing the beam.

$t_{fws} := 2in$ Future wearing surface is applied as dead laod to accuant for additional deck thickness if a thicker rigid overlay is placed on deck

$walk_{width} := 0ft + 0in$ sidewalk width

$walk_{thick} := 0in$ sidewalk thickness (0" indicates no separate sidewalk pour)

$S := 8ft + 0in$ Center to center beam spacing

$NO_{beams} := 8$ Total number of beams

$haunch := 0in$ Average haunch thickness for section properties and strength calculations

$haunch_d := 2.0in$ Average haunch thickness for Load calculations

$\text{overhang} := 2\text{ft} + 11.5\text{in}$ Deck overhang width (same vaLue on both overhangs is assumed)

$\text{barrier}_{\text{width}} := 1\text{ft} + 2.5\text{in}$ Barrier width; include offset from back of barrier to edge of deck

$S_{\text{exterior}} := 56\text{ft} + 0\text{in}$ Hz distance between center of gravity of two exterior girders

$\text{Lanes} := \text{floor}\left(\frac{\text{clear}_{\text{roadway}}}{12\text{ft}}\right) = 4.00$ The number of design traffic Lanes can be caLcuLated as

$\text{angle}_{\text{crossing}} := 90\text{deg}$ Angle measured from centerline of bridge to the reference line

$\theta_{\text{skew}} := 90\text{deg} - \text{angle}_{\text{crossing}} = 0.00\cdot\text{deg}$ Angle measured from a line perpendicular to the centerline of bridge to the reference line

Concrete Material Properties

$f_{c\_deck} := 5\text{ksi}$ Deck concrete compressive strength

$f_{c\_beam} := 8\text{ksi}$ FinaL beam concrete compressive strength

$f_{ci\_beam} := 0.8 f_{c\_beam} = 6.4\cdot\text{ksi}$ Beam concrete compressive strength at reLease

$\omega_{\text{conc}} := 0.150\frac{\text{kip}}{\text{ft}^3}$ Unit weight of reinforced concrete for load calculations

$\text{barrier}_{\text{weight}} := 0.475\frac{\text{kip}}{\text{ft}}$ Weight per foot of barrier (aesthetic parapet tube, see MDOT BDG 6.29.10)

Unit weights of concrete used for modulus of eLasticity calculations, AASHTO Table 3.5.1-1

$$\gamma_c(f'_c) := \begin{cases} 0.145\frac{\text{kip}}{\text{ft}^3} & \text{if } f'_c \le 5\text{ksi} \\ 0.140\frac{\text{kip}}{\text{ft}^3} + 0.001\cdot\left(\frac{f'_c}{\text{ksi}}\right)\frac{\text{kip}}{\text{ft}^3} & \text{otherwise} \end{cases}$$

$\gamma_{c.deck} := \gamma_c\left(f_{c\_deck}\right) = 145\cdot\text{pcf}$

$\gamma_{c.beam} := \gamma_c\left(f_{c\_beam}\right) = 148\cdot\text{pcf}$

$\gamma_{ci.beam} := \gamma_c\left(f_{ci\_beam}\right) = 146.4\cdot\text{pcf}$

Concrete Modulus of Elasticity

Elastic modulus for concrete is as specified by **AASHTO A 5.4.2.4** with a correction factor of 1.0

$E_{c.beam\_i} := 120000\cdot\left(\frac{\gamma_{ci.beam}}{\frac{\text{kip}}{\text{ft}^3}}\right)^{2.0}\cdot\left(\frac{f_{ci\_beam}}{\text{ksi}}\right)^{0.33}\cdot\text{ksi} = 4745.73\cdot\text{ksi}$ Beam concrete at reLease

$E_{c.beam} := 120000 \cdot \left(\frac{\gamma_{c.beam}}{\frac{kip}{ft^3}} \right)^{2.0} \cdot \left(\frac{f_{c\_beam}}{ksi} \right)^{0.33} \cdot ksi = 5220.65 \cdot ksi$ Beam concrete at 28 days

$E_{c.deck} := 120000 \cdot \left(\frac{\gamma_{c.deck}}{\frac{kip}{ft^3}} \right)^{2.0} \cdot \left(\frac{f_{c\_deck}}{ksi} \right)^{0.33} \cdot ksi = 4291.19 \cdot ksi$ Deck concrete at 28 days

CFCC Material Properties

$d_s := 15.2mm = 0.6 \cdot in$ Prestressing strand diameter

$A_{strand} := 0.179 \cdot in^2$ Effective cross sectionaL area

$E_p := 21000ksi$ Tensile elastic modulus

$T_{guts} := 60.70kip$ Guaranteed ultimate tensile capacity

$f'_{pu} := \frac{T_{guts}}{A_{strand}} = 339.11 \cdot ksi$ Calculated ultimate tensile stress

$C_{Ese} := 0.9$

$f_{pu.service} := C_{Ese} \cdot f'_{pu} = 305.2 \cdot ksi$

Environmental reduction factor for prestressed concrete exposed to weather for service limit state calculations

$C_{Est} := 0.9$

$f_{pu} := C_{Est} \cdot f'_{pu} = 305.2 \cdot ksi$

Environmental reduction factor for prestressed concrete exposed to weather for strength limit state calculations

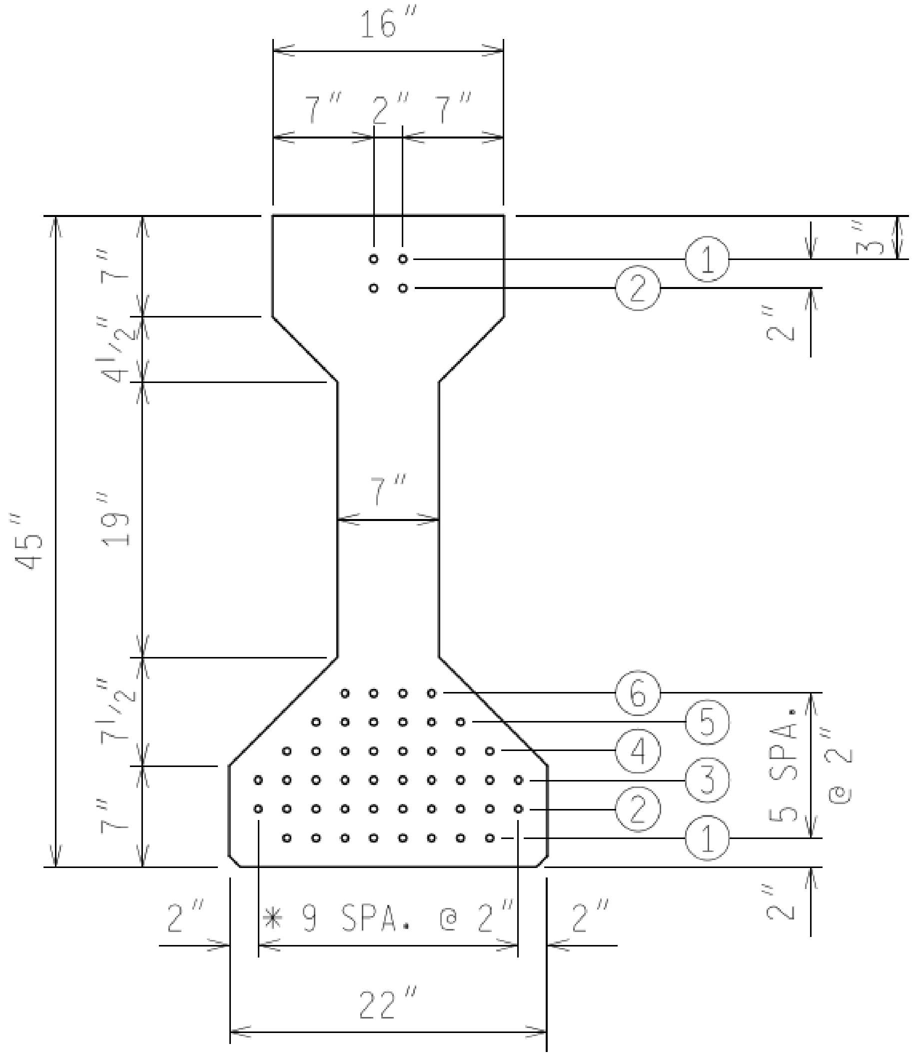
Modular Ratio

$n := \frac{E_{c.beam}}{E_{c.deck}} = 1.217$ Modular ratio for beam

$n_p := \frac{E_p}{E_{c.deck}} = 4.89$ Modular ratio for Prestressing CFCC

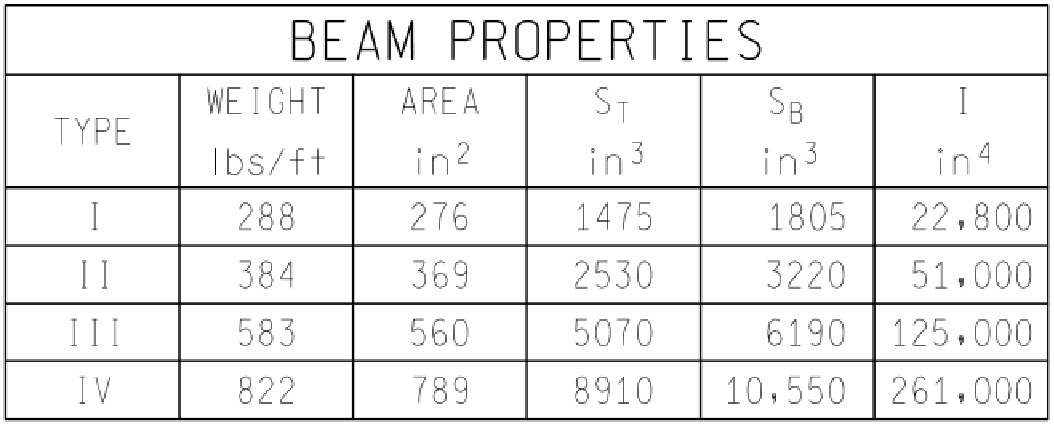
I Beam Section Properties:

TYPE I



TYPE III

* 7 SPA. @ 2" FOR ROW ① ONLY



BEAM PROPERTIES					
TYPE	WEIGHT lbs/ft	AREA in^2	S_T in^3	S_B in^3	I in^4
I	288	276	1475	1805	22,800
II	384	369	2530	3220	51,000
III	583	560	5070	6190	125,000
IV	822	789	8910	10,550	261,000

The values given in the Table are rounded. Exact values can be calculated manually and are given below for section III

$A_{beam} := 558.9375in^2$	Minimum area of beam section
$d := 45in$	Depth of beam
$b_{web} := 7in$	Minimum web thickness
$b_{web.max} := 7in$	Maximum web thickness
$b_{ft} := 16in$	Width of top flange
$d_{ft} := 7in$	Thickness of top flange
$d_h := 4.5\cdot in$	Depth of a first haunch under the top flange
$b_{fb} := 22in$	Width of bottom flange
$d_{fb} := 7in$	Thickness of bottom flange
$b_v := b_{web} = 7.00\cdot in$	Total web shear depth
$\omega_{beam} := A_{beam}\cdot(150pcf) = 582.23\cdot plf$	Beam weight per foot
$I_{beam} := 125164.6in^4$	Minimum moment of inertia
$y_t := 24.706in$	Depth from centroid to top of beam

$y_b := 20.2936in$ Depth from centroid to soffit of beam

$S_T := \frac{I_{beam}}{y_t} = 5066.16 \cdot in^3$ Minimum section modulus about top flange

$S_B := \frac{I_{beam}}{y_b} = 6167.69 \cdot in^3$ Minimum section modulus about bottom flange

Effective Flange Width of Concrete Deck Slab, AASHTO A 4.6.2.6

Beam_Design := "Interior" **Choose the design of the beam either "Interior" or "Exterior"**

$b_{eff.int} := S = 8.00\,ft$ Effective flange width of deck slab for interior beams

$b_{eff.ext} := \frac{1}{2} \cdot S + overhang = 6.96\,ft$ Effective flange width of deck slab for exterior beams

$$b_{eff} := \left| \begin{array}{ll} b_{eff.int} & \text{if } Beam\_Design = \text{"Interior"} \\ b_{eff.ext} & \text{if } Beam\_Design = \text{"Exterior"} \end{array} \right. = 2.438$$

$d_{total} := deck_{thick} + d = 54 \cdot in$ Total depth of section including deck

Dynamic load Allowance

Dynamic load allowance from **AASHTO Table 3.6.2.1-1** is applied as an increment to the static wheel loads to account for wheel load impacts from moving vehicles.

