Some Mathematical Measures for Determining if a Redistricting Plan Disproportionally Advantages a Political Party

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A number of objective mathematical measures have been developed to determine if an existing or proposed redistricting map disadvantages one political party relative to the other. In my presentation, I focused on three such measures. The reasons for my choice are as follows:

1. Each of the measures discussed are easy to understand and straightforward to calculate. They produce scores that indicate both the direction and the magnitude of any political bias in the redistricting map.

2. Because I easily calculated the scores for each of these measures in excel, I know it is possible to incorporate a report function into redistricting software that will provide these scores. (My understanding is that these measures are currently being added as available reports in AutoBound's Edge redistricting software.)

3. Although these three measures have only recently been developed, they have all have been introduced and accepted by federal and state courts as useful tools for determining if a redistricting map is politically fair.[1]

The three measures discussed here are the lobsided margins test, the mean-median difference, and the efficiency gap. All three use historical district election results to evaluate redistricting plans, but all three can be used in conjunction with reconfigured election results to evaluate proposed redistricting plans.[2]

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[1] These measures were introduced into court in the context of partisan gerrymandering challenges. While the adoption of an independent citizens redistricting commission such as the MICRC addresses the problem of intentional discrimination by removing the task from the hands of partisan politicians that may draw districts to disproportionally favor their own party, it is still possible for redistricting plan drawn by a nonpartisan or bipartisan commission to favor one political party over the other. The difference is that the bias is unintentional. And this is presumably why the Michigan State Constitution obliges the MICRC to use accepted measures of partisan fairness to ensure that the redistricting map adopted does not provide a disproportionate advantage to any political party.

[2] Both the efficiency gap and the mean-median difference have been used to evaluate computer simulated alternative redistricting maps for comparative purposes in partisan gerrymandering challenges. Election results for select statewide elections were reconfigured to determine how the candidates in these elections would have fared in the alternative districts.

Lobsided Margins Test

In a perfectly fair plan – at least in a state in which the two political parties are competitive (closely divided) – we would expect a mix of districts, some strongly partisan districts, some moderately reliable districts, and some tossups – but each party would have a roughly similar mix. If one party has a smaller number of victories with larger margins of victory that the other party, this is an indication that one party is being disfavored over the other in the map. This pattern of outcomes can be quantified by sorting the districts into two groups, by winning party. Each party's winning vote share can then be compared to see if one party has significantly higher margin of victories than the other.[3]

This measure was first discussed in Sam Wang, "Three Tests for Practical Evaluation of Partisan Gerrymandering," *Stanford Law Journal*, 16, June 2016. Available at: https://www.stanfordlawreview.org/print/article/three-tests-for-practical-evaluation-of-partisan-gerrymandering/)

Example:

				Percent of Votes			Party Wins	
District	**Party A**	**Party B**	**Total Votes**	**Party A**	**Party B**		**Party A**	**Party B**
1	279	120	399	69.9%	30.1%		69.9%	
2	172	198	370	46.5%	53.5%			53.5%
3	167	192	359	46.5%	53.5%			53.5%
4	148	212	360	41.1%	58.9%			58.9%
5	185	180	365	50.7%	49.3%		50.7%	
6	139	193	332	41.9%	58.1%			58.1%
7	169	201	370	45.7%	54.3%			54.3%
8	179	206	385	46.5%	53.5%			53.5%
9	234	99	333	70.3%	29.7%		70.3%	
10	178	199	377	47.2%	52.8%			52.8%
TOTAL	**1850**	**1800**	**3650**	**50.7%**	**49.3%**		**63.6%**	**54.9%**

Party A in the example is winning districts with a much higher average vote (63.6%) than Party B (54.9%) – and the difference between the two percentages is 8.7 (63.6 – 54.9). This indicates

[3] A t-test can be used to compare the two averages to determine if the difference is statistically significant but this is most relevant in the context of a partisan gerrymander challenge where the intent of the redistricters is an issue. I am not sure how relevant this is in the context of an independent citizens commission.

that Party A supporters are packed into a few districts that it wins by large margins. Party B, on the other hand, is winning substantially more districts with substantially lower vote margins.