$IM := 1 + 33\% = 1.33$

Design Factors

These factors are related to the ductility, redundancy and operational importance of the bridge structure components and are applied to the strength limit state.

Ductility

For Strength limit State, a factor of 1.05 is used for nonductile components and connections, 1.00 for conventional designs and details complying with these specifications, and 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by these specifications, **AASHTO A 1.3.3.**

$\eta_D := 1.00$

Redundancy

For Strength limit State, a factor of 1.05 is used for nonredundant members, 1.00 for conventional levels of redundancy, foundation elements where ϕ already accounts for redundancy as specified in **AASHTC A 10.5**, and 0.95 for exceptional levels of redundancy beyond girder continuity and a torsionally-closed cross-section, **AASHTO A 1.3.4.**

$\eta_R := 1.00$

Operational Importance

For the Strength limit State, a factor of 1.05 is used for critical or essential bridges, 1.00 for typical bridges, and 0.95 for relatively less important bridges, **AASHTO A 1.3.5.**

$\eta_I := 1.00$

Ductility, redundancy, and operational classification considered in the load modifier, **AASHTO Eqn. 1.3.2.1-2.**

$\eta_i := \eta_D \cdot \eta_R \cdot \eta_I = 1.00$

Composite Section Properties

This is the moment of inertia resisting superimposed dead loads.

Elastic Section Properties - Composite Section: k=2

$k_{sdl} := 2$

$A_{haunchkn} := \frac{b_{ft}}{k_{sdl}\, n} \cdot haunch = 0 \cdot in^2$ effective area of haunch

$d_{haunchkn} := d + \frac{haunch}{2} = 45 \cdot in$

Depth of centroid of haunch to bottom of beam

$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$

$b_{effkn} := \frac{b_{eff}}{k_{sdl} n} = 39.45 \cdot in$

Transformed deck width

$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 49.5 \cdot in$

Depth from center of deck to beam soffit

$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 355.09 \cdot in^2$

Area of transformed deck section

$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 17576.89 \cdot in^3$

Static moment of inertia of transformed section about soffit of beam

$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 31.64 \cdot in$

Depth of CG of composite section from beam soffit

$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\ 3}}{12} = 2396.85 \cdot in^4$

Moment of inertia of transformed deck about centroid

$I_{haunchkn} := \frac{\frac{b_{ft}}{k_{sdl} \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$

Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_{3n} := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \ldots = 312785.5 \cdot in^4$$
$$+ A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2$$

$y_{b3n} := d_k = 31.64 \cdot in$

Depth of CG of composite section from beam soffit

$S_{b3n} := \frac{I_{3n}}{y_{b3n}} = 9885.78 \cdot in^3$

Section modulus about bottom of beam

$y_{t.bm.3n} := d - y_{b3n} = 13.36 \cdot in$

Depth of CG of composite section from top of beam

$S_{t.bm.3n} := \frac{I_{3n}}{y_{t.bm.3n}} = 23412 \cdot in^3$

Section modulus about top of beam

$y_{t3n} := d + haunch + deck_{thick} - t_{wear} - y_{b3n} = 22.36 \cdot in$

Depth of CG of composite section from top of deck

$$S_{t3n} := \frac{I_{3n}}{y_{t3n}} = 13988.59 \cdot in^3$$

Section modulus about top of deck

Elastic Section Properties - Composite Section: k=1

These properties are used to evaluate the moment of inertia for resisting live loads

Assumed wearing surface not included in the structural design deck thickness, per **MDOT BDM 7.02.19.A.4**............

$$k := 1$$

$$A_{haunchkn} := \frac{b_{ft}}{kn} \cdot haunch = 0 \cdot in^2$$

effective area of haunch

$$d_{haunchkn} := d + \frac{haunch}{2} = 45 \cdot in$$

Depth of centroid of haunch to bottom of beam

$$Ad_{haunchkn} := d_{haunchkn} \cdot A_{haunchkn} = 0 \cdot in^3$$

$$b_{effkn} := \frac{b_{eff}}{kn} = 78.91 \cdot in$$

Transformed deck width

$$d_{slabkn} := d + haunch + \frac{deck_{thick} - t_{wear}}{2} = 49.5 \cdot in$$

Depth from center of deck to beam soffit

$$A_{slabkn} := deck_{thick} \cdot b_{effkn} = 710.18 \cdot in^2$$

Area of transformed deck section

$$Ad_{slabkn} := A_{slabkn} \cdot d_{slabkn} = 35153.77 \cdot in^3$$

Static moment of inertia of transformed section about soffit of beam

$$d_k := \frac{A_{beam} \cdot y_b + Ad_{slabkn} + Ad_{haunchkn}}{A_{beam} + A_{slabkn} + A_{haunchkn}} = 36.64 \cdot in$$

Depth of CG of composite section from beam soffit

$$I_{oslabkn} := \frac{b_{effkn} \cdot deck_{thick}^{\;3}}{12} = 4793.7 \cdot in^4$$

Moment of inertia of transformed deck about centroid

$$I_{haunchkn} := \frac{\frac{b_{ft}}{k \cdot n} \cdot haunch^3}{12} = 0 \cdot in^4$$

Effective moment of interia of the haunch

Moment of inertia of composite section to resist superimposed dead loads calculated using parallel axis theorem

$$I_n := I_{beam} + A_{beam} \cdot \left(d_k - y_b\right)^2 + I_{oslabkn} + A_{slabkn} \cdot \left(d_{slabkn} - d_k\right)^2 + I_{haunchkn} \cdots = 396757.9 \cdot in^4$$
$$+ A_{haunchkn} \cdot \left(d_{haunchkn} - d_k\right)^2$$

$y_{bn} := d_k = 36.637 \cdot in$ Depth of CG of composite section from beam soffit

$S_{bn} := \frac{I_n}{y_{bn}} = 10829.42 \cdot in^3$ Section modulus about bottom of beam

$y_{t.bm.n} := d - y_{bn} = 8.36 \cdot in$ Depth of CG of composite section from top of beam

$S_{t.bm.n} := \frac{I_n}{y_{t.bm.n}} = 47442.37 \cdot in^3$ Section modulus about top of beam

$y_{tn} := d + haunch + deck_{thick} - t_{wear} - y_{bn} = 17.36 \cdot in$ Depth of CG of composite section from top of deck

$S_{tn} := \frac{I_n}{y_{tn}} = 22850.84 \cdot in^3$ Section modulus about top of deck

live load lateral Distribution Factors

Cross-section classification... **Type K**

Distribution of live loads from the deck to the beams is evaluated based on the **AASHTO** specified distribution factors. These factors can only be used if generally, the following conditions are met;

- Width of deck is constant.
- Unless otherwise specified, the number of beams is not less than four.
- Beams are parallel and have approximately the same stiffness.
- Curvature in plan is less than the limit specified in **AASHTO A 4.6.1.2.4.**
- Unless otherwise specified, the roadway part of the overhang does not exceed 3.0 ft.
- Cross-section is consistent with one of the cross-sections shown in AASHTO **Table 4.6.2.2.1-1.**

Unless otherwise stated, stiffness parameters for area, moments of inertia and torsional stiffness used shall be taken as those of the cross-section to which traffic will be applied (composite section)

Distance between the centers of gravity of the basic beam and deck

$$e_g := d + \left(\frac{deck_{thick}}{2}\right) + haunch - y_b = 29.206 \cdot in$$

logitudinal stiffness parameter

$$K_g := n \cdot \left(I_{beam} + A_{beam} \cdot e_g^{\ 2}\right) = 732326.17 \cdot in^4$$

Distribution of live loads for Moment in Interior Beams, AASHTO Table 4.6.2.2.2b-1

Range of Applicability................................

$\text{if}(3.5\text{ft} < S \le 16\text{ft}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\text{if}\left(4.5\text{in} < deck_{thick} \le 12\text{in}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

$\text{if}(20\text{ft} < L \le 240\text{ft}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\text{if}\left(NO_{beams} \ge 4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

$\text{if}\left(10000\ \text{in}^4 < K_g \le 7000000\ \text{in}^4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

One lane loaded

$$M_{lane1\_int} := 0.06 + \left(\frac{S}{14\text{ft}}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12 \cdot L \cdot deck_{thick}^{\ 3}} \cdot \frac{\text{ft}}{\text{in}}\right)^{0.1} = 0.497$$

Two or more lanes loaded

$$M_{lane2\_int} := 0.075 + \left(\frac{S}{9.5\text{ft}}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot deck_{thick}^{\ 3}} \cdot \frac{\text{ft}}{\text{in}}\right)^{0.1} = 0.683$$

live load moment disribution factor for interior beam

$$M_{lane\_int} := \max\left(M_{lane1\_int}, M_{lane2\_int}\right) = 0.683$$

Distribution of live loads for Moment in Exterior Beams, AASHTO Table 4.6.2.2.2d-1

One lane loaded (using the lever rule)

The lever rule involves the use of statics to determine the lateral distribution to the exterior girder by summing moments about the adjacent interior girder to determine the wheel-load reaction at the exterio girder assuming the concrete deck is hinged at the interior girder. A wheel cannot be closer than 2'-0" to the toe of barrier, and the standard wheel spacing is 6'-0". The evaluated factor is multiplied by the multiple presence factor, **AASHTO Table 3.6.1.1.2-1**.

Summing moments about the center of the interior beam

$$R := \frac{\left(S + overhang - barrier_{width} - 2 \cdot \text{ft} - \frac{6 \cdot \text{ft}}{2}\right)}{S} = 0.594$$

This factor is based on the lever arm rule considring the wheel load and not the resultant of both wheel

Moment distribution factor for exterior beam, one load loaded. The 1.2 accounts for the multiple presence factor, *m* from **AASHTO Table 3.6.1.1.2-1** for one lane loaded

$$M_{lane1\_ext} := R \cdot 1.2 = 0.713$$

Two or more lanes loaded

Horizontal distance from the centerline of the exterior web of exterior beam at deck level to the interior web edge of curb or traffic barrier must be greater than 0'-0"

$d_e := \max\left(\text{overhang} - \text{barrier}_{\text{width}}, 0\text{ft}\right) = 1.75\text{ ft}$

Range of Applicability

$\text{if}\left(-1\text{ft} \le d_e \le 5.5\text{ft}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

lane fraction

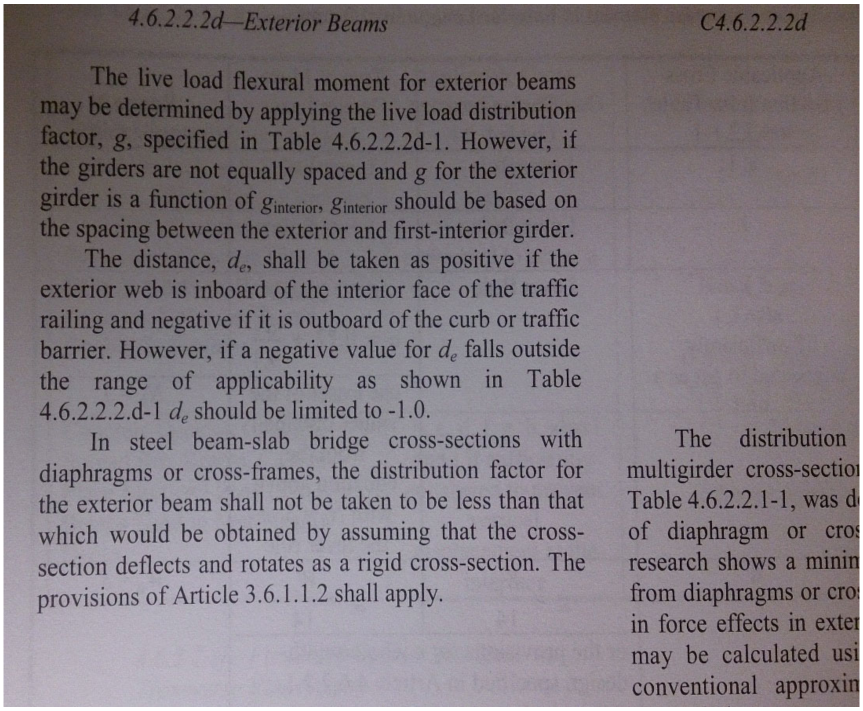
$$e := 0.77 + \frac{d_e}{9.1\text{ft}} = 0.962$$

Moment distribution factor for exterior beam, two or more lanes loaded

$M_{\text{lane2\_ext}} := M_{\text{lane\_int}} \cdot e = 0.658$

Distribution of live loads for Moment in Exterior Beams, AASHTO C4.6.2.2.2d

AASHTO LRFD 2014 recommends the rigid plate analysis only for **steel beam-slab bridges**. This was a change from ealier versions of AASHTO. It is up to the designed to ignore the rigid plate analysis or take it into consideration when calculating the DF for exterior beam



4.6.2.2.2d—Exterior Beams

C4.6.2.2.2d

The live load flexural moment for exterior beams may be determined by applying the live load distribution factor, *g*, specified in Table 4.6.2.2.2d-1. However, if the girders are not equally spaced and *g* for the exterior girder is a function of $g_{interior}$, $g_{interior}$ should be based on the spacing between the exterior and first-interior girder.

The distance, d_e, shall be taken as positive if the exterior web is inboard of the interior face of the traffic railing and negative if it is outboard of the curb or traffic barrier. However, if a negative value for d_e falls outside the range of applicability as shown in Table 4.6.2.2.2.d-1 d_e should be limited to -1.0.

In steel beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam shall not be taken to be less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section. The provisions of Article 3.6.1.1.2 shall apply.

The distribution
multigirder cross-sectio
Table 4.6.2.2.1-1, was d
of diaphragm or cros
research shows a minin
from diaphragms or cros
in force effects in exter
may be calculated usi
conventional approxin

Additional special analysis investigation is required because the distribution factor for multigirder in cross section was determined without consideration of diaphragm or cross frames. The multiple presence factors are used per **AASHTO Table 3.6.1.1.2-1.** This analysis should be done by sketching the cross section to determine the variables required for this example, the defined deck geometry is used. For any other geometry, these variables should be hand computed and input:

Horizontal distance from center of gravity of the pattern of girders to the exterior girder

$$X_{ext} := \frac{S_{exterior}}{2} = 28.00\,ft$$

Eccentricity of the center line of the standard wheel from the center of gravity of the pattern of girders

$$e_1 := X_{ext} + overhang - barrier_{width} - 2ft - \frac{6ft}{2} = 24.75\,ft$$

$$e_2 := e_1 - 12ft = 12.75\,ft$$

$$e_3 := e_2 - 12ft = 0.75\,ft$$

$$e_4 := e_3 - 12ft = -11.25\,ft$$

Summation of eccentricities for number of lanes considered:

$e_{NL1} := e_1 = 24.75\,ft$ One lane loaded

$e_{NL2} := e_1 + e_2 = 37.5\,ft$ Two lanes loaded

$e_{NL3} := e_{NL2} + e_3 = 38.25\,ft$ Three lanes loaded

$e_{NL4} := e_{NL3} + e_4 = 27\,ft$ Four lanes loaded

Horizontal distances from the center of gravity of the pattern of girders to each girder

$$X_{beams} := \left| \begin{array}{l} \text{for } i \in 0..NO_{beams} - 1 \\ \quad X_i \leftarrow X_{ext} - (i \cdot S) \\ X \end{array} \right. = \begin{pmatrix} 28.00 \\ 20.00 \\ 12.00 \\ 4.00 \\ -4.00 \\ -12.00 \\ -20.00 \\ -28.00 \end{pmatrix} ft$$

Summation of horizontal distances from the center of gravity of the pattern of girders to each girder

$$X_{NB} := \sum X_{beams}^{2} = 2688.00 \cdot ft^{2}$$

$$m_{1R} := 1.2 \cdot \left(\frac{1}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL1}}{X_{NB}} \right) = 0.459$$

Reaction on exterior beam when one lane is loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{2R} := 1.0 \cdot \left(\frac{2}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL2}}{X_{NB}} \right) = 0.641$$

Reaction on exterior beam when two lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{3R} := 0.85 \cdot \left(\frac{3}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL3}}{X_{NB}} \right) = 0.657$$

Reaction on exterior beam when three lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

$$m_{4R} := 0.65 \cdot \left(\frac{4}{NO_{beams}} + \frac{X_{ext} \cdot e_{NL4}}{X_{NB}} \right) = 0.508$$

Reaction on exterior beam when four lanes are loaded enhanced with the appropriate multiple lane factor from **AASHTO Table 3.6.1.1.2-1**

live load moment disribution factor for exterior beam

$$M_{lane\_ext} := \max\left(M_{lane1\_ext}, M_{lane2\_ext}, m_{1R}, m_{2R}, m_{3R}, m_{4R}\right) = 0.713$$

Reduction of load Distribution Factors for Moment in longitudinal Beams on Skewed Supports

When the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10 degrees, the bending moments and shear forces are reduced in accordance with **AASHTO Table 4.6.2.2.2e-1 and 4.6.2.2.3c-1** respectively.

Moment

Range of Applicability

$\text{if}\left(30deg \le \theta_{skew} \le 60deg, \text{"ok"}, \text{"Check below for adjustments of C1 and θskew"}\right) = \text{"Check below for adj}$

$\text{if}(3.5ft < S \le 16ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\text{if}(20ft < L \le 240ft, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$

$\text{if}\left(NO_{beams} \ge 4, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

$$\theta_{skew} := \begin{cases} \theta_{skew} & \text{if } \theta_{skew} \le 60 \cdot deg \\ 60 \cdot deg & \text{if } \theta_{skew} > 60 \cdot deg \end{cases} = 0 \cdot deg$$

$$C_1 := \begin{cases} 0 & \text{if } \theta_{skew} < 30 \cdot deg \\ \left[0.25 \cdot \left(\frac{K_g}{12.0 \cdot L \cdot deck_{thick}^{\ 3}} \cdot \frac{ft}{in} \right)^{0.25} \cdot \left(\frac{S}{L} \right)^{0.5} \right] & \text{otherwise} \end{cases} = 0$$

$$Mcorr_{factor} := 1 - C_1 \cdot \tan\left(\theta_{skew}\right)^{1.5} = 1$$

Correction factor for moment

Reduced distribution factors at strength limit state for interior girders due to skew

$DF_{strength\_moment\_int} := M_{lane\_int} \cdot Mcorr_{factor} = 0.683$ Moment

Reduced distribution factors at strength limit state for exterior girders due to skew

$DF_{strength\_moment\_ext} := M_{lane\_ext} \cdot Mcorr_{factor} = 0.713$ Moment

Design distribution factors for service and strength limit states

Distribution factor for moment at strength limit state

$$DF_{strength\_moment} := \begin{cases} DF_{strength\_moment\_int} & \text{if Beam\_Design} = \text{"Interior"} \\ DF_{strength\_moment\_ext} & \text{if Beam\_Design} = \text{"Exterior"} \end{cases} = 0.683$$

live load Analysis

Flexure

As per **AASHTO A 3.6.1.2.1**, vehicular live loading designated by the standard HI-93 truck shall be a combination of the design truck or design tandem, and the design lane load. To produce extreme force effects, the spacing between the two 32-kip axles are taken as 14 ft.

Calculate the maximum moment due to the truck load. Maximum truck load moment occurs when the middle axle is positioned at distance 2.33 ft from the midspan. Maximum momment occurs under the middle axle load. Moment due to distributed load occurs at midspan.

Unless more detailed analysis is performed to determine the location and value for the maximum moment under combined truck and distributed loads at both service and strength limit state, the maximum moment from the truck load at distance 2.33 ft from midspan can be assumed to occur at the midspan and combined with the maximum moment from other dead and live distributed loads

Calculate the reaction at the end of the span

$$R := \frac{8kip\cdot\left(\frac{L}{2} - 16.33ft\right) + 32kip\cdot\left(\frac{L}{2} - 2.33ft\right) + 32kip\cdot\left(\frac{L}{2} + 11.67ft\right)}{L} = 38.588\cdot kip$$

Calculate the maximum moment

$$M_{truck} := R\cdot\left(\frac{L}{2} + 2.33ft\right) - 32\cdot kip\cdot 14\cdot ft = 896.031\cdot kip\cdot ft$$

Maximum moment due to design lane load, **AASHTO A 3.6.1.2.4**

$$X := \frac{L}{2} = 32.5\,ft$$

$$M_{lane} := \frac{0.64klf\cdot L\cdot X}{2} - 0.64klf\cdot\frac{X^2}{2} = 338.00\cdot kip\cdot ft$$

Maximum moment due to design tandem, **MDOT BDM 7.01.04.A**

$$M_{tandem} := \frac{60kip\cdot L}{4} = 975\cdot kip\cdot ft$$

Maximum moment due to vehicular live loading by the modified HI-93 design truck and tandem per **MDOT BDM 7.01.04.A**. Modification is by multiplying the load effects by a factor of 1.20. Dynamic load allowance is considered only for the design truck and tandem, **AASHTO A 3.6.1.2.2, 3.6.1.2.3 & 3.6.1.2.4.**

$$M_{LLI} := \left[1.20M_{lane} + IM\cdot\left(1.20\cdot\max\left(M_{truck}, M_{tandem}\right)\right)\right]\cdot DF_{strength\_moment} = 1340.77\cdot kip\cdot ft$$

Dead load Analysis

Dead load calculations are slightly adjusted for exterior beam design.

Noncomposite Dead load (DC_1)

$$M_{swbeam} := \frac{\omega_{beam}\cdot L^2}{8} = 307.49\cdot kip\cdot ft$$

Total moment due to selfweight of beam

$deck := \left(deck_{thick} \cdot b_{eff} + haunch_{d} \cdot b_{ft}\right) \cdot 0.15 \frac{kip}{ft^{3}} = 0.93 \cdot klf$

Selfweight of deck and haunch on beam

$M_{deck} := \frac{deck \cdot L^{2}}{8} = 492.92 \cdot kip \cdot ft$

Moment due to selfweight of deck and haunch

$sip := 15psf \cdot \left(b_{eff} - b_{ft}\right) = 0.1 \cdot klf$

15 psf weight included for stay-in-place forms per MDOT BDM 7.01.04.I

$M_{sip} := \frac{sip \cdot L^{2}}{8} = 52.81 \cdot kip \cdot ft$

Moment due to stay-in-place forms

$dia_{int} := 0.5 \cdot kip$

Weight of steel diaphragms at mid-span per each interior beam

$dia_{ext} := 0.25 \cdot kip$

Weight of steel diaphragms at mid-span per each exterior beam

$$diaphragm := \begin{array}{|l} dia_{int} \ \text{ if } Beam\_Design = \text{"Interior"} \\ dia_{ext} \ \text{ if } Beam\_Design = \text{"Exterior"} \end{array} = 0.5 \cdot kip$$

$spa_{dia} := 2\left(S - b_{fb}\right) \cdot \tan\left(\theta_{skew}\right) = 0 ft$

One row of diaphragms at midspan are used.