Mean-Median Difference

This approach to ascertaining political bias in redistricting maps was proposed by Michael D. McDonald and Robin Best in "Unfair Partisan Gerrymanders in Politics and Law: A Diagnostic Applied to Six Cases," *Election Law Journal* 14(4), 2015 (available at: https://www.liebertpub.com/doi/abs/10.1089/elj.2015.0358). It was further quantified by Sam Wang in "Three Tests for Practical Evaluation of Partisan Gerrymandering."

The mean-median district vote share difference compares a party's mean district vote share to its median district vote share:

- Mean = average party vote share across all districts
- Median = party vote share in the median district when districts are sorted on share of party vote

The difference between the mean and median vote shares provides a measure of whether the redistricting map produces skewed election results.

Comparing a data set's mean and median is a common statistical analysis used to assess how skewed data set is – if the dataset is balanced, the mean will be very close in value to its median. As a dataset becomes more skewed and extreme values are added only on one side, the mean and median begin to diverge and looking at the difference between the two can be used determine the extent to which the data is skewed.

Example:

Party A	Percentages I
	41.1%
	41.9%
	45.7%
	46.5%
	46.5%
	46.5%
	47.2%
	50.7%
	69.9%
	70.3%
District median percentage	46.5%
Statewide mean percentage	50.7%
Mean-Median Difference	4.2%

In this example, Party A received 50.7% of the statewide vote. Party A's median vote share (46.5%) is 4.2% lower than its mean vote share of 50.7%. This indicates that Party A must win more districts than Party B to win half of the seats – the redistricting map in skewed in favor of Party B. In fact, Party A would have had to win 54.2% (50.0 + 4.2) of the statewide vote to win 50% of the seats.

Efficiency Gap

The efficiency gap, introduced by University of Chicago law professor Nick Stephanopoulos and Public Policy Institute of California research fellow Eric McGhee, looks at the number of "wasted votes" across districts. (Nicholas O. Stephanopoulos and Eric M. McGhee, "Partisan Gerrymandering and the Efficiency Gap," *University of Chicago Law Review*: Vol. 82 (2), 2015. Available at: https://chicagounbound.uchicago.edu/uclrev/vol82/iss2/4)

In any election, nearly 50 percent of votes are wasted: all votes cast for a losing candidate, and any votes cast for a winning candidate beyond the threshold needed to win (50 percent in a two candidate contest). In a hypothetical map with perfect partisan symmetry, both parties would waste the same number of votes. A large difference between the parties' wasted votes indicates one party is treated more favorably than the other by the redistricting map. This is because the plan packs and cracks one party's supporters more than the other party's supporters.

The efficiency gap is calculated by taking one party's total wasted votes in an election, subtracting the other party's total wasted votes, and dividing this by the total number of votes cast. It captures in a single number the extent to which district lines waste the two parties votes unequally.

$$\text{Efficiency Gap} = \frac{[\text{Party A wasted votes}] - [\text{Party B wasted votes}]}{\text{total number of votes cast statewide}}$$

Example:

				Lost Votes		minimum	Surplus Votes		Total Wasted Votes	
District	Party A	Party B	Total Votes	Party A	Party B	to win	Party A	Party B	Party A	Party B
1	279	120	399	0	120	200	79	0	79	120
2	172	198	370	172	0	185	0	13	172	13
3	167	192	359	167	0	180	0	12	167	12
4	148	212	360	148	0	180	0	32	148	32
5	185	180	365	0	180	183	2	0	2	180
6	139	193	332	139	0	166	0	27	139	27
7	169	201	370	169	0	185	0	16	169	16
8	179	206	385	179	0	193	0	13	179	13
9	234	99	333	0	99	167	67	0	67	99
10	178	199	377	178	0	189	0	10	178	10
TOTAL	1850	1800	3650	1152	399		148	123	1300	522

In this example, supporters of Party A cast 1152 votes for losing candidates and 148 surplus votes – votes beyond what was necessary to elect Party A candidates. Supporters of Party B, on the other hand cast only 399 of their votes for losing candidates and 522 surplus votes. Adding together these two sets of votes, Party A had a total of 1300 wasted votes; Party B had a total of only 522 votes. The efficiency gap is therefore calculated as 21.3% (1300-522/3650 = 778/3650 = .213). This efficiency gap in favor of Party B can be interpreted as the percentage of seats Party B won above what would be expected in a politically fair or neutral map.