$M_{dia} := diaphragm \cdot \frac{L}{4} = 8.125 \cdot kip \cdot ft$

$DC_{1} := \omega_{beam} + deck + sip = 1.616 \cdot klf$

Dead load (o.wt of beam+ deck+ SIP forms) acting on non-composite section

$M_{DC1} := M_{swbeam} + M_{deck} + M_{sip} + M_{dia} = 861.34 \cdot kip \cdot ft$

Total midspan moment acting on the non-composite section

Composite Dead load (DC_{2})

$util := \frac{1}{2} \cdot (0plf) = 0 \cdot klf$

No utilities are supported by the superstructure

$barrier1_{weight} := 0.475 \frac{kip}{ft}$

Weight per foot of first barrier (aesthetics parapet tube, **MDOT BDG 6.29.10**)

$barrier2_{weight} := 2.25 \cdot in \cdot 40 \cdot in \cdot \omega_{conc} + 0.475 \frac{kip}{ft} = 0.569 \cdot \frac{kip}{ft}$

Weight per foot of second barrier (modified aesthetics parapet tube, **MDOT BDG 6.29.10**)

$$sidewalk := \frac{2 \cdot walk_{width} \cdot walk_{thick} \cdot \omega_{conc}}{NO_{beams}} = 0.00 \cdot klf$$

Weight to due extra thickness of sidewalk per beam

$$barrier := \frac{barrier1_{weight} + barrier2_{weight}}{NO_{beams}} = 0.13 \cdot klf$$

Total barrier weight per beam

$$soundwall_{weight} := 0.0 \cdot \frac{kip}{ft}$$

Weight of the sound wall, if there is a sound wall

Weight of the sound wall **for exterior beam** design assuming lever arm and an inetremiate hinge on the first interior beam

$$soundwall := \left| \begin{array}{ll} 0 \cdot \frac{kip}{ft} & \text{if Beam\_Design} = \text{"Interior"} \\ \left[soundwall_{weight} \cdot \frac{(S + overhang)}{S} \right] & \text{if Beam\_Design} = \text{"Exterior"} \end{array} \right. = 0 \cdot \frac{kip}{ft}$$

$$DC_2 := sidewalk + barrier + util + soundwall = 0.13 \cdot klf$$

Total dead load acting on the composite section

$$M_{DC2} := \frac{DC_2 \cdot L^2}{8} = 68.90 \cdot kip \cdot ft$$

Total midspan moment acting on the composite section

(DW) Wearing Surface load

Self weight of future wearing surface

$$DW := \left(b_{eff}\right) \cdot 0.025 \frac{kip}{ft^2} = 0.2 \cdot klf$$

Maximum unfactored dead load moments

$$M_{DC} := M_{DC1} + M_{DC2} = 930.25 \cdot kip \cdot ft$$

Total midspan moment due to loads acting on the composite and non-composite section

$$M_{DW} := \frac{DW \cdot L^2}{8} = 105.63 \cdot kip \cdot ft$$

Midspan moment due to weight of future wearing surface

Wind load on the sound wall

$$M_{wind} := 0.0 \cdot ft \cdot \frac{kip}{ft}$$

Moment due to wind acting at the sound wall

$$W := \frac{M_{wind}}{S} = 0 \cdot \frac{kip}{ft}$$

Extra load on the interior beam due to wind load assuming lever arm analysis and an intermediate hinge at the first interior beam

$M_{WS} := \frac{W \cdot L^2}{8} = 0 \cdot kip \cdot ft$

Interior beam moment due to wind acting at the sound wall

load Combinations

Load Combinations: Strength, Extreme Event, Service and Fatigue load combinations are defined per AASHTO 3.4.1. Verify which combination are appropriate. For this concrete box beam design, wind load is not evaluated, and no permit vehicle is specified. However, the design live loading is MDOT HL-93 Modified which accounts for Michigan's inventory of legal and permit vehicles.

Strength I, III, IV and Strength V limit states are considered for the design of this beam. Load combinations factors according to AASHTO LRFD 2016 Interim revision are used

$M\_Strength_I := \eta_i \cdot (1.25M_{DC} + 1.50M_{DW} + 1.75M_{LLI}) = 3667.60 \cdot kip \cdot ft$

$M\_Strength_{III} := \eta_i \cdot (1.25M_{DC} + 1.50M_{DW} + 1.0M_{WS}) = 1321.25 \cdot kip \cdot ft$

$M\_Strength_{IV} := \eta_i \cdot [1.50 \cdot (M_{DC} + M_{DW})] = 1553.81 \cdot kip \cdot ft$

$M\_Strength_V := \eta_i \cdot (1.25M_{DC} + 1.50M_{DW} + 1.35M_{LLI} + 1.0 \cdot M_{WS}) = 3131.29 \cdot kip \cdot ft$

$M_{u\_strength} := \max(M\_Strength_I, M\_Strength_{III}, M\_Strength_{IV}, M\_Strength_V) = 3667.6 \cdot kip \cdot ft$

Number of Prestressing Strands

The theoretical number of strands required is calculated using the **Service III** limit state

$f_b := \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = 3.08 \cdot ksi$

Tensile stress in bottom flange due to applied loads

Allowable stress limits for concrete

$f_{ti} := 0.24 \cdot \sqrt{f_{ci\_beam} \cdot ksi} = 0.61 \cdot ksi$

Initial allowable tensile stress

$f_{ci} := -0.65 \cdot f_{ci\_beam} = -4.16 \cdot ksi$

Initial allowable compressive stress (according to AASHTO LRFD 2016 interim revision)

$f_{tf} := 0 \cdot \sqrt{f_{c\_beam} \cdot ksi} = 0.00 \cdot ksi$

Final allowable tensile stress (allowing no tension)

No tension is allowed under service III limit state to avoid potential cracks and shear action on the strands

$f_{cfp} := -0.45 \cdot f_{c\_beam} = -3.60 \cdot ksi$

Final allowable compressive stress in the beam due to sum of effective prestress and permanent loads

$f_{cf.deckp} := -0.45 \cdot f_{c\_deck} = -2.25 \cdot ksi$

Final allowable compressive stress in the slab due to permanent loads

$f_{cf} := -0.6 \cdot f_{c\_beam} = -4.80 \cdot ksi$

Final allowable compressive stress in the beam due to sum of effective prestress, permanent loads, & transient loads

$f_{cf.deck} := -0.6 \cdot f_{c\_deck} = -3.00 \cdot ksi$

Final allowable compressive stress in the beam due to sum of permanent loads and transient loads

$f_p := f_b - f_{tf} = 3.08 \cdot ksi$

Excess tension in the bottom flange due to applied loads

Assuming strand pattern center of gravity is midway between the bottom two rows of strands, i.e. the same number of strands are used in the top and bottom rows of the bottom flange.

$y_{bs} := 3in$

Distance from soffit of beam to center of gravity of strands

$e_{st} := y_b - y_{bs} = 17.29 \cdot in$

Eccentricity of strands from the centroid of beam

Final prestressing force required to counteract excess tension in the bottom flange. Set allowable stress equal to the excess tension, solve for P_e.

$$P_{et} := \frac{f_p}{\left(\frac{1}{A_{beam}} + \frac{e_{st}}{S_B}\right)} = 669.771 \cdot kip$$

$f_{j.max} := 0.65 \cdot f_{pu.service} = 198.377 \cdot ksi$

Maximum allowable Jacking stress, **ACI 440.4R Table 3.3**

$P_j := A_{strand} \cdot f_{j.max} = 35.51 \cdot kip$

Maximum Jacking prestressing force per strand

$f_t := 0.64 f_{pu.service} = 195.33 \cdot ksi$

Initial prestressing stress immediately **prior** to transfer. shall be less than or equal to the maximum jacking strength, and shall be adjusted accordingly to make sure the stress immedietely following transfer is not exceeding 0.6 times guaranteed strength as shown on the following page

$P_{in} := A_{strand} \cdot f_t = 34.96 \cdot kip$

Initial prestressing force per strand prior to transfer

$P_{pet} := A_{strand} \cdot f_t \cdot 0.75 = 26.22 \cdot kip$

Effective prestressing force assuming 25% final prestress losses per 0.6" diameter strand

$NO_{strands\_i} := ceil\left(\frac{P_{et}}{P_{pet}}\right) = 26$

Minimum number of strands required

Strand distribution per row. Row 0 is the bottom most row in the beam. Start adding strands from the bottom row going up until the number of strands is reached. do not skip rows inbetween. Extra rows with zero strands will be eliminated in the analysis.

$row_0 := 8$ $row_1 := 10$ $row_2 := 8$ $row_3 := 0$ $row_4 := 0$ $row_5 := 0$

$row_6 := 0$ $row_7 := 0$ $row_8 := 0$ $row_9 := 0$

$$row = \begin{pmatrix} 8 \\ 10 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
Row := | a ← 0
       | for  i ∈ 0 .. length(row) − 1
       |     | a ← a + 1  if row_i > 0
       |     | a ← a  otherwise
       | for  j ∈ 0 .. a − 1
       |     D_j ← row_j
       | D
```

$$Row = \begin{pmatrix} 8 \\ 10 \\ 8 \end{pmatrix}$$

$NO_{strands} := \sum Row = 26.00$

Total number of prestressing strands

$$d_{strand} := \left| \begin{array}{l} \text{for } i \in 0..\,\text{length}(Row) - 1 \\ \quad d_{s_i} \leftarrow d - (2in) - (2in)i \\ d_s \end{array} \right. = \begin{pmatrix} 43.00 \\ 41.00 \\ 39.00 \end{pmatrix} \cdot in$$

Depth of CFCC strands in each layer from the top of the beam section. This calculation assumes a 2" vertical spacing of the strand rows

$$CG := \frac{\left[\overrightarrow{Row \cdot \left(d - d_{strand}\right)}\right]}{\sum Row} = 4.00 \cdot in$$

Center of gravity of the strand group measured from the soffit of the beam section

$d_f := (d - CG) + haunch + deck_{thick} = 50.00 \cdot in$

Depth from extreme compression fiber to centroid of CFCC tension reinforcement

$e_s := y_b - CG = 16.29 \cdot in$

Eccentricity of strands from centroid of beam

$A_{ps} := A_{strand} \cdot NO_{strands} = 4.65 \cdot in^2$

Total area of prestressing CFCC strands

Prestress losses

loss due to Elastic Shortening, AASHTO Eqn. C5.9.5.2.3a-1

$$\Delta f_{PES} := \frac{A_{ps} \cdot f_t \cdot \left(I_{beam} + e_s^{\,2} \cdot A_{beam}\right) - e_s \cdot M_{swbeam} \cdot A_{beam}}{A_{ps} \cdot \left(I_{beam} + e_s^{\,2} \cdot A_{beam}\right) + \dfrac{A_{beam} \cdot I_{beam} \cdot E_{c.beam\_i}}{E_p}} = 12.59 \cdot ksi$$

$F_{pt} := f_t - \Delta f_{PES} = 182.74 \cdot ksi$

Prestressing stress immediately following transfer

$P_t := A_{ps} \cdot F_{pt} = 850.452 \cdot kip$

According to ACI 440.4R, Table 3.3, the allowable stress immediately after transfer shall not exceed 0.6 fpu

$0.6 \cdot f_{pu.service} = 183.117 \cdot ksi$

$\text{if}\left(F_{pt} \le 0.6 \cdot f_{pu.service}, \text{"Ok"}, \text{"Not Ok"}\right) = \text{"Ok"}$

Approximate Estimate of Time dependent losses, AASHTO A 5.9.5.3

$H := 75$

Average annual ambient relative humidity

$$\gamma_h := 1.7 - 0.01 \cdot H = 0.95$$

Correction factor for relative humidity of ambient air

$$\gamma_{st} := \frac{5}{1 + \frac{f_{ci\_beam}}{ksi}} = 0.68$$

Correction factor for specified concrete strength at time of prestress transfer to the concrete member

$$\Delta f_{pR} := f_t \cdot 1.75\% = 3.42 \cdot ksi$$

Relaxation loss taken as 1.75% of the initial pull per experimental results from Grace et. al based on 1,000,000 hours (114 years)

$$\Delta f_{pLT} := 10 \cdot \frac{f_t \cdot A_{ps}}{A_{beam}} \cdot \gamma_h \cdot \gamma_{st} + 12ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} = 21.56 \cdot ksi$$

long term prestress loss

Difference in thermal coefficient expansion between concrete and CFCC

$$\alpha := 6 \cdot 10^{-6} \cdot \frac{1}{F}$$

Difference in coefficient of thermal expansion between concrete and CFCC

$$t_{amb} := 68F$$

Ambient temperature

$$t_{low} := -10F$$

lowest temperature in Michigan according to AASHTO IRFD 3.12.2

$$\Delta t := t_{amb} - t_{low} = 78\,F$$

Change in the temperature

$$\Delta f_{pt} := \alpha \cdot \Delta t \cdot E_p = 9.83 \cdot ksi$$

Prestress losses due to temp. effect

$$f_{pe} := f_t - \Delta f_{pLT} - \Delta f_{PES} - \Delta f_{pt} = 151.35 \cdot ksi$$

Effective prestressing stress after all losses

$$P_e := A_{ps} \cdot f_{pe} = 704.37 \cdot kip$$

Effective prestressing force after all losses

$$f_t = 195.33 \cdot ksi$$

Initial prestress prior to transfer, not including anchorage losses

$$f_{pe} = 151.35 \cdot ksi$$

Prestress level after all losses

$$loss := \frac{f_t - f_{pe}}{f_t} = 22.52 \cdot \%$$

Total prestress loss

Debonding Criteria

Estimate the location from each beam end where top prestressing or debonding is no longer needed The vectors are developed for possible two different deboning lengths per row. Enter the number of debonded strands and the estimated debonding length in the vectors below per each row location

Location: number of strands: debonding length:

$$Row_{db} := \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad n_{db} := \begin{pmatrix} 3 \\ 0 \\ 4 \\ 0 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad l_{db} := \begin{pmatrix} 22 \\ 0 \\ 16 \\ 0 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot ft$$

For debonding pattern, follow staggering guidelines in MDOT BDM 7.02.18.A.2

$$row_{db} := \left| \begin{array}{l} \text{for } i \in 0..2\,\text{length}(Row) - 1 \\ \quad D_i \leftarrow Row_{db_i} \\ D \end{array} \right.$$

$$N_{db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right) - 1 \\ \quad D_i \leftarrow n_{db_i} \\ D \end{array} \right.$$

$$L_{db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}\left(row_{db}\right) - 1 \\ \quad D_i \leftarrow l_{db_i} \\ D \end{array} \right.$$

$$row_{db} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \end{pmatrix} \qquad N_{db} = \begin{pmatrix} 3 \\ 0 \\ 4 \\ 0 \\ 3 \\ 0 \end{pmatrix} \qquad L_{db} = \begin{pmatrix} 22 \\ 0 \\ 16 \\ 0 \\ 8 \\ 0 \end{pmatrix} ft$$

$$\sum N_{db} = 10$$

$$Debond_{tot} := \frac{\sum N_{db}}{NO_{strands}} = 38.46 \cdot \%$$

Portion of partially debonded strands in beam section

$$\text{if}\left(Debond_{tot} \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

Total number of debonded strands in rows

```
$N_{db.row} :=$ | for $i \in 0 .. \text{length}(Row) - 1$                    $= \begin{pmatrix} 3.00 \\ 4.00 \\ 3.00 \end{pmatrix}$
                |     $a_i \leftarrow 0$
                |     for $j \in 0 .. \text{length}(N_{db}) - 1$
                |         $a_i \leftarrow a_i + N_{db_j}$  if $row_{db_j} = i + 1$
                | $a$
```

```
$Debond_{row} :=$ | for $i \in 0 .. \text{length}(Row) - 1$               $= \begin{pmatrix} 37.50 \\ 40.00 \\ 37.50 \end{pmatrix} \cdot \%$
                  |     $a_i \leftarrow 0$
                  |     $a_i \leftarrow \frac{N_{db.row_i}}{Row_i}$  if $Row_i > 0$
                  |     $0$  otherwise
                  | $a$
```

$$\text{if}\left(\max\left(Debond_{row}\right) \le 40\%, \text{"ok"}, \text{"No Good"}\right) = \text{"ok"}$$

The limit of 40% is taken according to MDOT BDM 7.02.18. A2

Optional: only needed if debonding scheme is not sufficient to eliminate the tensile stresses at beam ends either at transfer or due to handling and shipping

CFCC strand transfer length, **ACI 440.4R Table 6.1** $L_t := 50d_s = 2.49\,ft$

Number of top prestressing strands in the top flange

$$Row_{top} := \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

Depth of the top prestressing strands from the top surface of the beam

$$d_{top} := \begin{pmatrix} 3 \\ 5 \end{pmatrix} \cdot in$$

Initial prestressing stress/force at the top prestressing strands

$$F_{p\_top} := 50 \cdot ksi$$

Distance from the end of the beam to the point where the top prestressing is no longer needed

$$x_{p\_top} := 10 \cdot ft$$

Top prestressing strands shall not extend the the middle third of the beam. Otherwise, it could affect the stresses at service limit state

$$Check\_Top\_prestressing\_Length := \left| \begin{array}{l} \text{"Okay"} \;\; \text{if } x_{p\_top} \le \frac{L_{beam}}{3} \\ \text{"Check service stress @ x.p\_top"} \;\; \text{if } x_{p\_top} > \frac{L_{beam}}{3} \end{array} \right. = \text{"Okay"}$$

Distance from the end of the beam to the pocket where top prestressing strand is cut after concrete pouring. The middle region between the cut pockets shall be dobonded to avoid force transfer to the middle region

$$x_{pocket} := x_{p\_top} + L_t = 12.493\,ft$$

Serviceability Checks

Stress check locations along the beam

Stress locations after the transfer length for bonded and de-bonded strands

$$X_{release} := \mathrm{sort}\left[\mathrm{stack}\left[L_t,\left(L_{db}+L_t\right),x_{p\_top},x_{pocket}\right]\right] = \begin{pmatrix} 2.493 \\ 2.493 \\ 2.493 \\ 2.493 \\ 10 \\ 10.493 \\ 12.493 \\ 18.493 \\ 24.493 \end{pmatrix} \mathrm{ft}$$

Extracting repreated X from the vector

```
$x_{release} :=$ | $k \leftarrow 0$
                 | $x_0 \leftarrow L_t$
                 | for $i \in 1 .. \mathrm{length}(X_{release}) - 1$
                 |     | $k \leftarrow k + 1$ if $(X_{release_i} \neq X_{release_{i-1}})$
                 |     | $x_k \leftarrow X_{release_i}$
                 | $x$
```

$$x_{release} = \begin{pmatrix} 2.493 \\ 10 \\ 10.493 \\ 12.493 \\ 18.493 \\ 24.493 \end{pmatrix} \cdot \mathrm{ft}$$

Area of strands in each row at each stress check location

```
$A_{db} :=$
    for $i \in 0..\text{length}(x_{release}) - 1$
        for $z \in 0..\text{length}(Row) - 1$
            $A_{i,z} \leftarrow Row_z \cdot A_{strand}$
        for $j \in 0..\text{length}(N_{db}) - 1$
            $n \leftarrow N_{db_j}$
            $row \leftarrow row_{db_j}$
            $L \leftarrow L_{db_j}$
            $A_{i,row-1} \leftarrow (A_{i,row-1} - n \cdot A_{strand}) \cdot \frac{x_{release_i}}{L_t}$ if $x_{release_i} < L_t$
            $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand}$ if $L_t \le x_{release_i} \le L$
            $A_{i,row-1} \leftarrow A_{i,row-1} - n \cdot A_{strand} \cdots + n \cdot A_{strand} \cdot \frac{(x_{release_i} - L)}{L_t}$ if $L < x_{release_i} \le L + L_t$
    $A$
```

$$A_{db} = \begin{pmatrix} 0.90 & 1.07 & 0.90 \\ 0.90 & 1.07 & 1.33 \\ 0.90 & 1.07 & 1.43 \\ 0.90 & 1.07 & 1.43 \\ 0.90 & 1.79 & 1.43 \\ 1.43 & 1.79 & 1.43 \end{pmatrix} \cdot in^2$$

Beam stresses at release due to prestressing only

Sign convention; negative and positive stresses/forces for compression and tension respectively

$$P_{ps} := -F_{pt} \cdot A_{db} = \begin{pmatrix} -163.55 & -196.26 & -163.55 \\ -163.55 & -196.26 & -242.26 \\ -163.55 & -196.26 & -261.68 \\ -163.55 & -196.26 & -261.68 \\ -163.55 & -327.10 & -261.68 \\ -261.68 & -327.10 & -261.68 \end{pmatrix} \cdot kip$$

Midspan moment due to prestressing at release

$$M_{ps} := P_{ps} \cdot \left(d_{strand} - y_t\right) = \begin{pmatrix} -710.629 \\ -804.385 \\ -827.517 \\ -827.517 \\ -1005.174 \\ -1154.771 \end{pmatrix} \cdot kip \cdot ft$$

Top and bottom concrete stresses at check locations due to prestressing ONLY

```
$f_{ps} :=$ for $i \in 0..\,\text{length}(x_{release}) - 1$
        $M \leftarrow M_{ps_i}$
        $P \leftarrow \sum_{j=0}^{\text{cols}(P_{ps})-1} P_{ps_{i,j}}$
        $A \leftarrow A_{beam}$
        $S_{top} \leftarrow S_T$
        $S_{bott} \leftarrow S_B$
        $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
        $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
    $f$
```

$$f_{ps} = \begin{array}{cc} \text{top} & \text{bottom} \\ \left(\begin{matrix} 747 \\ 828 \\ 848 \\ 848 \\ 1035 \\ 1214 \end{matrix}\right. & \left.\begin{matrix} -2319 \\ -2642 \\ -2722 \\ -2722 \\ -3302 \\ -3768 \end{matrix}\right) \end{array} \cdot psi$$

<u>Beam stresses at release due to selfweight</u>

Moment due to self weight of beam at check locations

$$M_{sw}(x) := \frac{\omega_{beam} \cdot x}{2} \cdot \left(L_{beam} - x\right)$$

Top and bottom concrete stresses at check locations due to beam self weight ONlY

```
$f_{sw} :=$
    for $i \in 0..\text{length}(x_{release}) - 1$
        $M \leftarrow M_{sw}(x_{release_i})$
        $f_{i,0} \leftarrow \frac{-M}{S_T}$
        $f_{i,1} \leftarrow \frac{M}{S_B}$
    $f$
```

$$f_{sw} = \begin{array}{cc} \text{top} & \text{bottom} \\ \end{array}$$

$$f_{sw} = \begin{pmatrix} -109 & 90 \\ -386 & 317 \\ -401 & 329 \\ -460 & 378 \\ -605 & 497 \\ -700 & 575 \end{pmatrix} \cdot psi$$

Area of **top prestressing** strands at distance X.release from the end