Court Acceptance of these Measures

These three measures have all been developed within the last five or six years and therefore do not have a long history of consideration by the courts. However, they have all been introduced recently in the context of partisan gerrymandering challenges. While recognizing each of the measures have some disadvantages, the courts in each instance relied on these measures (in addition to other measures introduced) to find the plans before them were politically biased towards one of the political parties at the expense of the other.

- **Michigan** The three judge federal court decision, *League of Women Voters of Michigan v. Benson*, discusses the efficiency gap and mean-median difference (referred to as median-mean difference) at length as all three of plaintiffs' experts (Jowei Chen, Christopher Warshaw and Kenneth Mayer) relied on these, as well as additional measures, to argue the congressional and state legislative plans in Michigan were partisan gerrymanders. The Court found that these measures provided convincing evidence of political bias and the criticisms of these measures by defendants to be unpersuasive. The court held that the plans were unconstitutional gerrymanders.

- **Ohio** The three judge federal court decision, *Ohio A. Philip Randolph Institute v. Householder*, discusses two of the measures, the efficiency gap and mean-median difference, as presented by plaintiffs' expert Christopher Warshaw. The Court found defendant's experts criticisms of these methods unconvincing and held the Ohio congressional map to be an unconstitutional partisan gerrymander.

- **Pennsylvania** The Pennsylvania State Supreme Court in *League of Women Voters of Pennsylvania v. Commonwealth of Pennsylvania* held the Pennsylvania congressional districts to be in violation of the Pennsylvania Constitution.[4] It found the testimony of plaintiffs' experts, including Jowei Chen and Christopher Warshaw, persuasive. Jowei

[4] A federal court found against plaintiffs challenging the Pennsylvania congressional map as violative of the U.S. Constitution in a separate suit, *Agre v. Wolf*. Plaintiff's experts in that case do not appear to have utilized the measures discussed here.

Chen relied in part of the mean-median difference score and Christopher Warshaw relied in part on the efficiency gap to argue that the maps were politically biased.

- **Wisconsin** The efficiency gap was first introduced in court by two of plaintiffs' experts, Kenneth Mayer and Simon Jackman, in a challenge to the Wisconsin state assembly districts. The three judge federal court decision in *Whitford v. Gill* discussed the measure at length. The court found the criticisms leveled against the measure by defendant's experts unpersuasive, determined that the efficiency gap provided convincing evidence that the plan was politically biased, and held that the state assembly districts were unconstitutional.

- **North Carolina** The three judge federal court in *Common Cause v. Rucho* found the efficiency gap as utilized by Jowei Chen and Simon Jackman, and the mean-median difference measure used by Simon Jackman, to be persuasive evidence that the state's remedial congressional district plan adopted in 2016 was an unconstitutional partisan gerrymander. (These were two of several statistical measures introduced and relied on by the Court.)

  This North Carolina decision, along with the Maryland case, *Lamone v. Benisek*,[5] was later overturned by the U.S. Supreme Court on unrelated grounds, but grounds that served to moot all of the federal decisions discussed above. However, in a separate challenge before the North Carolina Superior Court, *Common Cause v. Lewis*, the court held that the state legislative districts violated the North Carolina State Constitution. Jowei Chen was one of the plaintiffs' experts. Another of the plaintiffs' experts, Christopher Cooper, introduced the lopsided margins test. The court found the evidence offered by these and other plaintiffs' experts convincing and held the state legislative district maps to be violative of the state constitution.

Conclusion

I have discussed only three measures of political fairness—many more have been developed and several have been accepted by the courts. As I noted above, I focused only on these three measures because they seemed to me to be the easiest to understand and the simplest to calculate. I would use all three of the measures in evaluating the partisan fairness of draft redistricting maps because, while they are all related, they measure different aspects of political fairness.

[5] The federal court in the Maryland case does not appear to have considered measures of partisan fairness in rendering its decision.