```
$A_{top} :=$
    for $i \in 0..\text{length}(x_{release}) - 1$
        for $z \in 0..\text{length}(Row_{top}) - 1$
            $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} \cdot \frac{x_{release_i}}{L_t}$  if $x_{release_i} \le L_t$
            $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand}$  if $L_t < x_{release_i} \le x_{p\_top}$
            $A_{i,z} \leftarrow Row_{top_z} \cdot A_{strand} - \frac{x_{release_i} - x_{p\_top}}{L_t} \cdot (Row_{top_z} \cdot A_{strand})$  if $x_{p\_top} < x_{release_i} \le x$
            $A_{i,z} \leftarrow 0$  if $x_{release_i} > x_{p\_top} + L_t$
    $A$
```

$$A_{top} = \begin{pmatrix} 0.358 & 0.358 \\ 0.358 & 0.358 \\ 0.287 & 0.287 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \cdot in^2 \qquad x_{release} = \begin{pmatrix} 2.493 \\ 10 \\ 10.493 \\ 12.493 \\ 18.493 \\ 24.493 \end{pmatrix} ft$$

$$P_{p\_top} := -F_{p\_top} \cdot A_{top} = \begin{pmatrix} -17.90 & -17.90 \\ -17.90 & -17.90 \\ -14.36 & -14.36 \\ -0.00 & -0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \end{pmatrix} \cdot kip$$

$$M_{p\_top} := P_{p\_top} \cdot \left(d_{top} - y_t\right) = \begin{pmatrix} 61.773 \\ 61.773 \\ 49.548 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot kip \cdot ft$$

```
$f_{p\_top} :=$ | for $i \in 0 .. length(x_{release}) - 1$
               |     $M \leftarrow M_{p\_top_i}$
               |     $P \leftarrow \sum_{j=0}^{cols(P_{p\_top})-1} P_{p\_top_{i,j}}$
               |     $A \leftarrow A_{beam}$
               |     $S_{top} \leftarrow S_T$
               |     $S_{bott} \leftarrow S_B$
               |     $f_{i,0} \leftarrow \frac{-M}{S_{top}} + \frac{P}{A}$
               |     $f_{i,1} \leftarrow \frac{M}{S_{bott}} + \frac{P}{A}$
               | $f$
```

Stresses in the beam due to the top prestressing strands only

$$f_{p\_top} = \begin{pmatrix} -210.369 & 56.137 \\ -210.369 & 56.137 \\ -168.738 & 45.028 \\ -7.406 \times 10^{-14} & 1.976 \times 10^{-14} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} psi$$

<u>Check for beam stresses at release against allowable stresses</u>

Beam stresses at release top bottom

$$f_{c.release} := f_{ps} + f_{sw} + f_{p\_top} = \begin{pmatrix} 427.481 & -2173.247 \\ 232.212 & -2269.343 \\ 278.436 & -2347.505 \\ 387.969 & -2343.902 \\ 430.179 & -2804.936 \\ 514.094 & -3193.636 \end{pmatrix} \cdot psi \qquad x_{release} = \begin{pmatrix} 2.49 \\ 10.00 \\ 10.49 \\ 12.49 \\ 18.49 \\ 24.49 \end{pmatrix} ft$$

$f_{ti.release} := \max\left(f_{c.release}\right) = 514\,psi$ Maximum tensile stress at release

$f_{ci.release} := \min\left(f_{c.release}\right) = -3194\,psi$ Maximum compressive stress at release

$\text{if}\left(f_{ti} \geq f_{ti.release}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable tension check $f_{ti} = 607\,psi$

$\text{if}\left(-f_{ci} \geq -f_{ci.release}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable compression check $f_{ci} = -4160\,psi$

Camber immediately after transfer

Camber due to prestressing assuming constant maximum force (not including debonding effect)

$$\frac{-\min\left(M_{ps}\right) \cdot L_{beam}^{2}}{8 \cdot E_{c.beam\_i} \cdot I_{beam}} = 1.825 \cdot in$$

Deflection due to top prestressing assuming constant maximum force (including debonding transfer length)

$$\delta_{p\_top} := \frac{M_{p\_top_0} \cdot x_{p\_top}^{2}}{2 \cdot \left(E_{c.beam\_i} \cdot I_{beam}\right)} = 8.985 \times 10^{-3} \cdot in$$

Deflection due to selfweight of the beam

$$\frac{-5 \cdot \omega_{beam} \cdot L_{beam}^{4}}{384 \cdot E_{c.beam\_i} \cdot I_{beam}} = -0.416 \cdot in$$

Considering the reduced camber due to the effect of debonding

$$d_{strand.db} := \left| \begin{array}{l} \text{for } i \in 0..\text{length}(row_{db})-1 \\ \quad d_{s_i} \leftarrow d-(2in)row_{db_i} \\ d_s \end{array} \right. = \begin{pmatrix} 43.00 \\ 43.00 \\ 41.00 \\ 41.00 \\ 39.00 \\ 39.00 \end{pmatrix} \cdot in$$

$$\delta_{db} := \frac{\overrightarrow{\left[N_{db}\cdot A_{strand}\cdot F_{pt}\cdot\left(d_{strand.db}-y_t\right)\cdot\left(L_{db}+L_t\right)^2\right]}}{2\cdot E_{c.beam\_i}\cdot I_{beam}} = \begin{pmatrix} 0.131 \\ 0 \\ 0.088 \\ 0 \\ 0.019 \\ 0 \end{pmatrix} \cdot in$$

$$\sum \delta_{db} = 0.238\cdot in$$

$$Camber_{tr} := \frac{-\min\left(M_{ps}\right)\cdot L_{beam}{}^2}{8\cdot E_{c.beam\_i}\cdot I_{beam}} - \frac{5\cdot \omega_{beam}\cdot L_{beam}{}^4}{384\cdot E_{c.beam\_i}\cdot I_{beam}} - \sum \delta_{db} - \delta_{p\_top} = 1.162\cdot in$$

Positive sign indicates camber upwards. Negative sign indeicates deflection

Check the stresses of the beam during shipping and handling, where the supports are not at the ends of the beam (Find the exact location of the supports during shipping and handling)

Moment due to self weight of beam at check locations

$$M_{sw.ship}(x) := \left| \begin{array}{ll} \dfrac{-\omega_{beam}\cdot x^2}{2} & \text{if } 0\cdot in \le x \le l_{ship} \\ \dfrac{\omega_{beam}\cdot L_{beam}\cdot\left(x-l_{ship}\right)}{2} - \dfrac{\left(\omega_{beam}\cdot x^2\right)}{2} & \text{if } l_{ship} \le x \le \dfrac{L_{beam}}{2} \end{array} \right.$$

Top and bottom concrete stresses at check locations due to beam self weight ONIY

```
$f_{sw.ship} :=$ for $i \in 0 .. \text{length}(x_{release}) - 1$
    $M \leftarrow M_{sw.ship}(x_{release_i})$
    $f_{i,0} \leftarrow \frac{-M}{S_T}$
    $f_{i,1} \leftarrow \frac{M}{S_B}$
$f$
```

$$f_{sw.ship} = \begin{array}{cc} \text{top} & \text{bottom} \\ \left(\begin{array}{cc} -18 & 15 \\ -295 & 242 \\ -310 & 255 \\ -369 & 303 \\ -514 & 422 \\ -609 & 500 \end{array}\right) \end{array} \cdot \text{psi}$$

<u>Check for beam stresses during handling & shipping against allowable stresses</u>

Beam stresses during shipping @ handling

$$f_{c.ship} := f_{ps} + f_{sw.ship} + f_{p\_top} = \begin{array}{cc} \text{top} & \text{bottom} \\ \left(\begin{array}{cc} 518.387 & -2247.917 \\ 323.118 & -2344.013 \\ 369.341 & -2422.175 \\ 478.875 & -2418.572 \\ 521.084 & -2879.606 \\ 604.999 & -3268.306 \end{array}\right) \end{array} \cdot \text{psi} \qquad x_{release} = \left(\begin{array}{c} 2.49 \\ 10.00 \\ 10.49 \\ 12.49 \\ 18.49 \\ 24.49 \end{array}\right) \text{ft}$$

$f_{ti.ship} := \max(f_{c.ship}) = 605\,\text{psi}$ Maximum tensile stress at release

$f_{ci.ship} := \min(f_{c.ship}) = -3268\,\text{psi}$ Allowable tension check

$\text{if}(f_{ti} \geq f_{ti.ship}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$ Allowable compression check $f_{ti} = 607\,\text{psi}$

$\text{if}(-f_{ci} \geq -f_{ci.ship}, \text{"ok"}, \text{"not ok"}) = \text{"ok"}$ Maximum compressive stress at release $f_{ci} = -4160\,\text{psi}$

<u>Service I limit State - Check for compressive stresses at top of deck at service conditions due to permanent loads only</u>

Compressive stress at top of deck due to loads on composite section

$$f_{cf\_actual\_mid} := \frac{-(M_{DC2} + M_{DW})}{S_{t3n} \cdot k_{sdl} \cdot n} = -62\,\text{psi}$$

$$\text{if}(-f_{cf.deckp} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"no good"}) = \text{"ok"}$$

Service I limit State - Check for compressive stresses at top flange of beam at service conditions due to prestress and permament loads only

Compressive stress at top flange of beam due to prestressing and permanent loads

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} = -1125\,psi$$

$\text{if}\left(-f_{cfp} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Service I limit State - Check for compressive stresses at top of deck at service conditions

Compressive stress at top of deck due to loads on composite section **including wind effect according to AASHTO LRFD 2016 Interim revision**

$$f_{cf\_actual\_mid} := \frac{-\left(M_{DC2} + M_{DW}\right)}{S_{t3n} \cdot k_{sdl} \cdot n} - \frac{1.0M_{LLI}}{S_{tn} \cdot k \cdot n} - \frac{1.0M_{WS}}{S_{tn} \cdot k \cdot n} = -640\,psi$$

$\text{if}\left(-f_{cf.deck} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$ Allowable stress check

Service I limit State - Check for compressive stresses at top flange of beam at service conditions

Compressive stress at top flange of beam due to prestressing and all loads................

$$f_{cf\_actual\_mid} := \frac{-P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_T} - \frac{M_{DC1}}{S_T} - \frac{M_{DC2} + M_{DW}}{S_{t.bm.3n}} - \frac{M_{LLI}}{S_{t.bm.n}} - \frac{1.0 \cdot M_{WS}}{S_{t.bm.n}} = -1464\,psi$$

$\text{if}\left(-f_{cf} > -f_{cf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Service III limit State - Check for tensile stresses at bottom flange of beam at service conditions

Tensile stress at bottom flange of beam due to prestressing and all loads

$$f_{tf\_actual\_mid} := \frac{-P_e}{A_{beam}} - \frac{P_e \cdot e_s}{S_B} + \frac{M_{DC1}}{S_B} + \frac{M_{DC2} + M_{DW}}{S_{b3n}} + \frac{0.8M_{LLI}}{S_{bn}} = -45 \cdot psi$$

$\text{if}\left(f_{tf} > f_{tf\_actual\_mid}, \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$ Allowable stress check

Calculate bar area required to resist tension in the top flange at release, AASHTO Table 5.9.4.1.2-1:

$f_{ti.ship} = 604.999\,psi$

Maximum top flange tensile stress at release or handling, whichever is larger (usually, handling stresses are larger)

$f_c := vlookup\left(f_{ti.ship}, f_{c.ship}, 1\right)_0 = -3.268 \times 10^3\,psi$

Bottom flange compressive stress corresponding to the maximum top flange tensile stress at release/shipping

$$slope_m := \frac{f_{ti.ship} - f_c}{d} = 86.073 \cdot \frac{psi}{in}$$

Slope of the section stress over the depth of the beam

$$x_o := \frac{f_{ti.ship}}{slope_m} = 7.029 \cdot in$$

Distance measured from the top of the beam to the point of zero stress

Calculate the width of the beam where the tensile stresses are acting

```
$b_{ten} :=$ for $i \in 0 .. ceil\left(\frac{x_o}{in}\right)$
        $x_i \leftarrow \frac{x_o \cdot i}{ceil\left(\frac{x_o}{in}\right)}$
        $b_i \leftarrow b_{ft}$  if $0 \le x_i \le d_{ft}$
        $b_i \leftarrow \left[b_{ft} - \frac{x_i - d_{ft}}{d_h} \cdot \left(b_{ft} - b_v\right)\right]$  if $d_{ft} < x_i \le d_{ft} + d_h$
        $b_i \leftarrow b_v$  if $d_{ft} + d_h < x_i$
    $b$
```

Calculate the tensile stress values every inch of depth starting from the top surface of the beam

$$f := \left| \begin{array}{l} \text{for } i \in 0 .. \operatorname{ceil}\left(\frac{x_o}{in}\right) \\ \quad \left| \begin{array}{l} x_i \leftarrow \dfrac{x_o \cdot i}{\operatorname{ceil}\left(\frac{x_o}{in}\right)} \\ f_i \leftarrow f_{ti.ship} - slope_m \cdot x_i \end{array} \right. \\ f \end{array} \right.$$

$$f = \begin{pmatrix} 604.999 \\ 529.374 \\ 453.75 \\ 378.125 \\ 302.5 \\ 226.875 \\ 151.25 \\ 75.625 \\ 0 \end{pmatrix} psi \qquad b_{ten} = \begin{pmatrix} 16 \\ 16 \\ 16 \\ 16 \\ 16 \\ 16 \\ 16 \\ 16 \\ 15.942 \end{pmatrix} \cdot in$$

Calculate the tensile force that shall be resisted by top reinforcement

$$T := \sum_{i=0}^{length(f)-2} \left[\frac{1}{4} \cdot \left(f_i + f_{i+1}\right) \cdot \left(b_{ten_i} + b_{ten_{i+1}}\right) \cdot \frac{x_o}{\operatorname{ceil}\left(\frac{x_o}{in}\right)} \right] = 34.019 \cdot kip$$

$$A_{s.top} := \frac{T}{30 \cdot ksi} = 1.134 \cdot in^2$$

Calculate area of tensile reinforcement required in the top of the beam. The stress in bars is limited to 30ksi per AASHTO 5.9.4.1.2. See Figure C.5.9.4.1.2-1 which is based upon .5 f.y of steel rebar

$$A_{bar.top} := 0.44 \cdot in^2$$

Cross sectional area of No. 6 steel rebars

$$n_{bar.release} := \operatorname{Ceil}\left(\frac{A_{s.top}}{A_{bar.top}}, 1\right) = 3$$

number of No. 6 bars provided in the top flange to resist tension at release in the beam ends.

Calculation of minimum length of top tensile reinforcement

AASHTO LRFD Table 5.9.4.1.2-1 specifies a maximum concrete tensile stress of

$0.0948 \cdot \sqrt{f_{ci\_beam}} \le 0.2\ \text{ksi}$ for tensile zones without bonded reinforcement

$$f_{t.max} := \min\left(0.0948 \cdot \sqrt{\frac{f_{ci\_beam}}{\text{ksi}}}, 0.2\right) \cdot \text{ksi} = 0.2 \cdot \text{ksi}$$

Calculate the minimum required length of top reinforcement based on the stress calculated at distanc
x.release during release or shipping and handling, whichever is greater. If all the stresses are larger th
f.t.max, estimate the stress after the last point of debonding

```
L_topr :=  | $h \leftarrow x_{release}$
           | $f \leftarrow f_{c.ship}^{\langle 0 \rangle}$
           | $i \leftarrow \text{length}(f) - 1$
           | while $f_i < f_{t.max}$
           |     | break if $i = 0$
           |     | $i \leftarrow i - 1$
           | $x \leftarrow 1 \cdot ft$
           | $f_{ps} \leftarrow f_{ps_{\text{rows}(f_{ps})-1,0}}$
           | $S(x) \leftarrow f_{ps} - f_{t.max} - \dfrac{\dfrac{\omega_{beam} \cdot L_{beam} \cdot (x - l_{ship})}{2} - \dfrac{(\omega_{beam} \cdot x^2)}{2}}{S_T}$
           | $g \leftarrow \text{root}(S(x), x)$
           | $g$  if $f_{\text{length}(f)-1} > f_{t.max}$
           | $\dfrac{L_{beam}}{2}$  if $\text{Im}(g) \ne 0 \wedge f_{\text{length}(f)-1} > f_{t.max}$
           | $h_{i+1}$  otherwise
```

$L_{topr} = 32.958\,\text{ft}$

$$l_d := 1.4 \cdot \frac{1.25 \cdot \dfrac{\pi \cdot \left(\dfrac{6}{8}\right)^2}{4} \cdot 60}{\sqrt{\dfrac{f_{c\_beam}}{\text{ksi}}}} \cdot \text{in} = 1.367 \cdot \text{ft}$$

Calculate the tension development length required for the tensile reinforcement in the top of the beam. As provided AASHTO 5.11.2.1.1 taking into account 1.4 modification factor per AASHTO 5.11.2.1.2

$L_{topR} := L_{topr} + l_d = 34.325\,\text{ft}$

Minimum length required for the top reinforcement from each end, **if larger than half the length of the beam, then the top reinforcement shall continue through the enitre beam length from end to end.**

Flexural Capacity

Stress block factor, **AASHTO 5.7.2.2.** Assuming depth of neutral axis lies within the deck

$$\beta_1 := \begin{cases} 0.65 & \text{if } f_{c\_deck} \geq 8000psi \\ 0.85 & \text{if } f_{c\_deck} \leq 4000psi \\ \left[0.85 - \left(\dfrac{f_{c\_deck} - 4000psi}{1000psi}\right)0.05\right] & \text{otherwise} \end{cases} = 0.8$$

$\varepsilon_{cu} := 0.003$ Maximum usable concrete compressive strain

$\varepsilon_{pu} := \dfrac{f_{pu}}{E_p} = 0.0145$ Ultimate tensile strain of CFCC strand

$\varepsilon_{pe} := \dfrac{f_{pe}}{E_p} = 0.0072$ Effective CFCC prestressing strain

$\varepsilon 0 := \varepsilon_{pu} - \varepsilon_{pe} = 0.0073$ Reserve strain in CFCC

$$d_i := d_{strand} + haunch + deck_{thick} = \begin{pmatrix} 52.00 \\ 50.00 \\ 48.00 \end{pmatrix} \cdot in$$

Depth of prestressing strands
from top of concrete deck

$$A_f := A_{strand} \cdot Row = \begin{pmatrix} 1.43 \\ 1.79 \\ 1.43 \end{pmatrix} \cdot in^2$$

Area of strands in rows

$$P_{row} := A_f \cdot f_{pe} = \begin{pmatrix} 216.73 \\ 270.91 \\ 216.73 \end{pmatrix} \cdot kip$$

Effective prestressing force of strands in rows

$$s_i := \left| \begin{array}{l} \text{for } i \in 0..\,\text{length}(Row) - 1 \\ \quad s_i \leftarrow d_{i_0} - d_{i_i} \\ s \end{array} \right. = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} \cdot in$$

Distance from each layer of prestressing strands to the bottom prestressting layer

$$deck_{eff} := deck_{thick} - t_{wear} = 9 \cdot in$$

Effective deck thickness (total thickness minus assumed sacrificial wearing surface thickness)

Balanced reinforcement ratio

$$c_{bal} := \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_0} \cdot d_{i_0} = 15.107 \cdot in$$

Depth of neutral axis at balanced failure

Balanced reinforcement ratio assuming **Rectangular section**

$$\rho_{R\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{eff} \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} = 0.0055$$

Balanced reinforcement ratio assuming **Flanged section**

$$\rho_{Fl\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot deck_{eff} \cdot \left(b_{eff} - b_{ft}\right) + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{ft} \cdot c_{bal} - P_e}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}} = 0.0041$$

Balanced reinforcement ratio assuming **Double Flanged section**

$$\rho_{DFl\_bal} := \frac{0.85 \cdot f_{c\_deck} \cdot deck_{eff} \cdot \left(b_{eff} - b_{web}\right) + 0.85 \cdot f_{c\_deck} \cdot d_{ft} \cdot \left(b_{ft} - b_{web}\right) + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{web} \cdot c_{b}}{E_p \cdot \varepsilon_0 \cdot b_{eff} \cdot d_{i_0}}$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Tension contorlled** section

```
$Fl\_T :=$
    $c \leftarrow 1.0\cdot in$
    $A_{eq\_s} \leftarrow 1.0\cdot in^2$
    $A_{eq\_f} \leftarrow 2.0\cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
        $c \leftarrow \frac{E_p\cdot\varepsilon_0\cdot A_{eq\_s} + P_e - 0.85\cdot f_{c\_deck}\cdot(b_{eff} - b_{ft})\cdot deck_{eff}}{0.85\cdot f_{c\_deck}\cdot\beta_1\cdot b_{ft}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_T = \begin{pmatrix} -30.4585 \\ 0.0009 \end{pmatrix}$$

$$c_{Fl\_T} := Fl\_T_0\cdot in = -30.459\cdot in$$

$$\rho_{Fl\_T} := Fl\_T_1 = 0.0009$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Tension contorlled** section

```
$R\_T :=$
    $c \leftarrow 1.0 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        $c \leftarrow \frac{E_p \cdot \varepsilon_0 \cdot A_{eq\_s} + P_e}{0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot b_{eff}}$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$(R\_T) = \begin{pmatrix} 4.259743 \\ 0.000893 \end{pmatrix}$$

$$c_{R\_T} := R\_T_0 \cdot in = 4.26 \cdot in$$

$$\rho_{R\_T} := R\_T_1 = 0.0009$$

Depth of the N.A. and reinforcement ratio assuming **Double-Flanged Tension contorlled** section. The depth of the stress block is deeper than the depth of the deck and the top flange together.

```
DFl_T := | $c \leftarrow 1.0\cdot in$
         | $A_{eq\_s} \leftarrow 1.0\cdot in^2$
         | $A_{eq\_f} \leftarrow 2.0\cdot in^2$
         | $N \leftarrow \text{length}(d_i) - 1$
         | while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^2$
         |     $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
         |     $c \leftarrow \frac{E_p\cdot\varepsilon_0\cdot A_{eq\_s} + P_e - 0.85\cdot f_{c\_deck}\cdot\left(b_{eff} - b_{web}\right)\cdot deck_{eff} - 0.85\cdot f_{c\_deck}\cdot\left(b_{ft} - b_{web}\right)\cdot d_{ft}}{0.85\cdot f_{c\_deck}\cdot\beta_1\cdot b_{web}}$
         |     $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
         | $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
         | $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$DFl\_T = \begin{pmatrix} -95.0130 \\ 0.0009 \end{pmatrix}$$

$$c_{DFl\_T} := DFl\_T_0\cdot in = -95.013\cdot in$$

$$\rho_{DFl\_T} := DFl\_T_1 = 0.0009$$

Depth of the N.A. and reinforcement ratio assuming **Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu} \cdot \left(\frac{d_{i_0} - c}{c} \right)$$

```
$Fl\_C :=$
    $c \leftarrow 1 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $|A_{eq\_s} - A_{eq\_f}| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
        $f(c) \leftarrow 0.85 \cdot f_{c\_deck} \cdot (b_{eff} - b_{ft}) \cdot deck_{eff} + 0.85 \cdot f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{ft} \ldots$
            $+ (-E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e)$
        $c \leftarrow root(f(c), c, 0.1 \cdot in, d_{i_0})$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[ \left( 1 - \frac{s_{i_i}}{d_{i_0} - c} \right) \cdot A_{f_i} \right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$Fl\_C = \begin{pmatrix} 5.017458 \\ 0.000893 \end{pmatrix}$$

$$c_{Fl\_C} := Fl\_C_0 \cdot in = 5.017 \cdot in$$

$$\rho_{Fl\_C} := Fl\_C_1 = 0.0009$$

Depth of the N.A. and reinforcement ratio assuming **Rectangular Compression contorlled** section

```
$R\_C :=$
    $c \leftarrow 1 \cdot in$
    $A_{eq\_s} \leftarrow 1.0 \cdot in^2$
    $A_{eq\_f} \leftarrow 2.0 \cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01 \cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
        $g(c) \leftarrow 0.85 f_{c\_deck} \cdot \beta_1 \cdot c \cdot b_{eff} - E_p \cdot \varepsilon_0(c) \cdot A_{eq\_s} - P_e$
        $c \leftarrow root\left(g(c), c, 0.1 \cdot in, d_{i_0}\right)$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N} \left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right) \cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff} \cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$R\_C = \begin{pmatrix} 7.3612 \\ 0.0009 \end{pmatrix}$$

$$c_{R\_C} := R\_C_0 \cdot in = 7.361 \cdot in$$

$$\rho_{R\_C} := R\_C_1 = 0.0009$$

Depth of the N.A. and reinforcement ratio assuming **Double Flanged Compression contorlled** section

$$\varepsilon_0(c) := \varepsilon_{cu}\cdot\left(\frac{d_{i_0} - c}{c}\right)$$

```
$DFl\_C :=$
    $c \leftarrow 1\cdot in$
    $A_{eq\_s} \leftarrow 1.0\cdot in^2$
    $A_{eq\_f} \leftarrow 2.0\cdot in^2$
    $N \leftarrow length(d_i) - 1$
    while $\left|A_{eq\_s} - A_{eq\_f}\right| > 0.01\cdot in^2$
        $A_{eq\_s} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
        $f(c) \leftarrow 0.85\cdot f_{c\_deck}\cdot(b_{eff} - b_{web})\cdot deck_{eff} + 0.85\cdot f_{c\_deck}\cdot(b_{ft} - b_{web})\cdot d_{ft} \ldots$
            $+ 0.85\cdot f_{c\_deck}\cdot\beta_1\cdot c\cdot b_{web} - E_p\cdot\varepsilon_0(c)\cdot A_{eq\_s} - P_e$
        $c \leftarrow root\left(f(c), c, 0.1\cdot in, d_{i_0}\right)$
        $A_{eq\_f} \leftarrow \sum_{i=0}^{N}\left[\left(1 - \frac{s_{i_i}}{d_{i_0} - c}\right)\cdot A_{f_i}\right]$
    $\rho \leftarrow \frac{A_{eq\_f}}{b_{eff}\cdot d_{i_0}}$
    $\begin{pmatrix} \frac{c}{in} \\ \rho \end{pmatrix}$
```

$$DFl\_C = \begin{pmatrix} 4.357204 \\ 0.000893 \end{pmatrix}$$

$$c_{DFl\_C} := DFl\_C_0\cdot in = 4.357\cdot in$$

$$\rho_{DFl\_C} := DFl\_C_1 = 0.0009$$

Check the mode of failure

$$\text{Section\_Mode} := \left| \begin{array}{l} \text{"Rectangular\_Tension"} \ \text{ if } \ \beta_1 \cdot c_{R\_T} \le deck_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ \text{"Rectangular\_Compression"} \ \text{ if } \ \beta_1 \cdot c_{R\_C} \le deck_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ \text{"Flanged\_Tension"} \ \text{ if } \ \beta_1 \cdot c_{Fl\_T} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_T} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_T} < \rho_{Fl\_b} \\ \text{"Flanged\_Compression"} \ \text{ if } \ \beta_1 \cdot c_{Fl\_C} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_C} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_C} > \\ \text{"Double\_Flanged\_Tension"} \ \text{ if } \ \beta_1 \cdot c_{DFl\_T} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_T} < \rho_{DFl\_bal} \\ \text{"Double\_Flanged\_Compression"} \ \text{ if } \ \beta_1 \cdot c_{DFl\_C} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_C} > \rho_{DFl\_bal} \end{array} \right.$$

$$(\text{Section\_Mode}) = \text{"Rectangular\_Tension"}$$

Select the correct depth of the N.A.

$$c := \left| \begin{array}{l} c_{R\_T} \ \text{ if } \ \beta_1 \cdot c_{R\_T} \le deck_{eff} \wedge \rho_{R\_T} < \rho_{R\_bal} \\ c_{R\_C} \ \text{ if } \ \beta_1 \cdot c_{R\_C} \le deck_{eff} \wedge \rho_{R\_C} > \rho_{R\_bal} \\ c_{Fl\_T} \ \text{ if } \ \beta_1 \cdot c_{Fl\_T} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_T} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_T} < \rho_{Fl\_bal} \\ c_{Fl\_C} \ \text{ if } \ \beta_1 \cdot c_{Fl\_C} > deck_{eff} \wedge \beta_1 \cdot c_{Fl\_C} \le deck_{eff} + d_{ft} \wedge \rho_{Fl\_C} > \rho_{Fl\_bal} \\ c_{DFl\_T} \ \text{ if } \ \beta_1 \cdot c_{DFl\_T} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_T} < \rho_{DFl\_bal} \\ c_{DFl\_C} \ \text{ if } \ \beta_1 \cdot c_{DFl\_C} > deck_{eff} + d_{ft} \wedge \rho_{DFl\_C} > \rho_{DFl\_bal} \end{array} \right.$$

$$c = 4.26 \cdot in$$

Disclaimer: The design of the section as a dobule flanged section, while theoretically possible, indicates that the depth of the N.A. is in the web of the beam. That could lead to an over-reinforced section that has little or no ductility. Designer is advised to avoid designing the section as a dobule flanged section if possible to ensure proper ducitliy and significant cracking.deflection before failure

Calculate the strain in the extreme CFRP based on the mode of failure

$$\varepsilon_0 := \left| \begin{array}{l} \varepsilon_{pu} - \varepsilon_{pe} \ \text{ if } \ \text{Section\_Mode} = \text{"Rectangular\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} \ \text{ if } \ \text{Section\_Mode} = \text{"Flanged\_Tension"} \\ \varepsilon_{pu} - \varepsilon_{pe} \ \text{ if } \ \text{Section\_Mode} = \text{"Double\_Flanged\_Tension"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \ \text{ if } \ \text{Section\_Mode} = \text{"Rectangular\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \ \text{ if } \ \text{Section\_Mode} = \text{"Flanged\_Compression"} \\ \varepsilon_{cu} \cdot \dfrac{d_{i_0} - c}{c} \ \text{ if } \ \text{Section\_Mode} = \text{"Double\_Flanged\_Compression"} \end{array} \right. = 7.326 \times 10^{-3}$$

$$\varepsilon := \left| \begin{array}{l} \text{for } \ i \in 0 .. \text{length(Row)} - 1 \\ \quad \varepsilon_i \leftarrow \varepsilon_0 \cdot \left(\dfrac{d_{i_i} - c}{d_{i_0} - c} \right) \\ \varepsilon \end{array} \right. = \begin{pmatrix} 0.0073 \\ 0.0070 \\ 0.0067 \end{pmatrix}$$

strain in ith layer of prestressing strands

$$\varepsilon_c := \varepsilon_0 \cdot \left(\frac{c}{d_{i_0} - c} \right) = 0.00065$$

strain in the concrete top of the deck

Strength limit state Flexural Resistance:

$$M_n := \left| \begin{array}{l} E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ldots \qquad \text{if } \ deck_{eff} < \beta_1 \cdot c \le deck_{eff} + d_{ft} \\ + 0.85 f_{c\_deck} \cdot \left(b_{eff} - b_{ft} \right) \cdot deck_{eff} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - \dfrac{deck_{eff}}{2} \right) \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \ldots \qquad \text{if } \ \beta_1 \cdot c > deck_{eff} + d_{ft} \\ + 0.85 f_{c\_deck} \cdot \left(b_{eff} - b_{web} \right) \cdot deck_{eff} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - \dfrac{deck_{eff}}{2} \right) \ldots \\ + 0.85 f_{c\_deck} \cdot \left(b_{ft} - b_{web} \right) \cdot d_{ft} \cdot \left(\dfrac{\beta_1 \cdot c}{2} - deck_{eff} - \dfrac{d_{ft}}{2} \right) \\ E_p \cdot \overrightarrow{(\varepsilon \cdot A_f)} \cdot \left(d_i - \dfrac{\beta_1 \cdot c}{2} \right) + P_e \cdot \left(d_f - \dfrac{\beta_1 \cdot c}{2} \right) \quad \text{if } \ \beta_1 \cdot c \le deck_{eff} \end{array} \right.$$

$$M_n = 5598.92 \cdot kip \cdot ft$$

Nominal moment capacity

$$\phi := \begin{vmatrix} 0.85 \ \text{ if } \ \varepsilon_0 \geq 0.005 \\ 0.5167 + 66.67 \cdot \varepsilon_0 \ \text{ if } \ 0.002 \leq \varepsilon_0 \leq 0.005 \\ 0.65 \ \text{ if } \ \varepsilon_0 \leq 0.002 \end{vmatrix} = 0.85$$

$M_r := \phi \cdot M_n = 4759.08 \cdot \text{kip} \cdot \text{ft}$

$M_{u\_strength} = 3667.60 \cdot \text{kip} \cdot \text{ft}$

$\text{if}\left(M_r > M_{u\_strength}, \text{"ok"}, \text{"no good"}\right) = \text{"ok"}$

$$\frac{M_r}{M_{u\_strength}} = 1.30$$

Minimum reinforcement against cracking moment

$f_r := 0.24 \cdot \sqrt{f_{c\_beam} \cdot \text{ksi}} = 678.823 \, \text{psi}$ Modulus of rupture of beam concrete, **AASHTO A 5.4.2.6**

$\gamma_1 := 1.6$ Flexural variability factor

$\gamma_2 := 1.1$ Prestress viariability factor

$\gamma_3 := 1.0$ Reinforcement strength ratio

$$f_{cpe} := \frac{P_e}{A_{beam}} + \frac{P_e \cdot e_s}{S_B} = 3120.97 \, \text{psi}$$

Compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$$M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r + \gamma_2 \cdot f_{cpe}\right) \cdot S_{bn} - M_{DC1} \cdot \left(\frac{S_{bn}}{S_B} - 1\right)\right] = 3427.32 \cdot \text{kip} \cdot \text{ft}$$

Cracking moment

$\text{if}\left(M_r > \min\left(M_{cr}, 1.33 \cdot M_{u\_strength}\right), \text{"ok"}, \text{"not ok"}\right) = \text{"ok"}$

Approximate mid-span deflection at failure

The deflection calculations follows the approach outlined in the paper " Flexural behaviour of CFRP precast Decked Bulb T beams " by Grace et al. in May/June 2012, Journal of Composites for Construction. In order to calculate the deflection at failure, the moment capacity of the composite section is used as the bending moment. The stress level in the bottom most row is used to calculate the flexural rigidity. The deflection calculated below is approximate, but will give an indication of the deformbility and the level of warning exhibited near failure of the beam.

$d_{i_0} = 52.00 \cdot \text{in}$ Depth of the bottom row of strands to the extreme compression fiber

$c = 4.26 \cdot \text{in}$ Depth of the neutral axis to the extreme compression fiber

$y_s := d_{i_0} - c = 47.74 \cdot in$

Distance from neutral axis to the bottom row of strands

$$EI := \frac{M_n \cdot y_s}{\varepsilon_0} = 437820996.50 \cdot kip \cdot in^2$$

Flexural rigidity of the beam/deck section based on the stress level in the bottom row of prestressing strands

$$\omega_f := 8 \cdot \frac{M_n}{L^2} = 10.602 \cdot \frac{kip}{ft}$$

Failure load (dead and live loads) uniformly dirstibuted over the entire span

$$\delta_f := \frac{5 \cdot \omega_f \cdot L^4}{384EI} = 9.725 \cdot in$$

Midspan deflection at strength limit state

ustments of C1 and θskew"

pocket

$$\frac{al - P_e}{} = 0.0043$$

al

Fl_bal